Math 422 **Estimating θ** **February 5, 2018**

The point of these problems is to estimate the parameter θ for a population with a Bernoulli distribution (e.g. to estimate the percentage of voters who will vote for a given candidate, the percentage of Americans who support a given policy, etc.). We'll use a random sample $X_1, X_2, \ldots, X_n$ from our Bernoulli population. The values of the random sample are $x_1, x_2, \ldots, x_n$.

1. Find the maximum likelihood estimator for θ based on the random sample.

2. (Do this problem last) Determine if your estimator from problem 1 is a minimum variance, unbiased estimator for θ (assume the C-R inequality holds; use the Bernoulli pdf $f(x) = \theta^x(1-\theta)^{(1-x)}$ for $x = 0$ or 1).

3. Suppose that we know that θ can be seen as the value of a random variable Θ having a beta distribution. This gives us a prior distribution to use in Bayesian estimation.

a) Identify the distribution of $X = \sum_{i=1}^{n} X_i$.

b) Identify the posterior distribution of Θ given $X = x$.

c) Find the expected value of Θ given $X = x$.

d) Suppose now that $\alpha = 10$, $\beta = 30$, and $n = 40$. Use a computer to calculate $E(\Theta|x)$ for $x = 1, 2, \ldots, 20$. Compare with the prior expectation $E(\Theta)$ and the estimate given by your estimator from problem 1.