1. You showed on homework problem 8.3 that if $\overline{X}_1$ and $\overline{X}_2$ are the means of independent random samples of sizes n_1 and n_2 from normally distributed populations with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, then $\overline{X}_1 - \overline{X}_2$ is normally distributed with mean $\mu_1 - \mu_1$ and variance $\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}$. Use this to find a $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$.

2. Homework problem 8.3 together with theorem 8.12 allow us to address the case when σ_1^2 and σ_2^2 are unknown but equal. Under these circumstances we may use the *pooled estimator*

$$S_p^2 = \frac{(n_1-1)S_1^2 - (n_2-1)S_2^2}{n_1+n_2-2}$$

and the test statistic

$$T = \frac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}},$$

which has a t-distribution with $n_1 + n_2 - 2$ degrees of freedom, to find a confidence interval for $\mu_1 - \mu_2$.

a) Show that S_p is an unbiased estimator for σ^2 (where σ^2 is the common variance of the two populations).

b) Find a $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$.

3. Theorem 8.11 tells us that if S^2 is the variance of a random sample of size n from a normally distributed population with variance σ^2, then

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

has a chi-square distribution with $n-1$ degrees of freedom. Use this to find a $100(1-\alpha)\%$ confidence interval for σ^2.