Math 422 **Exam 1** **February 14, 2018**

Instructions: Solve 6 of the following 7 problems and write your solutions on separate sheets of paper. Clearly label each solution and show the steps leading to your answer. Even correct solutions may receive little or no credit if your work does not support your solution. Indicate clearly which problem you are skipping (do not turn in 7 solutions). You may ask Dr. Axon for help, but you may not work with any other people on these problems. You may use any other resources you find helpful (e.g. calculators, notes, books, and web sites). Turn in your solutions by 5:00 PM on Tuesday, February 20.

1. Let Y_1, Y_2, and Y_3 be the order statistics of a random sample of size 3 from a population having a uniform continuous distribution on the interval $[0, \beta]$. You proved on worksheet 2 that $\frac{4}{3}Y_3$ is an unbiased estimator for β. Show that $4Y_1$ is also an unbiased estimator for β and compare its variance with the variance of $\frac{4}{3}Y_3$. Which estimator is better?

2. Suppose that a population is uniformly distributed on the interval $[-\alpha, \alpha]$ (where α is a positive constant). Use the method of moments to find an estimator for α.

3. Let $X_1, X_2, \ldots X_n$ be a random sample from a population with probability density function $f(x) = \dfrac{\theta}{(1+x)^{(\theta+1)}}$ for $x > 0$ (with $\theta > 0$). Find the maximum likelihood estimator for θ.

4. Suppose that a Bernoulli population with parameter $0 < \theta < 1$ is sampled at random until the first success is found. Let N be the trial on which the first success occurs. Find an unbiased estimator for θ based on N (your estimator should be a function of N).

5. Generic Electric Co. is studying the use of programmable thermostats among its customers. They have found that the proportion of households using a programmable thermostat in each substation service zone follows a Beta distribution with parameters $\alpha = 6$ and $\beta = 4$. A random sample of 12 households in a new zone finds 10 that have programmable thermostats. Estimate the true proportion of households with programmable thermostats in this zone using:

a) Only the prior information (not the sample);

b) Only the sample (not the prior distribution);

c) Bayesian estimation.

6. You have been employed to determine if a Bernoulli distribution has parameter $\theta = \frac{1}{3}$ or $\theta = \frac{2}{3}$. You will be able to make one observation before committing to a guess (you'll observe either a success or a failure; remember that θ is the probability of success). If you guess right, you get \$12, but if you guess wrong, you must pay a fine of \$15.

a) List all 4 possible decision functions.

b) Make a table giving the values of the risk function for the 4 decision functions and the 2 possible values of θ.

c) Identify the decision function that minimizes your maximum risk.

7. Let $\overline{X}$ be the mean of a random sample of size n from an infinite population with density $f(x) = e^{-(x-\alpha)}$ for $x > \alpha$. It follows that $\hat{\Theta} = \overline{X} - 1$ is an unbiased estimator of α and $var(\hat{\Theta}) = \frac{1}{n}$ (you do not need to verify these calculations). Show that $\hat{\Theta}$ is a minimum variance unbiased estimator for α.