

DIRECTIONS: Answer the following 20 questions and mark your solution with an **X** on this answer sheet. Turn in the answer sheet and take the questions with you.

NAME:

1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e
15	a	b	c	d	e
16	a	b	c	d	e
17	a	b	c	d	e
18	a	b	c	d	e
19	a	b	c	d	e
20	a	b	c	d	e

1. If S is a plane in Euclidean 3-space containing $(0, 0, 0)$, $(2, 0, 0)$, and $(0, 0, 1)$, then S is the

- a) xy -plane
- b) xz -plane
- c) yz -plane
- d) plane $y - z = 0$
- e) plane $x + 2y - 2z = 0$

2. $\int_0^1 \int_0^x xy dy dx =$

- a) 0
- b) $\frac{1}{8}$
- c) $\frac{1}{3}$
- d) 1
- e) 3

3. All functions f defined on the xy -plane such that

$$\frac{\partial f}{\partial x} = 2x + y \text{ and } \frac{\partial f}{\partial y} = x + 2y$$

are given by $f(x, y) =$

- a) $x^2 + xy + y^2 + C$
- b) $x^2 + 2xy + y^2 + C$
- c) $x^2 - xy + y^2 + C$
- d) $x^2 - 2xy + y^2 + C$
- e) $x^2 - xy - y^2 + C$

4. For $x \geq 0$, $\frac{d}{dx}(x^e \cdot e^x) =$

- a) $x^e \cdot e^x + x^{e-1} \cdot e^{x+1}$
- b) $x^e \cdot e^x + x^{e+1} \cdot e^{x-1}$
- c) $x^e \cdot e^x$
- d) $x^{e-1} \cdot e^{x+1}$
- e) $x^{e+1} \cdot e^{x-1}$

5. $\sum_{n=1}^{\infty} \frac{n}{n+1} =$

- a) $\frac{1}{e}$
- b) $\log 2$
- c) 1
- d) e
- e) ∞

6. k digits are to be chosen at random (with repetitions allowed) from $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the probability that 0 will **not** be chosen?

- a) $\frac{1}{k}$
- b) $\frac{1}{10}$
- c) $\frac{k-1}{k}$
- d) $\left(\frac{1}{10}\right)^k$
- e) $\left(\frac{9}{10}\right)^k$

7. In order to send an undetected message to an agent in the field, each letter in the message is replaced by the number of its position in the alphabet and that number is entered in a matrix M . Thus, for example, "DEAD" becomes the matrix

$M = \begin{pmatrix} 4 & 5 \\ 1 & 4 \end{pmatrix}$. In order to further avoid detection, each message with four letters is sent to an agent encoded as MC ,

where $C = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$. If the agent receives the matrix $\begin{pmatrix} 51 & -3 \\ 31 & -8 \end{pmatrix}$, then the message is

- a) RUSH
- b) COME
- c) ROME
- d) CALL
- e) not uniquely determined by the information given

8. If $\sin^{-1} x = \frac{\pi}{6}$, then the acute angle value of $\cos^{-1} x$ is

- a) $\frac{5\pi}{6}$
- b) $\frac{\pi}{3}$
- c) $\sqrt{1 - \frac{\pi^2}{6^2}}$
- d) $1 - \frac{\pi}{6}$
- e) 0

9. $\int_0^\pi e^{\sin^2 x} e^{\cos^2 x} dx =$

- a) π
- b) $e\pi$
- c) e^π
- d) $e^{\sin^2 \pi}$
- e) $e^\pi - 1$

10. A newscast contained the statement that the total use of electricity in Spokane had declined in one billing period by 5 percent, while household use had declined by 4 percent, and all other uses had increased by 25 percent. Which of the following must be true about the billing period (or statement)?

- a) The statement was in error
- b) The ratio of all other uses to household use was $\frac{29}{1}$
- c) The ratio of all other uses to household use was $\frac{29}{16}$
- d) The ratio of all other uses to household use was $\frac{29}{19}$
- e) None of the above

11. If f is a linear transformation from the plane to the real numbers and if $f(1, 1) = 1$ and $f(-1, 0) = 2$, then $f(3, 5) =$

- a) -6
- b) -5
- c) 0
- d) 8
- e) 9

12. If for all $x > 0$, $f(\log x) = \sqrt{x}$, then $f(x) =$

- a) $e^{\frac{x}{2}}$
- b) $\log \sqrt{x}$
- c) $e^{\sqrt{x}}$
- d) $\sqrt{\log x}$
- e) $\frac{\log x}{2}$

13. $S(n)$ is a statement about positive integers n such that whenever $S(k)$ is true, $S(k+1)$ must also be true. Furthermore, there exists some positive integer n_0 such that $S(n_0)$ is **not** true. Of the following, which is the strongest conclusion that can be drawn?

- a) $S(n_0 + 1)$ is not true
- b) $S(n_0 - 1)$ is not true
- c) $S(n)$ is not true for any $n \leq n_0$
- d) $S(n)$ is not true for any $n \geq n_0$
- e) $S(n)$ is not true for any n

14. Let f be a function such that the graph of f is a semicircle S with end points $(a, 0)$ and $(b, 0)$ where $a < b$. Then

$$\left| \int_a^b f(x) dx \right| =$$

- a) $f(b) - f(a)$
- b) $\frac{f(b) - f(a)}{b - a}$
- c) $(b - a) \frac{\pi}{4}$
- d) $(b - a)^2 \pi$
- e) $(b - a)^2 \frac{\pi}{8}$

15. The dimension of the subspace spanned by the real vectors

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \\ 9 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

is

- a) 2
- b) 3
- c) 4
- d) 5
- e) 6

16. The shortest distance from the curve $xy = 8$ to the origin is

- a) 4
- b) 8
- c) 16
- d) $2\sqrt{2}$
- e) $4\sqrt{2}$

17. If M is the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, then M^{100} is

a) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

e) none of the above

18. If $f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, then $\int_{-1}^1 f(x)dx$ is

a) -2

b) 0

c) 2

d) not defined

e) none of the above

19. Suppose f is a real function such that $f'(x_0)$ exists. Which of the following is the value of $\lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0 - h)}{h}$?

a) 0

b) $2(f'(x_0))$

c) $f'(-x_0)$

d) $-f'(x_0)$

e) $-2f'(x_0)$

20. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{e^n}{n!} x^n$ is

a) 0

b) $\frac{1}{e}$

c) 1

d) e

e) ∞