

**COMMENTS/WARNING:** This document was hastily written and may contain errors, but it should be correct in at least its broad outlines. Do not use this guide to limit the topics you review: material from any section listed on the website may appear on the final. The exercises referenced are in the 7th edition of the textbook (the same problems are generally in the 8th edition, often with the same number or one number greater) or in the actuarial exam P sample problems available here: <https://www.soa.org/Files/Edu/edu-exam-p-sample-quest.pdf> (with solutions here: <https://www.soa.org/Files/Edu/edu-exam-p-sample-sol.pdf>).

**Basic definitions/postulates.** Sample space, events, and the 3 postulates.

**Basic results.** See section 2.5, in particular theorems 2.3 and 2.7. Exercises 2.8, 14, 34, 66, 69, 71 and actuarial practice problems 1–9, 14 (there's some conditional probability in here).

1.  $P(A \cup B) = 0.4$  and  $P(A \cup B') = 0.8$ . Calculate  $P(A)$ .

**Counting and its connection to probability.**

2. A hat contains 20 identical slips numbered 1 to 20. Five slips are drawn at random and without replacement. What is the probability that all will have numbers greater than 5? What is the probability that all will be divisible by 3? What is the probability that all will be either divisible by 3 or greater than 5? What is the probability that all will be divisible by 3 and greater than 5?

3. A hand of 5 cards is dealt from a standard deck of 52 cards. What is the probability of being dealt a flush (5 cards all of the same suit)? What is the probability of being dealt 2 or more face cards? What is the probability of being dealt a full house?

**Conditional probability and the multiplication rule.** See definition 2.1 and theorem 2.9. Also section 3.7 for conditional distributions. Exercises 2.17, 21, 28, 89, 91 and actuarial practice problems 12, 13, 14.

4. Shanielle O'Queal shoots 4 free-throws. She makes the first shot with probability 0.5. Every time she makes a shot her probability of making the next shot increases by 0.1. Every time she misses a shot her probability of making the next shot decreases by 0.1. So if she makes her first shot, Shanielle will make the second shot with probability 0.6. On the other hand, if she misses her first shot, Shanielle will make the second shot with probability 0.4. Find the probability that Shanielle makes 2 or more free throws.

**Independence.** See section 2.7 for basics and section 3.7 for independence for random variables. Exercises 2.23, 2.27, 93, 95, 97 and actuarial practice problems 11, 16, 17.

**Bayes' rule.** Section 2.8. Exercises 2.103, 105, 107, 109, 111 and actuarial practice problems 19, 20, 21, 22.

5. A university uses an automatic system to detect plagiarism in student essays. This system correctly identifies 90% of plagiarized essays but it also incorrectly identifies 2% of non-plagiarized essays as being plagiarized. It is known that 4% of essays will be plagiarized. What is the probability that an essay is plagiarized given that the system has flagged the essay?

**Random variables, probability distributions, probability densities, and (cumulative) distribution functions.** See sections 3.1–3.4. Exercises 3.5, 9, 11, 13, 24, 25, 29, 93.

**Multivariate distributions, marginal distributions, conditional distributions.** See sections 3.5–3.7. Exercises 3.44, 45, 47, 53, 69, 76, 97b, 99, 105.

**Expected value and other moments.** Including fair games and covariance. Sections 4.2, 3, 6, 7. Exercises 4.7, 9, 61, 63, 67, 80.

**Chebyshev's theorem.** Exercises 4.73, 74, 75.

**Moment-generating functions.** Exercises 4.34, 37, 38.

**Special distributions.** See the special distributions sheet as well as chapters 5 and 6. Exercises 5.41, 43, 49, 53, 57, 59, 67, 69, 71, 73, 75, 77 and 6.51, 53, 55, 57, 65, 71, 73, 75–80. Also, actuarial practice problem 18.

**Approximating binomial random variables with Poisson random variables and normal random variables.**

6. An American roulette wheel gives a theoretical probability of winning with a bet on red of  $\frac{9}{19}$ . Use a normal approximation to a binomial to estimate the probability that you will win more than 50 times if you bet on red 100 times.

**The Central Limit Theorem.** Exercises 8.63, 64, 66, 67, 68, 69.

7. Suppose that canned Albacore tuna has a mean mercury content of 0.35 ppm with a standard deviation of 0.128 ppm. What is the probability that the mean mercury content of a random sample of 49 cans of tuna is greater than 0.375 ppm?