Name(s):

Math 421 Counting September 4, 2015

Today's problems deal with playing cards. A standard deck of cards contains 52 cards: 13 cards of each of 4 suits (spades, hearts, diamonds, and clubs) with each card numbered $2, 3, \ldots, 10, J, Q, K,$ or A. Generally, order doesn't matter for hands of cards.

1. How many different ways are there to shuffle a deck of cards?

2. How many different 5-card hands are there?

3. How many 5-card hands are flushes (all one suit, counting the straight flushes)?

4. How many 5-card hands contain at least one heart? Hint: how many poker hands do not contain any hearts?

5. Find the mistake in the following calculation of the number of 5-card hands that contain at least one heart:

> Order doesn't matter for the cards, so we can start by putting any one of the 13 hearts in the hand. We can do this in 13 ways. We then add any 4 other cards from the remaining 51 to the hand. We can do this in $\binom{51}{4}$ ways. Therefore the total number of hands containing at least one heart is
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> $$13\binom{51}{4} = 3,248,700.$$

6. How many 5-card hands contain four of a kind (four 2s, four 3s, ..., or four aces)?

Challenge. How many 5-card hands are straight flushes (all of the same suit and consecutive)? Note that aces can count as 1s for the purposes of straights, so $A, 2, 3, 4, 5$ of clubs is a straight flush.

Challenge. How many 5-card hands are straights (consecutive but not necessarily all of the same suit)?