

INSTRUCTIONS: Solve the following 8 problems and write your solutions clearly on blank paper. Number your solutions clearly and explain proofs or other arguments using English words and sentences. You may use a calculator, notes, your textbook, and the internet, but no other people. Please provide a reference to any source that is not a standard part of the course. Turn in your solutions by 11:59 PM on Tuesday, November 24.

1. Let Y be the difference between two rolls of a fair die. Compute $E(|Y|)$.
2. A random variable X has a mean of 3 and a variance of 1. Calculate $E[(X - 1)^2]$.
3. The moment-generating function of a random variable X is $M_X(t) = \frac{1}{(1-t)^2}$. Calculate the variance of X .
4. Let X be a discrete random variable with probability distribution function $f(x) = 3\left(\frac{1}{2}\right)^x$ for $x = 2, 4, 6, 8, \dots$. Find a closed form for the moment-generating function of X (your final answer should not be a series).
5. Calculate the covariance of continuous random variables X and Y with joint density

$$f(x, y) = \begin{cases} 2 & \text{if } 0 < y < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

6. Let N be the number of people selected at random that you must ask in order to find someone with the same birthday as yours. Assuming each day of the year is equally likely (and ignoring leap years) calculate the following:
 - a) The mean and variance of N ;
 - b) $P(N \geq 300)$.
7. Let $X_1, X_2, \dots, X_n$ be independent random variables such that X_i is exponentially distributed with parameter $\theta = i$. Let $Y = X_1 + X_2 + \dots + X_n$. Calculate $E(Y)$ and $\text{Var}(Y)$.
8. Suppose the actual amount of milk in a half-gallon container is normally distributed with a variance of 0.64 oz. How large can the mean fill be if less than 5.5% of the containers contain more than 66 oz of milk?