INSTRUCTIONS: Solve 5 of the following problems now and write your solutions on the provided paper, clearly labeling each solution. Take this sheet with you and solve the remaining 4 problems at home. Those solutions are due at the beginning of class on Monday, February 24. Write your solutions clearly and explain proofs or other arguments using English words and sentences. You may use a calculator during all portions of the exam. Books, notes, and other non-living resources may be used on the take-home portion of the exam, but you should cite any sources that are not normal class material (e.g. websites).

1. A random sample of size $n = 100$ is to be taken from a Poisson population with mean $\lambda = 8$. Estimate $P(\overline{X} > 6)$.

2. Let S^2 be the variance of a random sample of size 5 from a normally distributed population with variance $\sigma^2 = 20$. Calculate $P(S^2 > 47.44)$.

3. My tax return involves 32 different numbers, each rounded to the nearest dollar and then added together. Assuming that the errors made by rounding are uniformly distributed on the interval $(-\frac{1}{2}, \frac{1}{2})$, estimate the probability that the sum of the rounded amounts differs from the true sum by less than \$1. Hint: this is really a question about the sum of the rounding errors.

4. Let $X_1, X_2, \ldots, X_3$ be a random sample from an exponentially distributed population with parameter θ. Prove that $Y = X_1 + X_2 + \cdots + X_n$ has a gamma distribution with parameters $\alpha = n$ and $\beta = \theta$.

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from a geometric population with parameter θ. Use the method of maximum likelihood to find an estimator for θ.

6. Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponentially distributed population with mean θ. Prove that the random variable $\hat{\Theta}^2 = \dfrac{1}{2n}\displaystyle\sum_{i=1}^{n} X_i^2$ is an unbiased estimator for the variance of the population.

7. Let $X_1, X_2, \ldots, X_n$ be a random sample from an infinite population with density function $f(x)$ and distribution function $F(x)$. Let Y_1 be the smallest value in the sample (the *first order statistic*). Express the density of Y_1 in terms of n, f, and F. Hint: $P(Y_1 \leq y) = 1 - P(Y_1 > y)$.

8. The random variables X_1 and X_2 have joint density

$$f(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)} & \text{for } 0 < x_1 \text{ and } 0 < x_2 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate the density of $Y = X_1 + 2X_2$.

9. Alex and Brett play the following game: each writes either a 1 or 2 on a slip of paper and then they reveal their choices. If the sum of the two numbers is odd, then Alex wins that amount. If the sum of the two numbers is even, then Brett wins that amount.

a) Construct a payoff matrix for the game.

b) What randomized strategy should Alex employ to minimize her maximum expected loss? (Find $0 < x < 1$ so that if Alex writes 1 with probability x, then her expected loss to Brett is the same for either of Brett's choices).