Definition. A random variable has a *Poisson distribution* with parameter $\lambda > 0$ if its probability distribution function is $p(x;\lambda) = \dfrac{\lambda^x e^{-\lambda}}{x!}$ for $x = 0, 1, 2, \ldots$.

Proposition 1. *If X has a Poisson distribution with parameter $\lambda > 0$, then $E(X) = \lambda$ and $Var(X) = \lambda$.*

1. Suppose that a (radioactive) neutron source emits 3 neutrons every 1 millisecond on average. The actual number emitted is the result of complicated (quantum) physics: experience has shown that Poisson distributions can be useful here. Let N_t be the number of neutrons emitted over t milliseconds. We'll assume that N_t has a Poisson distribution.

a) What parameter λ does N_1 have?

b) What parameter does N_t have?

(Technically, N_t is a family of random variables known as a Poisson process).

Definition. A random variable has an *exponential distribution* with parameter $\theta > 0$ if its density is

$$g(x;\theta) = \begin{cases} \frac{1}{\theta}e^{-\frac{x}{\theta}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Proposition 2. *If X is exponentially distributed with parameter θ, then $E(X) = \theta$ and $Var(X) = \theta^2$.*

2. We'll continue to work with the same neutron sources as in the first problem. Let T be the duration between the emission of neutrons by the neutron source, that is the time t at which N_t changes from 0 to 1.

a) $T > t$ if and only if N_t is equal to what?

b) Find the cumulative distribution function for T.

c) Differentiate the cdf for T to find a probability density function for T. Name the distribution.

d) How long do you have to wait to be 99% sure than at least one neutron has been emitted?

e) Show that $P(T \geq t + t_0 | T \geq t_0) = P(T \geq t)$ for any $t, t_0 > 0$.

Challenge. Let T_2 be the time to the second emission from the neutron source. Find the probability density function of T_2.