The first problems on this worksheet continue our analysis of the gambling game from last class. You pay \$1 to play the game. Player B then selects a coin which is either 2-headed or normal and flips it. You then guess whether the coin is 2-headed or normal. If your guess is right, you get \$2 (for a net profit of \$1). If your guess is wrong, you get nothing (for a net loss of \$1).

Our analysis left us with two admissible decision functions:

- d_1: guess 2-headed if the flip is heads, otherwise guess normal;
- d_2: guess normal regardless of the flip.

The values of the risk function (expected loss) for our two decision functions and Player B's two choices of coin are in the following table:

	d_1	d_2
HH	-1	1
TH	0	-1

1. What is the minimax strategy for Player A? Explain why this strategy isn't spyproof.

2. Find a randomized strategy for Player A that minimizes the maximum expected risk.

3. Suppose now that you have observed that Player B picks a 2-headed coin with probability p. Determine which values of p make the expected risk of d_1 lower than the expected risk for d_2.

Definition. The expected risk $E[R(d, \Theta)]$ (the expected value is taken over the values of Θ) is known as the **Bayes risk**. Minimizing this quantity is known as the **Bayes criterion**. Note that this requires prior knowledge of the distribution of Θ.

4. Suppose now that the game is modified and Player B flips the coin n times before we have to make our guess.

a) How many decision functions are there?

b) Explain why only 2 of the decision functions are admissible.

5. Now you must decide on the basis of one observation whether the parameter θ of a Bernoulli distribution is 0, 0.5, or 1. Your loss in dollars is 100 times the absolute value of your error.

a) Construct a payoff matrix showing the 9 values of your loss function.

b) List the 9 possible decision functions and construct a table showing the values of the corresponding risk function.

c) Eliminate inadmissible decision functions and show that the remaining decision functions are all equally good according the the minimax criterion.

d) Which decision function is best according to the Bayes criterion if the possible values of the parameter θ are equally likely?