Name(s):

Math 422 Confidence Intervals February 24, 2014

1. Let Z be a standard normal random variable.

a) Find a number z such that $P(|Z| < z) = 0.95$.

b) Rearrange the inequality to fill in the blanks in the following expression: $P(Z - \_\_\_ < 0 < Z + \_\_\_) = 0.95$.

c) Translate the expression of part b into an English sentence.

2. Let X be a normally distributed random variable with mean $\mu = 50$ and standard deviation $\sigma = 8$.

a) Find a number x such that $P(|X - 50| < x) = 0.95$.

b) Rearrange the inequality to fill in the blanks in the following expression: $P(X - \_\_\_ < 50 < X + \_\_\_) = 0.95$.

c) Translate the expression of part b into an English sentence.

3. Let $\overline{X}$ be the mean of a random sample of size $n = 64$ from a population with mean μ (unknown) and standard deviation $\sigma = 8$.

a) Find a number x such that $P(|\overline{X} - \mu| < x) \approx 0.95$.

b) Rearrange the inequality to fill in the blanks in the following expression: $P(\overline{X} - \_\_\_ < \mu < \overline{X} + \_\_\_) \approx 0.95$.

c) Samples are taken and you find $\overline{x} = 50$. Substitute this value in for $\overline{X}$ in the $\overline{X} \pm \_\_\_$ part of your answers of part b to find an interval (called a *95% confidence interval* for the mean of the population).

d) What's wrong with this expression: $P(48.04 < \mu < 51.96) \approx 0.95$?

4. Suppose that another researcher samples the same population as in problem 3 and finds $\overline{x} = 54$ with a sample size of $n = 64$.

a) What is this researcher's 95% confidence interval for the population mean?

b) Can you put together your different results to find a new, better (i.e. smaller) 95% confidence interval, or would some sort of a duel better resolve your differences?

5. Suppose a third researcher takes a sample of size $n = 97$ and finds $\overline{x} = 49$. Find an improved 95% confidence interval including these new results