Instructions: Write in complete, grammatically correct sentences. When incorporating a mathematical equation, treat it as an object in the sentence, e.g., a careful examination of $2x = 8$ reveals that $x = 4$.

1. Use the quadratic formula to find the solutions of $x^2 + x - 2 = 0$.

   Solution: Recall the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives the two solutions of the quadratic equation $ax^2 + bx + c = 0$. Thus, the solutions of $x^2 + x + 1 = 0$ are
   
   $$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2}.$$

   That is, $x = 1$ and $x = -2$ are the two solutions of $x^2 + x - 2 = 0$.

2. Find the determinant of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

   Solution: The determinant of the matrix $A$ is
   
   $$\det(A) = \left| \begin{array}{cc} 1 & 2 \\ 3 & 4 \end{array} \right| = 1 \cdot 4 - 3 \cdot 2 = -2.$$

3. Suppose $X \sim \mathcal{N}(\mu, \sigma^2)$, what is the distribution of $2X + 3$?

   Solution: Applying the linearity property of the expectation and that $\text{Var}(aX + b) = a^2 \text{Var}(X)$, we have
   
   $$E(2X + 3) = 2 E(X) + 3 = 2\mu + 3 \quad (1)$$

   and
   
   $$\text{Var}(2X + 3) = 4\sigma^2. \quad (2)$$

   Thus, combining (1) and (2), we have $2X + 3 \sim \mathcal{N}(2\mu + 3, 4\sigma^2)$.

4. Suppose $a \in \mathbb{R}$ and $n \in \mathbb{Z}$. Determine $\frac{d^{4n}\sin(ax)}{dx^{4n}}$.

   Solution: First notice that
   
   $$f(x) = \sin(ax)$$
   
   $$\frac{df}{dx} = a\cos(ax)$$
   
   $$\frac{d^2f}{dx^2} = -a^2\sin(ax)$$
   
   $$\frac{d^3f}{dx^3} = -a^3\cos(ax)$$
   
   $$\frac{d^4f}{dx^4} = a^4\sin(ax)$$
   
   $$\vdots$$

   $$\frac{d^8f}{dx^8} = a^8\sin(ax)$$
   
   $$\vdots$$

   Consequently, $\frac{d^{4n}\sin(ax)}{dx^{4n}} = a^{4n} \sin(ax)$. 