MATH 499 PRACTICE EXAM OCTOBER 11, 2017

DIRECTIONS: Take 1 hour to answer the following 20 questions. Mark your solution with an **X** on this answer sheet. Return your answer sheet to Herak 307A and take the questions with you. Presentations next class will be based on these problems.

NAME:

| | | | | | |
|---|---|---|---|---|---|
| 1 | a | b | c | d | e |
| 2 | a | b | c | d | e |
| 3 | a | b | c | d | e |
| 4 | a | b | c | d | e |
| 5 | a | b | c | d | e |
| 6 | a | b | c | d | e |
| 7 | a | b | c | d | e |
| 8 | a | b | c | d | e |
| 9 | a | b | c | d | e |
| 10 | a | b | c | d | e |
| 11 | a | b | c | d | e |
| 12 | a | b | c | d | e |
| 13 | a | b | c | d | e |
| 14 | a | b | c | d | e |
| 15 | a | b | c | d | e |
| 16 | a | b | c | d | e |
| 17 | a | b | c | d | e |
| 18 | a | b | c | d | e |
| 19 | a | b | c | d | e |
| 20 | a | b | c | d | e |

**1.** The function $f$ is differentiable on the interval $(0, 4)$. If $f(1) = 1$ and $f(3) = 7$, then there is at least one $c$ in $(1, 3)$ such that $f'(c) =$

a) $-1$

b) $0$

c) $1$

d) $2$

e) $3$

**2.** In the $xy$-plane, the line tangent to the graph of $x^2 + xy + y^2 = 3$ at the point $(1, 1)$ has a slope of

a) $-3$

b) $-1$

c) $0$

d) $\frac{1}{3}$

e) $1$

**3.** What are the eigenvalues of $\begin{pmatrix} 6 & -3 \\ 1 & 2 \end{pmatrix}$?

a) 1 and 15

b) 2 and 6

c) 3 and 5

d) $\frac{5}{2} + \frac{i\sqrt{15}}{2}$ and $\frac{5}{2} - \frac{i\sqrt{15}}{2}$

e) $4 + i$ and $4 - i$

**4.** What is the area of the portion of the surface $z = x^2 + y^2$ lying inside the cylinder $x^2 + y^2 = 4$ in $xyz$-space?

a) $21\pi$

b) $\frac{21\pi}{2}$

c) $\frac{\pi}{3}\left(17^{\frac{3}{2}}\right)$

d) $\frac{\pi}{2}\left(17^{\frac{3}{2}} - 1\right)$

e) $\frac{\pi}{6}\left(17^{\frac{3}{2}} - 1\right)$

**5.** If $S$ is a plane in Euclidean 3-space containing the points $(0,0,0)$, $(2,0,0)$, and $(0,0,1)$, then $S$ is the

a) $xy$-plane

b) $xz$-plane

c) $yz$-plane

d) plane $y - z = 0$

e) plane $x + 2y - 2z = 0$

**6.** $\int_0^1 \int_0^x xy \, dy \, dx =$

a) 0

b) $\frac{1}{8}$

c) $\frac{1}{3}$

d) 1

e) 3

**7.** All functions $f$ defined on the $xy$-plane such that $\dfrac{\partial f}{\partial x} = 2x + y$ and $\dfrac{\partial f}{\partial y} = x + 2y$ are given by $f(x,y) =$

a) $x^2 + xy + y^2 + C$

b) $x^2 + 2xy + y^2 + C$

c) $x^2 - xy + y^2 + C$

d) $x^2 - 2xy + y^2 + C$

e) $x^2 - xy - y^2 + C$

**8.** For $x \geq 0$, $\dfrac{d}{dx}\left[(x^e)(e^x)\right] =$

a) $(x^e)(e^x) + (x^{e-1})(e^{x+1})$

b) $(x^e)(e^x) + (x^{e+1})(e^{x-1})$

c) $(x^e)(e^x)$

d) $(x^{e-1})(e^{x+1})$

e) $(x^{e+1})(e^{x-1})$

**9.** $k$ digits are to be chosen at random and with repetitions allowed from $\{0,1,2,3,4,5,6,7,8,9\}$. What is the probability that 0 will **not** be chosen?

a) $\dfrac{1}{k}$

b) $\dfrac{1}{10}$

c) $\dfrac{k-1}{k}$

d) $\left(\dfrac{1}{10}\right)^k$

e) $\left(\dfrac{9}{10}\right)^k$

**10.** In order to send an undetected message to an agent in the field, each letter in the message is replaced by the number of its position in the alphabet and that number is entered in a matrix $M$. Thus, for example, "DEAD" becomes the matrix $M = \begin{pmatrix} 4 & 5 \\ 1 & 4 \end{pmatrix}$. In order to further avoid detection, each message with four letters is sent to an agent encoded as $MC$, where $C = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$. If the agent receives the matrix $\begin{pmatrix} 51 & -3 \\ 31 & -8 \end{pmatrix}$, then the message is

a) RUSH

b) COME

c) ROME

d) CALL

e) not uniquely determined by the information given

**11.** If $\sin^{-1} x = \frac{\pi}{6}$, then the acute angle value of $\cos^{-1} x$ is

a) $\dfrac{5\pi}{6}$

b) $\dfrac{\pi}{3}$

c) $\sqrt{1 - \dfrac{\pi^2}{6^2}}$

d) $1 - \dfrac{\pi}{6}$

e) 0

**12.** A newscast contained the statement that the total use of electricity in Spokane had declined in one billing period by 5 percent, while household use had declined by 4 percent, and all other uses had increased by 25 percent. Which of the following must be true?

a) The statement was in error

b) The ratio of all other uses to household use was $\frac{29}{1}$

c) The ratio of all other uses to household use was $\frac{29}{16}$

d) The ratio of all other uses to household use was $\frac{29}{19}$

e) None of the above

**13.** If $f : \mathbb{R}^2 \to \mathbb{R}$ is a linear transformation from the plane to the real numbers and if $f(1,1) = 1$ and $f(-1,0) = 2$, then $f(3,5) =$

a) $-6$

b) $-5$

c) $0$

d) $8$

e) $9$

**14.** $S(n)$ is a statement about positive integers $n$ such that whenever $S(k)$ is true, $S(k+1)$ must also be true. Furthermore, there exists some positive integer $n_0$ such that $S(n_0)$ is **not** true. Of the following, which is the strongest conclusion that can be drawn?

a) $S(n_0 + 1)$ is not true

b) $S(n_0 - 1)$ is not true

c) $S(n)$ is not true for any $n \leq n_0$

d) $S(n)$ is not true for any $n \geq n_0$

e) $S(n)$ is not true for any n

**15.** Let $f$ be a function such that the graph of $f$ is a semicircle with end points $(a,0)$ and $(b,0)$ where $a < b$. Then $\left|\int_a^b f(x)dx\right| =$

a) $f(b) - f(a)$

b) $\dfrac{f(b) - f(a)}{b - a}$

c) $(b-a)\dfrac{\pi}{4}$

d) $(b-a)^2\pi$

e) $(b-a)^2\dfrac{\pi}{8}$

**16.** The dimension of the subspace spanned by the real vectors $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ 0 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -2 \\ 0 \\ 9 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ is

a) 2

b) 3

c) 4

d) 5

e) 6

**17.** If $M$ is the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, then $M^{100}$ is

a) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

e) none of the above

**18.** If $f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, then $\int_{-1}^{1} f(x)dx$ is

a) $-2$

b) 0

c) 2

d) not defined

e) none of the above

**19.** The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{e^n}{n!} x^n$ is

a) 0

b) $\frac{1}{e}$

c) 1

d) $e$

e) $\infty$

**20.** $\int_0^{\pi} e^{\sin^2 x} e^{\cos^2 x} dx =$

a) $\pi$

b) $e\pi$

c) $e^{\pi}$

d) $e^{\sin^2 \pi}$

e) $e^{\pi} - 1$