Definition. A random variable X has a **discrete uniform distribution** if it is equally likely to assume any one of a finite set of possible values.

Examples. Roll a single die. Choose a winning number in a lottery.

Definition. A random variable X has a **Bernoulli distribution** with parameter θ (with $0 < \theta < 1$) if its probability distribution is

$$\boxed{f(x;\theta) = \begin{cases} 1-\theta & \text{if } x = 0 \\ \theta & \text{if } x = 1 \end{cases}}$$

The outcome 1 is often referred to as "success" while 0 is "failure" and the experiment is often called a Bernoulli trial.

Proposition. *The mean and variance of a Bernoulli random variable are* $\mu = \theta$ *and* $\sigma^2 = \theta(1-\theta)$.

Examples. Flip a coin and count the number of heads. Ask one person a yes or no question.

Definition. The total number of successes in n independent, identically distributed (iid) Bernoulli trials is a random variable with a **Binomial distribution**. A random variable X has a binomial distribution with parameters n and θ if its probability distribution function is

$$\boxed{b(x;n,\theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x} \text{ for } x = 0, 1, \ldots, n}$$

Proposition. *The mean and variance of a binomial distribution are* $\mu = n\theta$ *and* $\sigma^2 = n\theta(1-\theta)$. *The moment-generating function of a binomial distribution is* $M_X(t) = [1 + \theta(e^t - 1)]^n$.

Examples. Flip n identical coins and count the number of heads. Ask n randomly selected people a yes or no question on a survey.

Definition. Let $X_1, X_2, \ldots$ be a sequence of independent, identically distributed (iid) Bernoulli trials, all with probability of success θ. Let N be the trial on which the first success occurs. The random variable N is said to have a **geometric distribution** with parameter θ and its probability distribution function is

$$\boxed{g(n:\theta) = \theta(1-\theta)^{n-1} \text{ for } n = 1, 2, 3, \ldots}$$

Proposition. *The mean and variance of a geometric distribution are* $\mu = \dfrac{1}{\theta}$ *and* $\sigma^2 = \dfrac{1}{\theta}\left(\dfrac{1}{\theta} - 1\right)$.

Examples. Flip a coin until the first heads appears. Roll a pair of dice until you first get a pair of sixes. Ask randomly selected people a yes or no question until you first get a yes.

Definition. Let $X_1, X_2, \ldots$ be a sequence of independent, identically distributed (iid) Bernoulli random variables, all with probability of success θ. Let N be the trial on which the k^{th} success occurs (so the possible values for N are $k, k+1, k+2, \ldots$). The random variable N is said to have a **negative binomial (or binomial waiting-time or Pascal) distribution** with parameters k and θ and its probability distribution function is

$$\boxed{b^*(n;k,\theta) = \binom{n-1}{k-1}\theta^k(1-\theta)^{n-k} \text{ for } n = k, k+1, k+2, \ldots}$$

Proposition. *The mean and variance of a negative binomial distribution are* $\mu = \dfrac{k}{\theta}$ *and* $\sigma^2 = \dfrac{k}{\theta}\left(\dfrac{1}{\theta} - 1\right)$.

Definition. A random variable with the probability distribution function

$$\boxed{p(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, \ldots}$$

is said to have a **Poisson distribution** with parameter $\lambda > 0$.

Proposition. *The mean and variance of a Poisson distribution are* $\mu = \lambda$ *and* $\sigma^2 = \lambda$. *The moment-generating function of a Poisson random distribution is* $M_X(t) = e^{\lambda(e^t - 1)}$.

Definition. Suppose n elements are to be selected without replacement from a population of size N of which M are successes. The number of successes selected is a **hypergeometric** random variable and its probability distribution function is

$$\boxed{h(x; nN, M) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}}$$

Proposition. *The mean and variance of a hypergeometric distribution are* $\mu = \dfrac{nM}{N}$ *and* $\sigma^2 = \dfrac{nM(N-M)(N-n)}{N^2(N-1)}$.