DERIVATIVES

Definition. Let I be an interval and let $f : I \to \mathbb{R}$ and $c \in I$. The **derivative** of f at c is $\boxed{\lim_{x\to c} \frac{f(x)-f(c)}{x-c}}$. If the limit converges, then we say f is differentiable at c. If f is differentiable at every $c \in I$, then we say that f is differentiable. The quantity $\frac{f(x)-f(c)}{x-c}$ is called the difference quotient.

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Prove that f is differentiable (let $c \in \mathbb{R}$ and prove that $\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$ converges).

2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = |x|$. Prove that f is not differentiable at 0.

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3. Prove the product rule: if f and g are both differentiable at c, then $(fg)'(c) = g(c)f'(c) + f(c)g'(c)$ (and thus fg is differentiable at c).

Definition. Let $f : S \to \mathbb{R}$. The function f has a **relative maximum** at $c \in S$ if there is $\delta > 0$ such that for all $x \in S \cap (c - \delta, c + \delta)$, $f(x) \leq f(c)$. Reverse the last inequality to get the definition of a **relative minimum**.

4. Suppose $f : [a, b] \to \mathbb{R}$ is differentiable at $c \in (a, b)$ and f has a relative minimum at c. Prove that $f'(c) = 0$. Hint: break the interval $(c - \delta, c + \delta)$ into $(c - \delta, c]$ and $[c, c + \delta)$.