1. Let X and Y be independent, identically distributed exponential random variables with common parameter $\theta = 1$. Let $Z = X - Y$. Let $F(z)$ be the CDF of Z.

a) Find $F(z)$ for $z > 0$

b) Find $F(z)$ for $z < 0$

c) Differentiate $F(z)$ to find a probability density function for Z

2. Let X be a random variable with probability density function f and let $Y = |X|$. Show that the following function is a probability density for Y :

$$g(y) = \begin{cases} f(y) + f(-y) & \text{if } y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

3. Let Z have a standard normal distribution and let $Y = |Z|$.

a) Show that $g(y) = 2n(y; 0, 1)$ if $y > 0$ is a probability density function for Y

b) Prove that $x = y^2$ is an increasing function on the values of y for which $g(y) > 0$

c) Apply the transformation technique to find a probability density function for $X = Y^2 = Z^2$