Let $X_1,\ X_2,\ldots,\ X_n$ be a random sample from a population with mean μ and variance σ. We will use the sample mean $\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ and the sample variance $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\overline{x})^2$ to decide whether or not to reject a hull hypothesis in favor of an alternative. We must have a simple null hypothesis (for example $H_0:\mu=\mu_0$ or $H_0:\sigma^2=\sigma_0^2$) and will usually have a composite alternative hypothesis (either one-tailed or two-tailed).

In our tests we can make either the **type I error** of rejecting the null hypothesis when in fact it is true or the **type II error** of not rejecting the null hypothesis when in fact it is false. The probability of a type I error is α. The probability of a type II error is β. Usually you should choose the largest acceptable value for α since this will minimize β. In all cases the **rejection region** is chosen so that the test statistic lands in the rejection region with probability α (when H_0 is true). Remember that your rejection region should be two-tailed for a two-tailed test and one-tailed for a one-tailed test. It may also be useful to find the ***P*-value** (or **observed significance level**) of your data. This is the smallest value for α that allows you to reject H_0 with your data.

For **tests about the mean** ($H_0:\mu=\mu_0$) test statistics are:

- $z=\dfrac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$ for samples from a population with a known variance σ^2 (all sample sizes if the population is normal, otherwise just for large samples);
- $z=\dfrac{\overline{x}-\mu_0}{\frac{S}{\sqrt{n}}}$ for large samples;
- $t=\dfrac{\overline{x}-\mu_0}{\frac{S}{\sqrt{n}}}$ for small samples ($n\leq 30$ or as big as your t-table goes) from a normally distributed population.

For **tests about a population proportion** ($H_0:\theta=\theta_0$) we call use the sample proportion $\hat{\Theta}$ or the sample total $X=n\hat{\Theta}$ and the test statistic is:

- x (X is binomial with parameters n and θ_0, works best for small samples);
- $z=\dfrac{\hat{\Theta}-\theta_0}{\sqrt{\frac{\theta_0(1-\theta_0)}{n}}}=\dfrac{X-n\theta_0}{\sqrt{n\theta_0(1-\theta_0)}}$ for large samples ($n\theta_0\geq 10$ and $n(1-\theta_0)\geq 10$).

For **tests about the variance** ($H_0:\sigma^2=\sigma_0^2$) the test statistic is

- $\chi^2=\dfrac{(n-1)s^2}{\sigma_0^2}$

For **tests about the difference of two means** ($H_0:\mu_1-\mu_2=\delta$) the test statistics are:

- $z=\dfrac{\overline{x}_1-\overline{x}_2-\delta}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$ for samples from populations with a known variances σ_1^2 and $sigma_2^2$ (all sample sizes if the populations are normal, otherwise just for large samples);
- $z=\dfrac{\overline{x}_1-\overline{x}_2-\delta}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$ for large samples;
- $t=\dfrac{\overline{x}_1-\overline{x}_2-\delta}{s_p^2\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ for small samples from normally distributed populations with the same variance.

Recall that

$$s_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{s_1+n_2-2}$$

For **tests about the ratio of two variances** ($H_0:\frac{\sigma_1^2}{\sigma_2^2}=1$) the test statistic is

- $\dfrac{s_1^2}{s_2^2}$ ($\frac{S_1^2}{S_2^2}$ has an F distribution with n_1-1 and n_2-1 degrees of freedom).

1. Newly purchased tires are supposed to be filled to a pressure of 30 lb/in^2. Let μ denote the true average pressure. We wish to test the hypothesis $H_0 : \ \mu = 30$ against the alternate hypothesis $H_a : \mu \neq 30$. Find the P-value of each measurement of the sample mean and standard deviation of a random sample of size $n = 100$.

a) $\overline{x} = 28.2, \ s = 8$

b) $\overline{x} = 28.2, \ s = 4$

c) $\overline{x} = 30.6, \ s = 4$

2. The article "Analysis of Reserve and Regular Bottlings: Why Pay for a Difference Only the Critics Claim to Notice?" reported on an experiment to determine if wine tasters could correctly distinguish between reserve and regular versions of a wine. In each trial tasters were given 4 indistinguishable containers of wine, two of which contained the regular version and two of which contained the reserve version of the wine. The taster then selected 3 of the containers, tasted them, and was asked to identify which one of the 3 was different from the other 2. In $n = 855$ trials, 346 resulted in correct distinctions. Does this provide compelling evidence that wine tasters can distinguish between regular and reserve wines?

3. The sample average unrestrained compressive strength for 45 specimens of a particular type of brick was 3107 psi, and the sample standard deviation was 188 psi.

a) Does the data strongly indicate that the true average unrestrained compressive strength is less than the design value of 3200? Test using $\alpha = 0.001$. What does it mean to use such a small value for α?

b) Is this strong evidence that $\sigma < 200$ psi?

4. A survey of 948 American college football fans finds that 597 would prefer playoffs instead of the current bowl system. Is this strong evidence that more than half of all such individuals would prefer playoffs?

5. It is known that roughly $\frac{2}{3}$ of all people have a dominant right foot and $\frac{2}{3}$ have a dominant right eye. Do people also kiss to the right? The article "Human Behavior: Adult Persistence of Head-Turning Asymmetry" reported that in a random sample of 124 kissing couples, 80 of the couples tended to lean more to the right than left. Does this result suggest that more than half of all couples lean right when kissing? Does this result provide evidence against the hypothesis that $\frac{2}{3}$ of all kissing couples lean right?

6. Minor surgery on horses under field conditions requires a reliable short-term anaesthetic. The article "A Field Trial of Ketamine Anaesthesia in the Horse" reports that for a sample of 73 horses to which ketamine was administered the average lateral recumbency time was 18.86 minutes with a standard deviation of 8.6 minutes.

a) Does this data suggest that the true average lateral recumbency time is less than 20 minutes?

b) Does this data suggest that the true average lateral recumbency time is more than 15 minutes?

c) Does this data suggest that the true variance of lateral recumbency time is more than 64 minutes?

7. The weights of a random sample of 10 Black Angus steers have a standard deviation of 238 pounds. Test $H_0 : \sigma = 250$ against $H_1 : \sigma \neq 250$.