Name(s):
Math 422 | Order Statistics and Estimators | January 25, 2016

Let $X_1, X_2, \ldots, X_n$ be a random sample from an infinite population. Let $Y_1, Y_2, \ldots, Y_n$ be the same sample arranged in increasing order: $Y_1$ is the smallest number from $X_1, X_2, \ldots, X_n$; $Y_2$ is the second smallest number from $X_1, X_2, \ldots, X_n$; et cetera. The random variable $Y_k$ is known as the $k^{\text{th}}$ order statistic.

**1.** Assume the population has a continuous distribution with cdf $F(x)$ and pdf $f(x)$.

a) The random variable $Y_n$ is the largest of the numbers $X_1, X_2, \ldots, X_n$. Use the distribution function technique to find the density of $Y_n$ (expressed in terms of $n$, $F$, and $f$).

b) Find the density of $Y_1$ (expressed in terms of $n$, $F$, and $f$).

c) Find the density of $Y_k$.

**2.** In this problem the population is uniformly distributed on $[0, \beta]$. This makes $f(x) = \frac{1}{\beta}$ for $0 < x < \beta$.

a) Find a constant $C$ such that $E(CY_n) = \beta$ (recall that the sample has size $n$).

b) Show that $E(2\overline{X}) = \beta$.