1. (5pts) Three devices $d_1$, $d_2$, and $d_3$ fail independently of each other. Let $R$ (running) and $F$ (failed) denote the status of the each of the devices so that

$$S = \{RRR, RRF, RFR, RFF, FRR, FRF, FFR, FFF\}.$$ 

Each device will be found on a given day to have failed with probability $1/5$.

a. Let $A$ be the event that at least two devices are running. List the outcomes in $A$.

$$A = \{RRR, RRF, RFR, RFF, FRR, FRF, FFR\}$$

b. Let $B$ be the event that the first device is running. List the outcomes in $B$.

$$B = \{RRR, RRF, RFR, RFF\}$$

c. Determine the outcomes in $A \cup B$.

$$A \cup B = \{RRR, RRF, RFR, RFF, FRR, FRF, FFR\}$$

d. Determine the outcomes in $A \cap B$ and then find $P(A \cap B)$.

$$A \cap B = \{RRR, RRF, RFR\}$$

$$P(A \cap B) = \left(\frac{4}{5}\right)^3 + \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^2 + \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^2 = \frac{96}{125}$$

2. (6pts) Consider two events $A$ and $B$ such that $P(A) = 0.6$, $P(B) = 0.7$, and $P(B \cap A') = 0.2$.

a. Find $P(A \cup B)$. hint: draw a Venn diagram

$$P(A \cup B) = 0.8$$

b. Are $A$ and $B$ mutually exclusive? Explain or show work.

No, $P(A \cap B) = 0.5 \neq 0$

c. An additional event $C$ with $P(C) = 0.2$ is such that $A$ and $C$ are independent. Find $P(A | C)$.

$$P(A | C) = P(A) = 0.6$$
3. (3pts) Select only one of the following counting problems to complete:

   a. Inhabitants of Dantooine have ID numbers branded on their foreheads when they are born. These ID numbers have five characters — the first three being numbers \([0, 1, \ldots, 9]\) and the last two being letters \(\{A, B, \ldots, Z\}\). What is the probability that a randomly selected individual has an ID number with exactly two of the same number and two identical letters, e.g., 141PR?

   \[
   3 \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{9}{10} \cdot \frac{1}{26} = \frac{27}{2600}
   \]

   b. There are 4 men and 7 women that are to randomly form a single file line and then enter a room. Thus, there are \(\binom{11}{3}\) different ways that men and women can enter the room. What is the probability exactly 3 out of the first 4 people to enter the room are men?

   \[
   4 \cdot \frac{4}{11} \cdot \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{7}{8} = \frac{28}{330}
   \]

4. (6pts) One out of ten bike frames manufactured by Kert\® are defective, whereas one out of five bike frames manufactured by other companies are defective. A bike store purchases 40 frames — 30 from Kert\® and 10 from other companies. One of the 40 frames is selected at random and tested for defects. Let \(D\) be the event the frame is defective and \(K\) be the event it is from Kert\®.

   a. Find the probability that the frame is defective, i.e., \(P(D)\).

   \[
P(D) = \frac{1}{10} \cdot \frac{3}{4} + \frac{1}{5} \cdot \frac{1}{4} = \frac{1}{8}
   \]

   \[
P(K) = \frac{3}{4} \quad P(K') = \frac{1}{4}
   \]

   \[
P(D|K) = \frac{1}{10} \quad P(D|K') = \frac{1}{5}
   \]

   b. Given that the frame is defective, find the probability that it came from Kert\®, i.e., \(P(K|D)\).

   \[
P(K|D) = \frac{P(D|K) \cdot P(K)}{P(D)} = \frac{\frac{1}{10} \cdot \frac{3}{4}}{\frac{1}{8}} = \frac{6}{10}
   \]