Name(s):

Math 421 Binomial, negative binomial, geometric, and Poisson distributions November 4, 2015

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent, identically distributed (iid) Bernoulli random variables with probability of success θ (with $0 < \theta < 1$). The total number of successes in the n trials is $T = X_1 + X_2 + \cdots + X_n$. The random variable T has a binomial distribution and its probability distribution function is

$$b(x; n, \theta) = \binom{n}{x} \theta^x (1-\theta)^{n-x} \text{ for } x = 0, 1, \ldots, n.$$

Applying Theorem 4.14, we find that $E(T) = n\theta$ and $V(T) = n\theta(1-\theta)$.

For the following two problems we'll consider another (possibly infinite) sequence of iid Bernoulli random variables $X_1, X_2, X_3, \ldots$.

1. Let X be the time of the first success (so the possible values for X are $1, 2, 3, \ldots$). Find the probability distribution function of X. This is the geometric distribution function with parameter θ, which the book calls $g(x; \theta)$.

2. Let Y be the time of the k^{th} success (so the possible values for Y are $k, k+1, k+2, \ldots$). Such a random variable is said to have a *negative binomial* distribution with parameters k and θ.

a) Find the probability distribution function of Y (the book calls this $b^*(x; k, \theta)$.

b) Calculate $E(Y)$. Hint: $\displaystyle\sum_{x=k}^{\infty} b^*(x+1; k+1, \theta) = 1$.

c) Prove that $b^*(x; k, \theta) = \left(\dfrac{k}{x}\right) b(k; x, \theta)$.

3. A random variable with the probability distribution function

$$p(x; \lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, \ldots$$

is said to have a *Poisson* distribution (with parameter $\lambda > 0$). Find the moment-generating function and use it to calculate the mean and variance of the distribution.