A short tour of algorithmic randomness

Combining computablility theoretic ideas and measure allows us to identify a collection of “random” binary sequences. “Random” because they are unpredictable, incompressible, and effectively average. The digits of any one random sequence behave pretty much as a sequence of independent identically distributed random variables (taking values 0 or 1).

Effective randomness has been extended to the space of continuous functions on the real interval, a probability space under the Wiener measure. A sequence of random variables taking values 0 or 1 determines a random walk. Taking a random sequence (random as above) determines a sequence of random walks. Limits of random walks have a well established connection with the Wiener measure so it is no surprise that random sequences correspond with “random” functions.

Recently some effort has been made to extend algorithmic randomness to the space of closed subsets of $2^{\mathbb{N}}$. Early efforts seemed unnatural and unsatisfying to some. Recently I have been trying to fit that work into the framework of probability theory for this space.