Random closed sets and probability

Barmpalias, Broadhead, Cenzer, Dashti, and Weber defined a notion of randomness for closed subsets of Cantor space by coding each infinite binary tree without dead ends as a ternary real. The set of paths through such a tree is said to be random if the ternary code for the tree is (Martin-Löf) random. Every closed set of Cantor space is the set of paths through a unique binary tree without dead ends and so we have a well founded definition of random closed set.

Probability theorists, on the other hand, have defined a random closed set to be something quite different. A random closed set as defined in the literature of probability theory is a measurable map from a probability space to the space of closed sets of a topological space (where this space is equipped with the Fell topology and the corresponding Borel σ-algebra). In particular a random closed set of Cantor space is simply a measurable map from any probability space to the space of closed sets of Cantor space.

This talk addresses one way of reconciling these two notions of random closed set. First we use the probability theory framework to develop a measure-based theory of randomness for the space of closed sets of Cantor space. Then we show that random closed sets in the sense of Barmpalias, et. al. are a specific example of randomness with respect to this new framework. To conclude we will look at some other examples of randomness for closed sets that arise from this framework.