Name(s):

Math 421 Advanced Moments October 18, 2017

Definition. If X and Y are jointly distributed random variables with means μ_X and μ_Y, respectively, then $E\left[(X-\mu_X)(Y-\mu_Y)\right]$ is called the *covariance* of X and Y and is denoted σ_{XY}, $cov(X,Y)$, or $C(X,Y)$.

Theorem 1. $C(X,Y) = E(XY) - E(X)E(Y)$

1. Calculate $C(X,Y)$ for X and Y with joint probability density $f(x,y) = \begin{cases} 4xy & \text{if } 0<x<1,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$

2. Calculate $C(X,Y)$ for X and Y with joint probability density $f(x,y) = \begin{cases} \frac{6}{5}(x^2+y) & \text{if } 0<x<1,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$

3. Let X be a random variable and let a and b be constants.

a) Show that $V(aX + b) = a^2 V(X)$

b) If $M_{aX+b}(t) = E\left[e^{(aX+b)t}\right]$, show that $M_{aX+b}(t) = e^{bt} M_X(at)$.