1. Continuous random variables X and Y have joint density $f(x, y) = \begin{cases} cxy & \text{if } 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$ where c is some constant.

a) Determine the value of c that makes f a joint density function

b) Calculate $P(Y > X^2)$

Definition. The joint cumulative distribution function for X and Y is $F(x, y) = P(X \leq x, Y \leq y)$.

2. Continue working with the same random variables as in problem 1.

a) Find the cdf of X and Y

b) Use the cdf to calculate $P(X \leq \frac{2}{5}, Y > \frac{1}{2})$

Definition. The marginal density of X is $f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy$ and the marginal density of Y is $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx$.

3. Calculate the marginal densities of the random variables from problem 1. Are these random variables independent?

Definition. The conditional density of X given $Y = y_0$ is $f_{X|Y}(x|y_0) = \dfrac{f(x,y_0)}{f_Y(y_0)}$ (where $f_Y(y_0) > 0$).

4. Let X and Y be continuous random variables with joint density $f(x,y) = \frac{6}{5}\left(x^2 + y\right)$ for $0 < x, y < 1$. As we have seen in class, the marginal distribution of Y is $f_Y(y) = \frac{6}{5}\left(y + \frac{1}{3}\right)$.

a) Find a formula for the conditional density of X given $Y = \frac{1}{3}$

b) Calculate $P(X < \frac{1}{2} | Y = \frac{1}{3})$

c) Find a formula for the conditional density of Y given $X = 1$