Definition. The $\mathbf{r}^{\text{th}}$ **moment about the origin** of a random variable X is $\mu_r' = E(X^r)$.

Definition. The first moment about the origin of a random variable is called the **mean** and is denoted by μ.

Example. For a random variable X with a uniform continuous distribution on the interval $(1,5)$, the mean is

$$\mu = E(X) = \int_1^5 x\left(\frac{1}{4}\right)dx = 3$$

and the second moment about the origin is

$$\mu_2' = E(X^2) = \int_1^5 x^2\left(\frac{1}{4}\right)dx = \frac{31}{3}$$

Proposition. *If a and b are constants, then $E(aX+b) = aE(X)+b$.*

Definition. The $\mathbf{r}^{\text{th}}$ **moment about the mean** of a random variable X is $\mu_r = E\left[(X-\mu)^r\right]$.

Definition. The second moment about the mean of a random variable is called the **variance** and is denoted by σ^2. The **standard deviation** of a random variable is $\sigma = \sqrt{\sigma^2}$.

Proposition (A calculating formula for the variance). $\boxed{\sigma^2 = \mu_2' - \mu^2 = E(X^2) - [E(X)]^2}$

Example. The variance of a random variable X with a uniform continuous distribution on the interval $(1,5)$ is $\sigma^2 = \dfrac{31}{3} - 3^2 = \dfrac{2}{3}$. The standard deviation of X is $\sigma = \sqrt{\frac{2}{3}}$.

Proposition. *If a and b are constants, then $V(aX+b) = a^2V(X)$.*

Proposition (A calculating formula for μ_3). $\mu_3 = \mu_3' - 3\mu_2'\mu + 2\mu^3$

Definition. The **moment-generating function** of a random variable X is $M_X(t) = E\left(e^{tX}\right)$.

Proposition. *If $M_X(t)$ is the moment-generating function of a random variable X, then* $\boxed{M_X^{(r)}(0) = \mu_r' = E(X^r)}$

Proposition. *If a and b are constants, then $M_{aX+b}(t) = e^{bt}M_X(at)$.*

Definition. If X and Y are jointly distributed random variables with means μ_X and μ_Y, respectively, then $E\left[(X-\mu_X)(Y-\mu_Y)\right]$ is called the **covariance** of X and Y and is denoted σ_{XY}, $\text{cov}(X,Y)$, or $C(X,Y)$. If σ_X and σ_Y are the standard deviations of X and Y, respectively, then $\text{cor}(X,Y) = \dfrac{\text{cov}(X,Y)}{\sigma_X\sigma_Y}$ is the **correlation** of X and Y.

See `http://www.randomservices.org/random/expect/Covariance.html`

Proposition (A calculating formula for the covariance). $\boxed{cov(X,Y) = E(XY) - E(X)E(Y)}$

1. Calculate the expected value and variance of a Match 4 lottery ticket (rules are available at `http://www.walottery.com/JackpotGames/Match4.aspx`). Start by finding the exact probabilities of winning each amount.

2. Calculate the mean, variance, and moment-generating function of a random variable with a uniform continuous distribution on the interval (a,b).

3. Calculate $\text{cor}(X,Y)$ for X and Y with joint probability density $f(x,y) = \begin{cases} \frac{1}{3}(x+y) & \text{if } 0 < x < 1,\ 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$