Directions: Spend 1 hour (60 minutes) working on the problems here. You may use blank scratch paper, but may not use any notes, books, or electronic devices. Mark each answer choice clearly on this cover sheet (which you should print ahead of time if you won't be working electronically). When you're done, submit this cover sheet in Gradescope (or give the paper copy to Dr. Axon).

Name:

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E

1. For which value of b is the value of $\int_b^{b+1}(x^2+x)\,dx$ a minimum?
   (a) 0
   (b) -1
   (c) -2
   (d) -3
   (e) -4

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2. Which of the following double integrals represents the volume of the solid bounded above by the graph of $z = 6 - x^2 - 2y^2$ and bounded below by the graph of $z = -2 + x^2 + 2y^2$?
   (a) $4\int_{x=0}^{x=2}\int_{y=0}^{y=\sqrt{2}}(8-2x^2-4y^2)\,dy\,dx$
   (b) $\int_{x=-2}^{x=2}\int_{y=-\sqrt{(4-x^2)/2}}^{y=\sqrt{(4-x^2)/2}}(8-2x^2-4y^2)\,dy\,dx$
   (c) $4\int_{y=0}^{y=\sqrt{2}}\int_{x=-\sqrt{4-2y^2}}^{x=\sqrt{4-2y^2}}dx\,dy$
   (d) $\int_{y=-\sqrt{2}}^{y=\sqrt{2}}\int_{x=-2}^{x=2}(8-2x^2-4y^2)\,dx\,dy$
   (e) $2\int_{y=0}^{y=\sqrt{2}}\int_{x=0}^{x=\sqrt{4-2y^2}}(8-2x^2-4y^2)\,dx\,dy$

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3. Let a be a number in the interval $[0,1]$ and let f be a function defined on $[0,1]$ by

$$f(x) = \begin{cases} a^2 & \text{if } 0 \le x \le a, \\ ax & \text{otherwise.} \end{cases}$$

   Then the value of a for which $\int_0^1 f(x)\,dx = 1$ is
   (a) $\frac{1}{4}$
   (b) $\frac{1}{3}$
   (c) $\frac{1}{2}$
   (d) 1
   (e) nonexistent

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4. If the curve in the yz-plane with equation $z = f(y)$ is rotated around the y-axis, an equation of the resulting surface of revolution is

   (a) $x^2 + z^2 = [f(y)]^2$

   (b) $x^2 + z^2 = f(y)$

   (c) $x^2 + z^2 = |f(y)|$

   (d) $y^2 + z^2 = |f(y)|$

   (e) $y^2 + z^2 = [f(x)]^2$

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5. If n apples, no two of the same weight, are lined up at random on a table, what is the probability that they are lined up in order of increasing weight from left to right?

   (a) $\dfrac{1}{2}$

   (b) $\dfrac{1}{n}$

   (c) $\dfrac{1}{n!}$

   (d) $\dfrac{1}{2^n}$

   (e) $\left(\dfrac{1}{n}\right)^n$

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6. If F is a function such that, for all positive integers x and y, $F(x, 1) = x + 1$, $F(1, y) = 2y$, and $F(x + 1, y + 1) = F(F(x, y + 1), y)$, then $F(2, 2) =$

   (a) 8

   (b) 7

   (c) 6

   (d) 5

   (e) 4

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7. If f and g are real-valued differentiable functions and if $f'(x) \geq g'(x)$ for all x in the closed interval $[0, 1]$, which of the following must be true?
   (a) $f(0) \geq g(0)$
   (b) $f(1) \geq g(1)$
   (c) $f(1) - g(1) \geq f(0) - g(0)$
   (d) $f - g$ has no maximum on $[0, 1]$
   (e) $\frac{f}{g}$ is a nondecreasing function on $[0, 1]$

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8. Consider the following procedure for determining whether a given name appears in an alphabetized list of n names.

   Step 1: Choose the name at the middle of the list (if $n = 2k$, choose the kth name); if that is the given name, you are done; if the list is only one name long, you are done. If you are not done, go to Step 2.

   Step 2: If the given name comes alphabetically before the name at the middle of the list, apply Step 1 to the first half of the list; otherwise, apply Step 1 to the second half of the list.

   If n is very large, the maximum number of steps required by this procedure is close to
   (a) n
   (b) n^2
   (c) $\log_2 n$
   (d) $n \log_2 n$
   (e) $n^2 \log_2 n$

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9. A city has square city blocks formed by a grid of north-south and east-west streets. One automobile route from City Hall to the main firehouse is to go exactly 5 blocks east and 7 blocks north. How many different routes from City Hall to the main firehouse traverse exactly 12 city blocks?
   (a) $5 \cdot 7$
   (b) $\frac{7!}{5!}$
   (c) $\frac{12!}{7!\,5!}$
   (d) 2^{12}
   (e) $7!\,5!$

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10. For all real numbers x and y, the expression $\frac{x+y+|x-y|}{2}$ is equal to
   (a) the maximum of x and y
   (b) the minimum of x and y
   (c) $|x+y|$
   (d) the average of $|x|$ and $|y|$
   (e) the average of $|x+y|$ and $x-y$

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11. A drawer contains 2 blue, 4 red, and 2 yellow socks. If 2 socks are to be randomly selected from the drawer, what is the probability that they will be the same color?
   (a) $\frac{2}{7}$
   (b) $\frac{2}{5}$
   (c) $\frac{3}{7}$
   (d) $\frac{1}{2}$
   (e) $\frac{3}{5}$

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12. In the Euclidean plane, point A is on a circle centered at point O, and O is on a circle centered at A. The circles intersect at points B and C. What is the measure of angle BAC?
    (a) 60°
    (b) 90°
    (c) 120°
    (d) 135°
    (e) 150°

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13. When 20 children in a classroom line up for lunch, Pat insists on being somewhere ahead of Lynn. If Pat's demand is to be satisfied, in how many ways can the children line up?
    (a) $20!$
    (b) $19!$
    (c) $18!$
    (d) $\frac{20!}{2}$
    (e) $20 \cdot 19$

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14. How many integers from 1 to $1,000$ are divisible by 30 but not by 16?
    (a) 29
    (b) 31
    (c) 32
    (d) 33
    (e) 38

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15. Suppose f is a differentiable function for which $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} f'(x)$ both exist and are finite. Which of the following must be true?

   (a) $\lim_{x\to\infty} f'(x) = 0$

   (b) $\lim_{x\to\infty} f''(x) = 0$

   (c) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} f'(x)$

   (d) f is a constant function

   (e) f' is a constant function

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16. Which of the following integrals on the inverval $\left[0, \frac{\pi}{4}\right]$ has the greatest value?

   (a) $\int_0^{\pi/4} \sin t\, dt$

   (b) $\int_0^{\pi/4} \cos t\, dt$

   (c) $\int_0^{\pi/4} \cos^2 t\, dt$

   (d) $\int_0^{\pi/4} \cos 2t\, dt$

   (e) $\int_0^{\pi/4} \sin t \cos t\, dt$

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17. A fair coin is to be tossed 8 times. What is the probability that more of the tosses will result in heads than will result in tails?

   (a) $\frac{1}{4}$

   (b) $\frac{1}{3}$

   (c) $\frac{87}{256}$

   (d) $\frac{23}{64}$

   (e) $\frac{93}{256}$

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18. An experimental car is found to have a fuel efficiency $E(v)$, in miles per gallon of fuel, where v is the speed of the car, in miles per hour. For a certain 4-hour trip, if $v = v(t)$ is the speed of the car t hours after the trip started, which of the following integrals represents the number of gallons of fuel that the car used on the trip?

(a) $\displaystyle\int_0^4 \frac{v(t)}{E(v(t))}\,dt$

(b) $\displaystyle\int_0^4 \frac{E(v(t))}{v(t)}\,dt$

(c) $\displaystyle\int_0^4 \frac{t\,v(t)}{E(v(t))}\,dt$

(d) $\displaystyle\int_0^4 \frac{t\,E(v(t))}{v(t)}\,dt$

(e) $\displaystyle\int_0^4 v(t)E(v(t))\,dt$

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19. How many continuous real-valued functions f are there with domain $[-1,1]$ such that $(f(x))^2 = x^2$ for each x in $[-1,1]$?

(a) One

(b) Two

(c) Three

(d) Four

(e) Infinitely many

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20. The inside of a certain water tank is a cube measuring 10 feet on each edge and having vertical sides and no top. Let $h(t)$ denote the water level, in feet, above the floor of the tank at time t seconds. Starting at time $t = 0$, water pours into the tank at a constant rate of 1 cubic foot per second, and simultaneously, water is removed from the tank at a rate of $0.25h(t)$ cubic feet per second. As $t \to \infty$, what is the limit of the volume of the water in the tank?

(a) 400 cubic feet

(b) 600 cubic feet

(c) $1,000$ cubic feet

(d) The limit does not exist.

(e) The limit exists, but it cannot be determined without knowing $h(0)$.
