3. If $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is invertible under matrix multiplication, then its inverse is

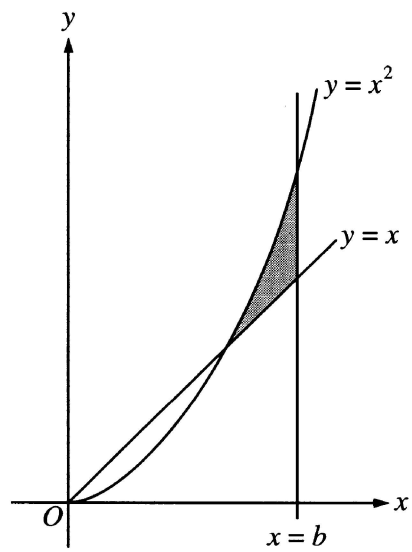
(A) $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(B) $\frac{1}{a^2 + b^2}\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(C) $\frac{1}{a^2 + b^2}\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(D) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(E) $\frac{1}{a^2 - b^2}\begin{pmatrix} -b & a \\ a & b \end{pmatrix}$



4. If $b > 0$ and if $\int_0^b x\,dx = \int_0^b x^2\,dx$, then the area of the shaded region in the figure above is

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

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16. For what value (or values) of m is the vector $(1, 2, m, 5)$ a linear combination of the vectors $(0, 1, 1, 1)$, $(0, 0, 0, 1)$, and $(1, 1, 2, 0)$?

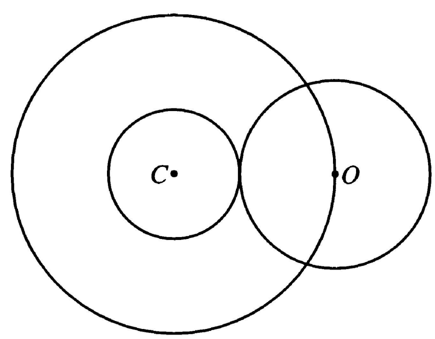
(A) For no value of m
(B) -1 only
(C) 1 only
(D) 3 only
(E) For infinitely many values of m

---

17. For a function f, the finite differences $\Delta f(x)$ and $\Delta^2 f(x)$ are defined by $\Delta f(x) = f(x+1) - f(x)$ and $\Delta^2 f(x) = \Delta f(x+1) - \Delta f(x)$. What is the value of $f(4)$, given the following partially completed finite difference table?

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	-1	4	
2		-2	6
3			
4			

(A) -5
(B) -1
(C) 1
(D) 3
(E) 5



18. In the figure above, the annulus with center C has inner radius r and outer radius 1. As r increases, the circle with center O contracts and remains tangent to the inner circle. If $A(r)$ is the area of the annulus and $a(r)$ is the area of the circular region with center O, then $\lim_{r \to 1^-} \frac{A(r)}{a(r)} =$

(A) 0
(B) $\frac{2}{\pi}$
(C) 1
(D) $\frac{\pi}{2}$
(E) ∞

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26. Let f be the function defined by

$$f(x) = \begin{cases} -x^2 + 4x - 2 & \text{if } x < 1, \\ -x^2 + 2 & \text{if } x \geq 1. \end{cases}$$

Which of the following statements about f is true?

(A) f has an absolute maximum at $x = 0$.
(B) f has an absolute maximum at $x = 1$.
(C) f has an absolute maximum at $x = 2$.
(D) f has no absolute maximum.
(E) f has local maxima at both $x = 0$ and $x = 2$.

---

27. Let f be a function such that $f(x) = f(1 - x)$ for all real numbers x. If f is differentiable everywhere, then $f'(0) =$

(A) $f(0)$
(B) $f(1)$
(C) $-f(0)$
(D) $f'(1)$
(E) $-f'(1)$

---

28. If V_1 and V_2 are 6-dimensional subspaces of a 10-dimensional vector space V, what is the smallest possible dimension that $V_1 \cap V_2$ can have?

(A) 0
(B) 1
(C) 2
(D) 4
(E) 6

---

29. Assume that p is a polynomial function on the set of real numbers. If $p(0) = p(2) = 3$ and $p'(0) = p'(2) = -1$, then $\int_0^2 xp''(x)\,dx =$

(A) -3
(B) -2
(C) -1
(D) 1
(E) 2

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30. Suppose B is a basis for a real vector space V of dimension greater than 1. Which of the following statements could be true?

(A) The zero vector of V is an element of B.
(B) B has a proper subset that spans V.
(C) B is a proper subset of a linearly independent subset of V.
(D) There is a basis for V that is disjoint from B.
(E) One of the vectors in B is a linear combination of the other vectors in B.

---

31. Which of the following CANNOT be a root of a polynomial in x of the form $9x^5 + ax^3 + b$, where a and b are integers?

(A) -9

(B) -5

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) 9

---

32. When 20 children in a classroom line up for lunch, Pat insists on being somewhere ahead of Lynn. If Pat's demand is to be satisfied, in how many ways can the children line up?

(A) 20!

(B) 19!

(C) 18!

(D) $\frac{20!}{2}$

(E) $20 \cdot 19$

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33. How many integers from 1 to 1,000 are divisible by 30 but not by 16 ?

(A) 29
(B) 31
(C) 32
(D) 33
(E) 38

---

34. Suppose f is a differentiable function for which $\lim_{x \to \infty} f(x)$ and $\lim_{x \to \infty} f'(x)$ both exist and are finite. Which of the following must be true?

(A) $\lim_{x \to \infty} f'(x) = 0$

(B) $\lim_{x \to \infty} f''(x) = 0$

(C) $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} f'(x)$

(D) f is a constant function.

(E) f' is a constant function.

---

35. In xyz-space, an equation of the plane tangent to the surface $z = e^{-x} \sin y$ at the point where $x = 0$ and $y = \frac{\pi}{2}$ is

(A) $x + y = 1$
(B) $x + z = 1$
(C) $x - z = 1$
(D) $y + z = 1$
(E) $y - z = 1$

---

36. For each real number x, let $\mu(x)$ be the mean of the numbers 4, 9, 7, 5, and x; and let $\eta(x)$ be the median of these five numbers. For how many values of x is $\mu(x) = \eta(x)$?

(A) None
(B) One
(C) Two
(D) Three
(E) Infinitely many

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39. Consider the function f defined by $f(x) = e^{-x}$ on the interval $[0, 10]$. Let $n > 1$ and let $x_0, x_1, \ldots, x_n$ be numbers such that $0 = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = 10$. Which of the following is greatest?

(A) $\sum_{j=1}^{n} f(x_j)(x_j - x_{j-1})$

(B) $\sum_{j=1}^{n} f(x_{j-1})(x_j - x_{j-1})$

(C) $\sum_{j=1}^{n} f\left(\frac{x_j + x_{j-1}}{2}\right)(x_j - x_{j-1})$

(D) $\int_0^{10} f(x)\,dx$

(E) 0

---

40. A fair coin is to be tossed 8 times. What is the probability that more of the tosses will result in heads than will result in tails?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{87}{256}$

(D) $\frac{23}{64}$

(E) $\frac{93}{256}$

---

41. The function $f(x, y) = xy - x^3 - y^3$ has a relative maximum at the point

(A) $(0, 0)$

(B) $(1, 1)$

(C) $(-1, -1)$

(D) $(1, 3)$

(E) $\left(\frac{1}{3}, \frac{1}{3}\right)$

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52. Consider the following system of linear equations over the real numbers, where x, y, and z are variables and b is a real constant.

$$\begin{aligned} x + y + z &= 0 \\ x + 2y + 3z &= 0 \\ x + 3y + bz &= 0 \end{aligned}$$

Which of the following statements are true?

I. There exists a value of b for which the system has no solution.
II. There exists a value of b for which the system has exactly one solution.
III. There exists a value of b for which the system has more than one solution.

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

---

53. In the complex plane, let C be the circle $|z| = 2$ with positive (counterclockwise) orientation. Then

$$\int_C \frac{dz}{(z-1)(z+3)^2} =$$

(A) 0

(B) $2\pi i$

(C) $\frac{\pi i}{2}$

(D) $\frac{\pi i}{8}$

(E) $\frac{\pi i}{16}$

---

54. The inside of a certain water tank is a cube measuring 10 feet on each edge and having vertical sides and no top. Let $h(t)$ denote the water level, in feet, above the floor of the tank at time t seconds. Starting at time $t = 0$, water pours into the tank at a constant rate of 1 cubic foot per second, and simultaneously, water is removed from the tank at a rate of $0.25h(t)$ cubic feet per second. As $t \to \infty$, what is the limit of the volume of the water in the tank?

(A) 400 cubic feet
(B) 600 cubic feet
(C) 1,000 cubic feet
(D) The limit does not exist.
(E) The limit exists, but it cannot be determined without knowing $h(0)$.

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