Directions: Spend 1 hour (60 minutes) working on the problems here. You may use blank scratch paper, but may not use any notes, books, or electronic devices. Mark each answer choice clearly on this cover sheet (which you should print ahead of time if you won't be working electronically). When you're done, submit this cover sheet in Gradescope (or give the paper copy to Dr. Axon).

Name:

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E

1. Which of the following is an equation of the line tangent to the graph of $y = x + e^x$ at $x = 0$?
   (a) $y = x$
   (b) $y = x + 1$
   (c) $y = x + 2$
   (d) $y = 2x$
   (e) $y = 2x + 1$

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2. If V and W are 2-dimensional subspaces of $\mathbb{R}^4$, what are the possible dimensions of the subspace $V \cap W$?
   (a) 1 only
   (b) 2 only
   (c) 0 and 1 only
   (d) 0, 1, and 2 only
   (e) 0, 1, 2, 3, and 4

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3. Let k be the number of real solutions of the equation $e^x + x - 2 = 0$ in the interval $[0, 1]$, and let n be the number of real solutions that are not in $[0, 1]$. Which of the following is true?
   (a) $k = 0$ and $n = 1$
   (b) $k = 1$ and $n = 0$
   (c) $k = n = 1$
   (d) $k > 1$
   (e) $n > 1$

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4. $\displaystyle\int_{-3}^{3} |x + 1| \, dx =$
   (a) 0
   (b) 5
   (c) 10
   (d) 15
   (e) 20

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5. What is the greatest possible area of a triangular region with one vertex at the center of a circle of radius 1 and the other two vertices on the circle?
   (a) $\frac{1}{2}$
   (b) 1
   (c) $\sqrt{2}$
   (d) π
   (e) $\frac{1 + \sqrt{2}}{4}$

6. Let A be a 2×2 matrix for which there is a constant k such that the sum of the entries in each row and each column is k. Which of the following must be an eigenvector of A?

I. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ II. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ III. $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(a) I only
(b) II only
(c) III only
(d) I and II only
(e) I, II, and III

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7. A total of x feet of fencing is to form three sides of a level rectangular yard. What is the maximum possible area of the yard, in terms of x?

(a) $\frac{x^2}{9}$
(b) $\frac{x^2}{8}$
(c) $\frac{x^2}{4}$
(d) x^2
(e) $2x^2$

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8. What is the units digit in the standard decimal expansion of the number 7^{25}?

(a) 1
(b) 3
(c) 5
(d) 7
(e) 9

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9. Let f be a continuous real-valued function defined on the closed interval $[-2, 3]$. Which of the following is NOT necessarily true?

(a) f is bounded.
(b) $\int_{-2}^{3} f(t)\,dt$ exists.
(c) For each c between $f(-2)$ and $f(3)$, there is an $x \in [-2, 3]$ such that $f(x) = c$.
(d) There is an M in $f([-2, 3])$ such that $\int_{-2}^{3} f(t)\,dt = 5M$.
(e) $\lim_{h \to 0} \frac{f(h) - f(0)}{h}$ exists.

10. How many real roots does the polynomial $2x^5 + 8x - 7$ have?

   (a) None

   (b) One

   (c) Two

   (d) Three

   (e) Five

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11. Let V be the real vector space of all real 2×3 matrices, and let W be the real vector space of all real 4×1 column vectors. If T is a linear transformation from V onto W, what is the dimension of the subspace $\{\mathbf{v} \in V : T(\mathbf{v}) = \mathbf{0}\}$?

   (a) 2

   (b) 3

   (c) 4

   (d) 5

   (e) 6

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12. Let f and g be twice-differentiable real-valued functions defined on $\mathbb{R}$. If $f'(x) > g'(x)$ for all $x > 0$, which of the following inequalities must be true for all $x > 0$?

   (a) $f(x) > g(x)$

   (b) $f''(x) > g''(x)$

   (c) $f(x) - f(0) > g(x) - g(0)$

   (d) $f'(x) - f'(0) > g'(x) - g'(0)$

   (e) $f''(x) - f''(0) > g''(x) - g''(0)$

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13. Let P_1 be the set of all primes, $\{2, 3, 5, 7, \ldots\}$, and for each integer n, let P_n be the set of all prime multiples of n, $\{2n, 3n, 5n, 7n, \ldots\}$. Which of the following intersections is nonempty?

   (a) $P_1 \cap P_{23}$

   (b) $P_7 \cap P_{21}$

   (c) $P_{12} \cap P_{20}$

   (d) $P_{20} \cap P_{24}$

   (e) $P_5 \cap P_{25}$

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14. For what value of b is the line $y = 10x$ tangent to the curve $y = e^{bx}$ at some point in the xy-plane?

   (a) $\frac{10}{e}$

   (b) 10

   (c) $10e$

   (d) e^{10}

   (e) e

15. Let h be the function defined by $h(x) = \displaystyle\int_0^{x^2} e^{x+t}\,dt$ for all real numbers x. Then $h'(1) =$

(a) $e - 1$

(b) e^2

(c) $e^2 - e$

(d) $2e^2$

(e) $3e^2 - e$

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16. Consider the two planes $x + 3y - 2z = 7$ and $2x + y - 3z = 0$ in $\mathbb{R}^3$. Which of the following sets is the intersection of these planes?

(a) $\emptyset$

(b) $\{(0, 3, 1)\}$

(c) $\{(x, y, z) : x = t, y = 3t, z = 7 - 2t, t \in \mathbb{R}\}$

(d) $\{(x, y, z) : x = 7t, y = 3 + t, z = 1 + 5t, t \in \mathbb{R}\}$

(e) $\{(x, y, z) : x - 2y - z = -7\}$

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17. Let A and B be nonempty subsets of $\mathbb{R}$ and let $f : A \to B$ be a function. If $C \subseteq A$ and $D \subseteq B$, which of the following must be true?

(a) $C \subseteq f^{-1}(f(C))$

(b) $D \subseteq f(f^{-1}(D))$

(c) $f^{-1}(f(C)) \subseteq C$

(d) $f^{-1}(f(C)) = f(f^{-1}(D))$

(e) $f(f^{-1}(D)) = f^{-1}(D)$

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18. Let A be a real 2×2 matrix. Which of the following statements must be true?

I. All of the entries of A^2 are nonnegative.

II. The determinant of A^2 is nonnegative.

III. If A has two distinct eigenvalues, then A^2 has two distinct eigenvalues.

(a) I only

(b) II only

(c) III only

(d) II and III only

(e) I, II, and III

19. Consider the theorem: If f and f' are both strictly increasing real-valued functions on the interval $(0, \infty)$, then $\lim_{x\to\infty} f(x) = \infty$. The following argument is suggested as a proof of this theorem.

(1) By the Mean Value Theorem, there is a c_1 in the interval $(1, 2)$ such that

$$f'(c_1) = \frac{f(2) - f(1)}{2 - 1} = f(2) - f(1) > 0.$$

(2) For each $x > 2$, there is a c_x in $(2, x)$ such that $\dfrac{f(x) - f(2)}{x - 2} = f'(c_x)$.

(3) For each $x > 2$, $\dfrac{f(x) - f(2)}{x - 2} = f'(c_x) > f'(c_1)$ since f' is strictly increasing.

(4) For each $x > 2$, $f(x) > f(2) + (x - 2)f'(c_1)$.

(5) $\lim_{x\to\infty} f(x) = \infty$

Which of the following statements is true?

(a) The argument is valid.

(b) The argument is not valid since the hypotheses of the Mean Value Theorem are not satisfied in (1) and (2).

(c) The argument is not valid since (3) is not valid.

(d) The argument is not valid since (4) cannot be deduced from the previous steps.

(e) The argument is not valid since (4) does not imply (5).

$$J = \int_0^1 \sqrt{1 - x^4}\, dx$$

$$K = \int_0^1 \sqrt{1 + x^4}\, dx$$

$$L = \int_0^1 \sqrt{1 - x^8}\, dx$$

20. Which of the following is true for the definite integrals shown above?

(a) $J < L < 1 < K$

(b) $J < L < K < 1$

(c) $L < J < 1 < K$

(d) $L < J < K < 1$

(e) $L < 1 < J < K$