Definition. A random variable X has a **uniform continuous distribution** with parameters α and β (with $\alpha < \beta$) if and only if the following function is a probability density for X: $\boxed{f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } \alpha < x < \beta \\ 0 & \text{elsewhere} \end{cases}}$.

Definition. The **gamma function** is defined as $\boxed{\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy}$ for $\alpha > 0$.

Proposition. *For any positive integer n, $\Gamma(n) = (n-1)!$*

Definition. A random variable X has a **gamma distribution** with parameters $\alpha > 0$ and $\beta > 0$ if and only if the following function is a probability density for X: $\boxed{g(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}}$.

Proposition. *A gamma distribution with parameters α and β has moment-generating function $M_X(t) = (1-\beta t)^{-\alpha}$, mean $\mu = \alpha\beta$, and variance $\sigma^2 = \alpha\beta^2$.*

Definition. A random variable X has an **exponential distribution** with parameter $\theta > 0$ if and only if the following function is a probability density for X: $\boxed{g(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}}$ (a gamma distribution with $\alpha = 1$ and $\beta = \theta$).

Proposition. *An exponential distribution with parameter θ has mean $\mu = \theta$ and variance $\sigma^2 = \theta^2$.*

Definition. A random variable X has a **chi-square distribution** with parameter $\nu > 0$ if and only if the following function is a probability density for X: $\boxed{f(x) = \begin{cases} \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\frac{\nu-2}{2}} e^{-\frac{\nu}{2}} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}}$ (a gamma distribution with $\alpha = \frac{\nu}{2}$ and $\beta = 2$).

Proposition. *A chi-square distribution with parameter ν has mean $\mu = \nu$ and variance $\sigma^2 = 2\nu$.*

Definition. A random variable X has a **beta distribution** with parameters $\alpha > 0$ and $\beta > 0$ if and only if the following function is a probability density for X: $\boxed{f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} & \text{if } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}}$.

Proposition. *A beta distribution with parameters α and β has mean $\mu = \dfrac{\alpha}{\alpha+\beta}$ and variance $\sigma^2 = \dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.*

Definition. A random variable X has a **normal distribution** with parameters μ and $\sigma > 0$ if and only if the following function is a probability density for the X: $\boxed{n(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \text{ for all } x \in \mathbb{R}}$.

Proposition. *A normal distribution with parameters μ and σ has a moment-generating function $M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$, mean $\mu = \mu$ and variance $\sigma^2 = \sigma^2$.*

Of these distributions, only the uniform continuous distribution and the exponential distribution allow one to compute probabilities by hand. Calculations of probabilities for the rest of the distributions generally rely on a table or computational device. For a normally distributed random variable X this usually means **standardizing** the random variable: $\boxed{Z = \dfrac{X-\mu}{\sigma}}$ has a **standard normal distribution** with mean 0 and variance 1 (cf. homework problem 4.23).