NAME(S):

MATH 422 CONFIDENCE INTERVALS FEBRUARY 1, 2016

1. Let Z be a standard normal random variable.

a) Find a number z such that $P(|Z| < z) = 0.95$.

b) Rearrange the inequality of part a to fill in the blanks in the following expression:

$$P(Z - \_\_\_\_\_\_\_ < 0 < Z + \_\_\_\_\_\_\_) = 0.95$$

2. Let X be a normally distributed random variable with mean μ and standard deviation σ. Using your work in problem 1 as a guide, fill in the blanks in the following expression:

$$P(X - \_\_\_\_\_\_\_ < \mu < X + \_\_\_\_\_\_\_) = 0.95$$

3. Let $\overline{X}$ be the mean of a random sample of size $n = 64$ from a population with mean μ (unknown) and standard deviation $\sigma = 8$.

a) Fill in the blanks in the following expression:

$$P(\overline{X} - \_\_\_\_\_\_\_ < \mu < \overline{X} + \_\_\_\_\_\_\_) \approx 0.95$$

b) Samples are taken and you find $\overline{x} = 50$. Substitute this value in for $\overline{X}$ in part a to find the *95% confidence interval* for the population mean μ.

c) What's wrong with the expression $P(48.04 < \mu < 51.96) \approx 0.95$?

d) Explain the meaning of your 95% confidence interval.

4. Suppose that another researcher samples the same population as in problem 3 and finds $\overline{x} = 54$ with a sample size of $n = 36$.

a) What is this researcher's 95% confidence interval for the population mean?

b) Can you put together your different results to find a new, better 95% confidence interval?

5. Suppose a sample from a (different) normally distributed population finds $\overline{x} = 10$ and $s^2 = 16$ with a sample size of $n = 25$. Use the sample statistic $T = \dfrac{\overline{X} - \mu}{S/\sqrt{n}}$ to find a 95% confidence interval for μ based on these results.