Math 422 **Hypothesis Tests** **March 7, 2018**

In class on Monday we started testing my hypothesis that taller people tend to sit at the back of the room. We divided all students into 2 populations: the front-sitters and the back-sitters. We then assumed that the 8 people in the front rows constituted a random sample from the population of front-sitters and the 6 people in the back row constituted a random sample from the population of back-sitters. This allowed us to test $H_0 : \mu_1 - \mu_2 = 0$ against $H_1 : \mu_1 - \mu_2 < 0$ (where μ_1 is the mean height of front-sitters and μ_2 is the mean height of back-sitters).

Assuming the 2 groups are random samples from 2 populations with the same variance allowed us to we use the pooled estimator

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_1 - 2}$$

and the test statistic

$$T = \frac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} - \frac{1}{n_2}}}$$

which has a t distribution with $n_1 + n_2 - 2$ degrees of freedom.

1. Out test statistic was $t = -0.81$. Calculate the P-value for the test. What conclusion should we draw?

2. Using the pooled estimator for the common variance of 2 populations is only justified if those populations actually have the same variance. Check to see if this is reasonable using $F = \frac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2}$, which has an F distribution with parameters $n_1 - 1$ and $n_2 - 1$ (order of these parameters matters), to test $H_0 : \sigma_1 = \sigma_2$ against $H_1 : \sigma_1 \neq \sigma_2$. Calculate the P-value for the test.

3. A different formulation of my original hypothesis is that taller people more often sit at the back of the class. Or maybe shorter people more often sit at the front. Formulate 2 hypothesis tests: one about the proportion of back-sitters who are taller than average (66 inches) and another about the proportion of front-sitters who are shorter than average. Use the data to test these hypothesis.

4. Another way of getting at my original hypothesis would be to consider paired data: if person A sits in front of person B then we record the difference in their heights. We can then formulate at test of my hypothesis as a test of $H_0 : \mu = 0$ (and heights of people sitting one in front of the other are independent) against $H_1 : \mu < 0$ where μ is the true mean difference in heights. Under H_0 the differences in heights should be normally distributed. Collect data from the class and test this hypothesis.