Work in groups of any size to solve the problems on this sheet (start with the problems on this side). Write your solutions on a separate piece of paper and turn in one set of solutions per group. I don't expect you to get to every problem–just turn in what you have done at the end of class.

The die-coin experiment gave us two random variables: the number rolled on the die, R, and the number of heads flipped on the coin, F. Each random variable has an associated *probability distribution function (pdf)* giving the probability of the random variable taking different values. You found the pdf of R on the first problem of the previous worksheet: $f(x) = P(R = x) = \frac{1}{6}$ if $x = 1, 2, \ldots, 6$ (we generally only care about the non-zero values of a pdf). The pdf of the random variable F is $g(y) = P(F = y)$, which we have noticed is hard to calculate. However, we do know that $g(6) = \frac{1}{384}$ and $g(5) = \frac{1}{48}$.

1. The function $f(x|5) = P(R = x|F = 5)$ is the *conditional pdf of R given $F = 5$*. Find the function $f(x|5)$.

2. The function $g(x|y) = P(F = x|R = y)$ is the *conditional pdf of F given $R = y$*.

a) Find the function $g(x|1)$.

b) Find the function $g(x|2)$.

c) Find a general formula for the function $g(x|y)$.

3. The function $f(x, y) = P(R = x \text{ and } F = y)$ is the *joint probability distribution function of R and F*. Give a formula for $f(x, y)$.

The following problems have all appeared on past versions of Exam 1.

4. Let A and B be events with $P(A) = 0.7$, $P(B) = 0.8$, and $P(A \cup B) = 0.9$. Are A and B independent? Prove that your answer is correct.

5. Events A, B, and C occur with probabilities 0.3, 0.5, and 0.6, respectively, and are independent. Calculate $P(A \cup B \cup C)$.

6. Suppose events A and B are independent, $A \subseteq B$, and $P(A) > 0$. Calculate $P(B)$.

7. A pair of dice is rolled. Given that the sum of the numbers is 6, what is the probability that (at least) one of the dice showed a 1?

8. Let X be the number of heads in 4 flips of a coin that is weighted to flip heads with probability 0.6. Calculate the probability that X is odd.

9. Cards are dealt from a well-shuffled deck until the first heart is dealt. Let X be the total number of cards dealt (including the heart). Calculate $P(X > 4)$.

10. Let X be the lesser of the two numbers rolled on a pair of dice (so $X(2,4) = 2$, $X(2,5) = 2$, $X(3,3) = 3$, and so on). Calculate $P(X \leq 5)$.

11. The joint probability distribution of discrete random variables X and Y is $f(x,y) = \dfrac{xy}{30}$ for $x = 1, 2, 3$ and $y = 2, 3$. Are X and Y independent?