7. $\int_{-3}^{3} |x+1|\, dx =$

(A) 0 (B) 5 (C) 10 (D) 15 (E) 20

---

8. What is the greatest possible area of a triangular region with one vertex at the center of a circle of radius 1 and the other two vertices on the circle?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) π (E) $\frac{1+\sqrt{2}}{4}$

$$J = \int_0^1 \sqrt{1-x^4}\, dx$$

$$K = \int_0^1 \sqrt{1+x^4}\, dx$$

$$L = \int_0^1 \sqrt{1-x^8}\, dx$$

9. Which of the following is true for the definite integrals shown above?

(A) $J < L < 1 < K$

(B) $J < L < K < 1$

(C) $L < J < 1 < K$

(D) $L < J < K < 1$

(E) $L < 1 < J < K$

GO ON TO THE NEXT PAGE.

13. A total of x feet of fencing is to form three sides of a level rectangular yard. What is the maximum possible area of the yard, in terms of x ?

(A) $\frac{x^2}{9}$ (B) $\frac{x^2}{8}$ (C) $\frac{x^2}{4}$ (D) x^2 (E) $2x^2$

---

14. What is the units digit in the standard decimal expansion of the number 7^{25} ?

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

---

15. Let f be a continuous real-valued function defined on the closed interval $[-2, 3]$. Which of the following is NOT necessarily true?

(A) f is bounded.

(B) $\int_{-2}^{3} f(t)\,dt$ exists.

(C) For each c between $f(-2)$ and $f(3)$, there is an $x \in [-2, 3]$ such that $f(x) = c$.

(D) There is an M in $f([-2, 3])$ such that $\int_{-2}^{3} f(t)\,dt = 5M$.

(E) $\lim_{h \to 0} \frac{f(h) - f(0)}{h}$ exists.

---

22. Let $C(\mathbb{R})$ be the collection of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Then $C(\mathbb{R})$ is a real vector space with pointwise addition and scalar multiplication defined by

$$(f+g)(x) = f(x) + g(x) \text{ and } (rf)(x) = rf(x)$$

for all $f, g \in C(\mathbb{R})$ and all $r, x \in \mathbb{R}$. Which of the following are subspaces of $C(\mathbb{R})$?

I. $\{f : f$ is twice differentiable and $f''(x) - 2f'(x) + 3f(x) = 0$ for all $x\}$

II. $\{g : g$ is twice differentiable and $g''(x) = 3g'(x)$ for all $x\}$

III. $\{h : h$ is twice differentiable and $h''(x) = h(x) + 1$ for all $x\}$

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

---

23. For what value of b is the line $y = 10x$ tangent to the curve $y = e^{bx}$ at some point in the xy-plane?

(A) $\frac{10}{e}$ (B) 10 (C) $10e$ (D) e^{10} (E) e

---

24. Let h be the function defined by $h(x) = \int_0^{x^2} e^{x+t}\,dt$ for all real numbers x. Then $h'(1) =$

(A) $e - 1$ (B) e^2 (C) $e^2 - e$ (D) $2e^2$ (E) $3e^2 - e$

GO ON TO THE NEXT PAGE.

27. Consider the two planes $x + 3y - 2z = 7$ and $2x + y - 3z = 0$ in $\mathbb{R}^3$. Which of the following sets is the intersection of these planes?

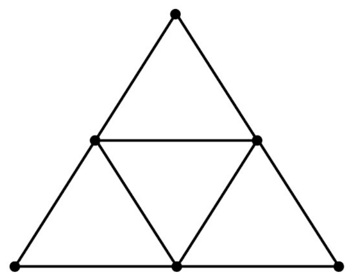
(A) $\varnothing$

(B) $\{(0, 3, 1)\}$

(C) $\{(x, y, z) : x = t,\ y = 3t,\ z = 7 - 2t,\ t \in \mathbb{R}\}$

(D) $\{(x, y, z) : x = 7t,\ y = 3 + t,\ z = 1 + 5t,\ t \in \mathbb{R}\}$

(E) $\{(x, y, z) : x - 2y - z = -7\}$



28. The figure above shows an undirected graph with six vertices. Enough edges are to be deleted from the graph in order to leave a spanning tree, which is a connected subgraph having the same six vertices and no cycles. How many edges must be deleted?

(A) One (B) Two (C) Three (D) Four (E) Five

---

33. The Euclidean algorithm is used to find the greatest common divisor (`gcd`) of two positive integers `a` and `b`.

```
input(a)
input(b)
while b > 0
  begin
    r := a mod b
    a := b
    b := r
  end
gcd := a
output(gcd)
```

When the algorithm is used to find the greatest common divisor of `a = 273` and `b = 110`, which of the following is the sequence of computed values for `r` ?

(A) `2, 26, 1, 0`

(B) `2, 53, 1, 0`

(C) `53, 2, 1, 0`

(D) `53, 4, 1, 0`

(E) `53, 5, 1, 0`

---

34. The minimal distance between any point on the sphere $(x-2)^2 + (y-1)^2 + (z-3)^2 = 1$ and any point on the sphere $(x+3)^2 + (y-2)^2 + (z-4)^2 = 4$ is

(A) 0 (B) 4 (C) $\sqrt{27}$ (D) $2(\sqrt{2}+1)$ (E) $3(\sqrt{3}-1)$

GO ON TO THE NEXT PAGE.

38. Let A and B be nonempty subsets of $\mathbb{R}$ and let $f : A \to B$ be a function. If $C \subseteq A$ and $D \subseteq B$, which of the following must be true?

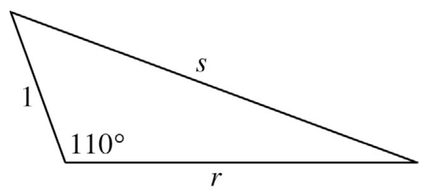
(A) $C \subseteq f^{-1}\left(f(C)\right)$

(B) $D \subseteq f\left(f^{-1}(D)\right)$

(C) $f^{-1}\left(f(C)\right) \subseteq C$

(D) $f^{-1}\left(f(C)\right) = f\left(f^{-1}(D)\right)$

(E) $f\left(f^{-1}(D)\right) = f^{-1}(D)$



39. In the figure above, as r and s increase, the length of the third side of the triangle remains 1 and the measure of the obtuse angle remains 110°. What is $\lim\limits_{\substack{s\to\infty \\ r\to\infty}} (s - r)$?

(A) 0

(B) A positive number less than 1

(C) 1

(D) A finite number greater than 1

(E) ∞

---

40. For which of the following rings is it possible for the product of two nonzero elements to be zero?

(A) The ring of complex numbers

(B) The ring of integers modulo 11

(C) The ring of continuous real-valued functions on $[0, 1]$

(D) The ring $\{a + b\sqrt{2} : a \text{ and } b \text{ are rational numbers}\}$

(E) The ring of polynomials in x with real coefficients

---

41. Let C be the circle $x^2 + y^2 = 1$ oriented counterclockwise in the xy-plane. What is the value of the line integral $\oint_C (2x - y)\,dx + (x + 3y)\,dy$?

(A) 0 (B) 1 (C) $\frac{\pi}{2}$ (D) π (E) 2π

---

42. Suppose X is a discrete random variable on the set of positive integers such that for each positive integer n, the probability that $X = n$ is $\frac{1}{2^n}$. If Y is a random variable with the same probability distribution and X and Y are independent, what is the probability that the value of at least one of the variables X and Y is greater than 3 ?

(A) $\frac{1}{64}$ (B) $\frac{15}{64}$ (C) $\frac{1}{4}$ (D) $\frac{3}{8}$ (E) $\frac{4}{9}$

---

43. If $z = e^{2\pi i/5}$, then $1 + z + z^2 + z^3 + 5z^4 + 4z^5 + 4z^6 + 4z^7 + 4z^8 + 5z^9 =$

(A) 0 (B) $4e^{3\pi i/5}$ (C) $5e^{4\pi i/5}$ (D) $-4e^{2\pi i/5}$ (E) $-5e^{3\pi i/5}$

---

44. A fair coin is to be tossed 100 times, with each toss resulting in a head or a tail. If H is the total number of heads and T is the total number of tails, which of the following events has the greatest probability?

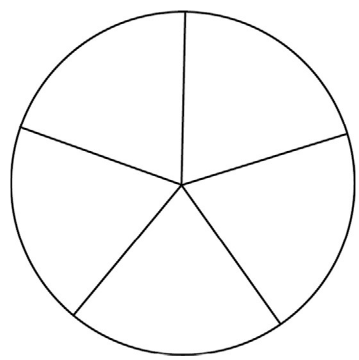
(A) $H = 50$

(B) $T \geq 60$

(C) $51 \leq H \leq 55$

(D) $H \geq 48$ and $T \geq 48$

(E) $H \leq 5$ or $H \geq 95$



45. A circular region is divided by 5 radii into sectors as shown above. Twenty-one points are chosen in the circular region, none of which is on any of the 5 radii. Which of the following statements must be true?

I. Some sector contains at least 5 of the points.

II. Some sector contains at most 3 of the points.

III. Some pair of adjacent sectors contains a total of at least 9 of the points.

(A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

GO ON TO THE NEXT PAGE.

49. Up to isomorphism, how many additive abelian groups G of order 16 have the property that $x + x + x + x = 0$ for each x in G?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

---

50. Let A be a real 2×2 matrix. Which of the following statements must be true?

I. All of the entries of A^2 are nonnegative.

II. The determinant of A^2 is nonnegative.

III. If A has two distinct eigenvalues, then A^2 has two distinct eigenvalues.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

---

51. If $\lfloor x \rfloor$ denotes the greatest integer not exceeding x, then $\int_0^{\infty} \lfloor x \rfloor e^{-x}\, dx =$

(A) $\dfrac{e}{e^2 - 1}$ (B) $\dfrac{1}{e - 1}$ (C) $\dfrac{e - 1}{e}$ (D) 1 (E) $+\infty$

---

52. If A is a subset of the real line $\mathbb{R}$ and A contains each rational number, which of the following must be true?

(A) If A is open, then $A = \mathbb{R}$.

(B) If A is closed, then $A = \mathbb{R}$.

(C) If A is uncountable, then $A = \mathbb{R}$.

(D) If A is uncountable, then A is open.

(E) If A is countable, then A is closed.

---

53. What is the minimum value of the expression $x + 4z$ as a function defined on $\mathbb{R}^3$, subject to the constraint $x^2 + y^2 + z^2 \leq 2$?

(A) 0 (B) -2 (C) $-\sqrt{34}$ (D) $-\sqrt{35}$ (E) $-5\sqrt{2}$

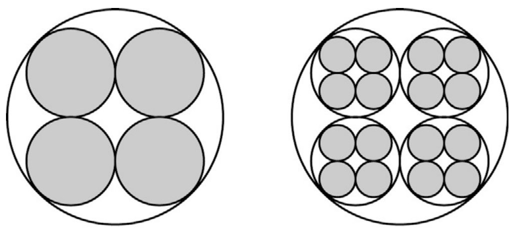


Figure 1 Figure 2

54. The four shaded circles in Figure 1 above are congruent and each is tangent to the large circle and to two of the other shaded circles. Figure 2 is the result of replacing each of the shaded circles in Figure 1 by a figure that is geometrically similar to Figure 1. What is the ratio of the area of the shaded portion of Figure 2 to the area of the shaded portion of Figure 1 ?

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{1+\sqrt{2}}$ (C) $\frac{4}{1+\sqrt{2}}$ (D) $\left(\frac{\sqrt{2}}{1+\sqrt{2}}\right)^2$ (E) $\left(\frac{2}{1+\sqrt{2}}\right)^2$

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58. Suppose A and B are $n \times n$ invertible matrices, where $n > 1$, and I is the $n \times n$ identity matrix. If A and B are similar matrices, which of the following statements must be true?

I. $A - 2I$ and $B - 2I$ are similar matrices.

II. A and B have the same trace.

III. A^{-1} and B^{-1} are similar matrices.

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

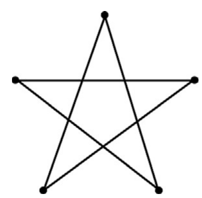
---

59. Suppose f is an analytic function of the complex variable $z = x + iy$ given by

$$f(z) = (2x + 3y) + ig(x, y),$$

where $g(x, y)$ is a real-valued function of the real variables x and y. If $g(2, 3) = 1$, then $g(7, 3) =$

(A) -14 (B) -9 (C) 0 (D) 11 (E) 18



60. The group of symmetries of the regular pentagram shown above is isomorphic to the

(A) symmetric group S_5

(B) alternating group A_5

(C) cyclic group of order 5

(D) cyclic group of order 10

(E) dihedral group of order 10
