Name(s):
Math 421 Counting Cards September 1, 2017

One of the first motivations for the development of Probability Theory was the effort to analyze games of chance (and gambling on those games). We'll start there, too. Today's problems involve applying the counting methods we've learned to playing cards. A standard deck of cards contains 52 cards. Each card has a suit (spades ♠, hearts ♡, diamonds ♢, or clubs ♣) and a rank ($2, 3, \ldots, 10$, Jack, Queen, King, or Ace). Two of the suits are black (spades ♠ and clubs ♣) and two are red (hearts ♡ and diamonds ♢). Generally, order doesn't matter for hands of cards.

1. How many different ways are there to shuffle a deck of cards?

2. How many different 5-card hands are there?

3. How many 5-card hands are all the same suit (in poker these are called flushes, or straight flushes if the cards have consecutive ranks)?

4. How many 5-card hands contain all four cards of some rank (four-of-a-kind)?

5. Identify the mistake in the following calculation of the number of 5-card hands that contain at least one heart:

> Order doesn't matter for the cards, so we can start by putting any one of the 13 hearts in the hand. We can do this in 13 ways. We then add any 4 other cards from the remaining 51 to the hand. We can do this in $\binom{51}{4}$ ways. Therefore the total number of hands containing at least one heart is
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> $$13\binom{51}{4} = 3,248,700.$$

6. How many 5-card hands contain at least one heart? Hint: how many don't contain any hearts?

Challenge. How many 5-card hands are straight flushes (all of the same suit and consecutive)? Note that aces can count as 1s for the purposes of straights, so $A♣, 2♣, 3♣, 4♣, 5♣$ is a straight flush. Aces can't be both high and low in one straight, so $Q\heartsuit, K\heartsuit, A\heartsuit, 1\heartsuit, 2\heartsuit$ is a flush, but not a straight.

Challenge. How many 5-card hands are straights (consecutive but not necessarily all of the same suit–include straight flushes)?