Math 499 Practice exam II November 21, 2017

Directions: Take 1 hour to answer the 20 questions below. Recall that all logarithms are natural logarithms unless a base is specified. Mark your solution on this answer sheet with an **X** or by highlighting your answer. Return your answer sheet to Herak 307A or email it to axon@gonzaga.edu.

Name:

1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e
15	a	b	c	d	e
16	a	b	c	d	e
17	a	b	c	d	e
18	a	b	c	d	e
19	a	b	c	d	e
20	a	b	c	d	e

1. If $f(g(x)) = 5$ and $f(x) = x + 3$ for all real x, then $g(x) =$

a) $x - 3$

b) $3 - x$

c) $\frac{5}{x+3}$

d) 2

e) 8

2. $\lim\limits_{x \to 0} \dfrac{\tan x}{\cos x} =$

a) $-\infty$

b) -1

c) 0

d) 1

e) $+\infty$

3. Let $A - B$ denote $\{x \in A : x \notin B\}$. If $(A - B) \cup B = A$, which of the following must be true?

a) B is empty

b) $A \subseteq B$

c) $B \subseteq A$

d) $(B - A) \cup A = B$

e) None of the above

4. For what value of b is the value of $\int_b^{b+1} (x^2 + x)dx$ a minimum?

a) 0

b) -1

c) -2

d) -3

e) -4

5. In how many of the eight standard octants of xyz-space does the graph of $z = e^{x+y}$ appear?

a) One

b) Two

c) Three

d) Four

e) Eight

6. Suppose that the function f is defined on an interval by the formula $f(x) = \sqrt{\tan^2 x - 1}$. If f is continuous, which of the following intervals could be its domain?

a) $\left(\frac{3\pi}{4}, \pi\right)$

b) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

c) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

d) $\left(-\frac{\pi}{4}, 0\right)$

e) $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right)$

7. If $f''(x) = f'(x)$ for all real x, and if $f(0) = 0$ and $f'(0) = -1$, then $f(x) =$

a) $1 - e^x$

b) $e^x - 1$

c) $e^{-x} - 1$

d) e^{-x}

e) $-e^x$

8. Which of the following double integrals represents the volume of the solid bounded above by the graph of $z = 6 - x^2 - 2y^2$ and bounded below by the graph of $z = -2 + x^2 + 2y^2$?

a) $4 \int_0^2 \int_0^{\sqrt{2}} (8 - 2x^2 - 4y^2)\, dy\, dx$

b) $\int_{-2}^2 \int_{-\sqrt{(4-x^2)/2}}^{\sqrt{(4-x^2)/2}} (8 - 2x^2 - 4y^2)\, dy\, dx$

c) $4 \int_0^2 \int_{-\sqrt{4-2y^2}}^{-\sqrt{4-2y^2}} 1\, dx\, dy$

d) $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-2}^2 (8 - 2x^2 - 4y^2)\, dx\, dy$

e) $2 \int_0^{\sqrt{2}} \int_0^{\sqrt{4-2y^2}} (8 - 2x^2 - 4y^2)\, dx\, dy$

9. If A and B are events in a probability space such that $0 < P(A) = P(B) = P(A \cap B) < 1$, which of the following CANNOT be true?

a) A and B are independent

b) $A \cap B = A \cup B$

c) A is a proper subset of B

d) $P(A)P(B) < P(A \cap B)$

e) $A \neq B$

10. Let f be a real-valued function with domain $[0, 1]$. If there is some $K > 0$ such that $f(x) - f(y) \leq K|x - y|$ for all x and y in $[0, 1]$, which of the following must be true?

a) f is discontinuous at each point of $(0, 1)$

b) f is not continuous on $(0, 1)$, but is discontinuous at only countably many points of $(0, 1)$

c) f is continuous on $(0, 1)$, but is differentiable at only countably many points of $(0, 1)$

d) f is continuous on $(0, 1)$, but may not be differentiable on $(0, 1)$

e) f is differentiable on $(0, 1)$

11. Let $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, and $\mathbf{k} = (0, 0, 1)$. The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal if $\mathbf{v}_1 = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{v}_2 =$

a) $\mathbf{i} + \mathbf{j} - \mathbf{k}$

b) $\mathbf{i} - \mathbf{j} + \mathbf{k}$

c) $\mathbf{i} + \mathbf{k}$

d) $\mathbf{j} - \mathbf{k}$

e) $\mathbf{i} + \mathbf{j}$

12. If the curve in the yz-plane with equation $z = f(y)$ is rotated about the y-axis, an equation of the resulting surface of revolution is

a) $x^2 + z^2 = [f(y)]^2$

b) $x^2 + z^2 = f(y)$

c) $x^2 + z^2 = |f(y)|$

d) $y^2 + z^2 = |f(y)|$

e) $y^2 + z^2 = [f(x)]^2$

13. Let A and B be subspaces of a vector space V. Which of the following must be subspaces of V?

I. $A + B = \{a + b : \ a \in A \text{ and } b \in B\}$

II. $A \cup B$

III. $A \cap B$

IV. $\{x \in V : \ x \notin A\}$

a) I and II only

b) I and III only

c) III and IV only

d) I, II, and III only

e) I, II, III, and IV

14. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{2^k} \right) =$

a) 0

b) 1

c) 2

d) 4

e) $+\infty$

15. if $f(x_1, x_2, \ldots, x_n) = \sum_{1\le i<j\le n} x_i x_j$, then $\frac{\partial f}{\partial x_n} =$

a) $n!$

b) $\sum_{1\le i<j<n} x_i x_j$

c) $\sum_{1\le i<j<n} (x_i + x_j)$

d) $\sum_{j=1}^{n} x_j$

e) $\sum_{j=1}^{n-1} x_j$

16. If $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, then the set of all vectors X for which $AX = X$ is

a) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \,\middle|\, a = 0 \text{ and } b \text{ is arbitrary} \right\}$

b) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \,\middle|\, a \text{ is arbitrary and } b = 0 \right\}$

c) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \,\middle|\, a = -b \text{ and } b \text{ is arbitrary} \right\}$

d) $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

e) the empty set

17. If n apples, no two of the same weight, are lined up at random on a table, what is the probability that they are lined up in order of increasing weight from left to right?

a) $\frac{1}{2}$

b) $\frac{1}{n}$

c) $\frac{1}{n!}$

d) $\frac{1}{2^n}$

e) $\left(\frac{1}{n}\right)^n$

18. $\frac{d}{dx}\int_0^{x^2} e^{-t^2}\,dt =$

a) e^{-x^2}

b) $2e^{-x^2}$

c) $2e^{-x^4}$

d) $x^2e^{-x^2}$

e) $2xe^{-x^4}$

19. An automorphism ϕ of a field F is a one-to-one mapping of F onto itself such that $\phi(a+b) = \phi(a) + \phi(b)$ and $\phi(ab) = \phi(a)\phi(b)$ for all $a, b \in F$. If F is the field of rational numbers, then the number of distinct automorphisms of F is

a) 0

b) 1

c) 2

d) 4

e) infinite

20. Let $r > 0$ and let C be the circle $|z| = r$ in the complex plane. If P is a polynomial function, then $\int_C P(z)\,dz =$

a) 0

b) πr^2

c) $2\pi i$

d) $2\pi P(0)i$

e) $P(r)$