422 S16 Ch 13: 1, 6, 29, 38 & 39, 47, 68, 76, 82

1. Under H_0 $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$ has standard normal distribution.

By theorem 8.7 Z^2 has chi-square dst with 1 df.

A critical region of size α is $Z^2 \geq \chi^2_{\alpha, 1}$.

6. For large n, $\dfrac{(n-1)s^2}{\sigma^2}$ is approximately normal with mean $n-1$ and variance $2(n-1)$. Hence an approximate critical region for testing $H_0: \sigma^2 = \sigma_0^2$ against $H_1: \sigma^2 > \sigma_0^2$ is $\dfrac{\frac{(n-1)s^2}{\sigma_0^2} - n-1}{\sqrt{2(n-1)}} \geq z_\alpha$.

Isolating s^2 gives $s^2 \geq \sigma_0^2 \left[1 + z_\alpha \sqrt{\dfrac{2}{n-1}} \right]$.

29. Solution in the book

38 & 39. $H_0: \mu_1 - \mu_2 = -.5$ $H_1: \mu_1 - \mu_2 < -.5$

Test stat: $Z = \dfrac{53.8 - 54.5 - (-.5)}{\sqrt{\frac{2.4}{400} + \frac{2.5}{500}}} \approx -1.906925$

(Large samples mean that this is reasonable)

P-value: .028.

Reject H_0 at level $\alpha = .05$.

47. Solution in the book

68. Pooled estimate $\hat{\theta} = \dfrac{74 + 92}{250 + 250} = \dfrac{166}{500}$

$$\chi^2 = \frac{\left(74 - 250\left(\frac{166}{500}\right)\right)^2}{250\left(\frac{166}{500}\right)\left(\frac{334}{500}\right)} + \frac{\left(92 - 250\left(\frac{166}{500}\right)\right)^2}{250\left(\frac{166}{500}\right)\left(\frac{334}{500}\right)} = \frac{(-9)^2 + (9)^2}{55.444} \approx 2.92$$

1 df. p-value: .0874.

Fail to reject $H_0: \theta_1 = \theta_2$.

76. Contingency table.

Test stat:

$$\chi^2 = \frac{\left(48 - \frac{100(160)}{360}\right)^2}{\frac{100(160)}{360}} + \frac{\left(40 - \frac{100(139)}{360}\right)^2}{\frac{100(139)}{360}} + \frac{\left(12 - \frac{100(61)}{360}\right)^2}{\frac{100(61)}{360}}$$

$$+ \frac{\left(55 - \frac{137(160)}{360}\right)^2}{\frac{137(160)}{360}} + \frac{\left(53 - \frac{137(139)}{360}\right)^2}{\frac{137(139)}{360}} + \frac{\left(29 - \frac{137(61)}{360}\right)^2}{\frac{137(61)}{360}}$$

$$+ \frac{\left(57 - \frac{123(160)}{360}\right)^2}{\frac{123(160)}{360}} + \frac{\left(46 - \frac{123(139)}{360}\right)^2}{\frac{123(139)}{360}} + \frac{\left(20 - \frac{123(61)}{360}\right)^2}{\frac{123(61)}{360}} \approx 3.97$$

with 4 df.

Critical region: $\chi^2 \geq 9.4877$. Fail to reject H_0.

Note: I don't trust myself to have done every part of this calculation correctly, so if your answer is a little different that may be because I'm wrong and you're right.

82. Goodness of fit. Estimate the parameter $3\hat{\theta} = \frac{\text{\# cakes sold in total}}{\text{\# days in total}} = \frac{810}{300}$

Hence $\hat{\theta} = \frac{810}{900} = 0.9$.

$e_0 = 300(.1)^3 = .3$

$e_1 = 300(.1)^2(.9)\binom{3}{1} = 8.1$

Group these: 8.4

$e_2 = 300(.1)(.9)^2\binom{3}{2} = 72.9$

$e_3 = 300(.9)^3 = 218.7$

Test stat: $\chi^2 = \frac{(17-8.4)^2}{8.4} + \frac{(55-72.9)^2}{72.9} + \frac{(228-218.7)^2}{218.7}$

≈ 13.60 (1 df since we estimated 1 parameter)

p-value: .0002

Critical region: $\chi^2 \geq 3.84$

Reject H_0; the dist is not binomial.