PRECALCULUS MODULE

**A Free Technology to Enrich the Teaching and Learning of Two- and Three-Dimensional Graphs**

**Abstract**

*Educational institutions are sorely in need of affordable technology to improve mathematical learning. The financial reality facing many schools prohibits the allocation of limited resources for appropriate mathematical software. The free add-in offered to license holders of Microsoft Word 2007 offers a prudent solution to this challenge. The Word add-in is a powerful computer algebra system and should be available in any mathematics classroom. With particular strengths of graphing in two- and three-dimensions, Word is a viable choice. This paper explores the usage of the animate command to improve student understanding via inquiry. Teachers with interactive whiteboard technology will find the graphics options well suited to their learning environment.*

1. **Introduction**

A viable solution for schools that are challenged with budgets that do not adequately support institutional investments for laboratory mathematics software is the free add-in found in *Microsoft Word 2007*. As well, students may download the free math add-in to their personal computer and have a valuable tool in learning mathematics outside of the classroom.

This paper is based on the presentation made at the 2009 National Educational Computing Conference (NECC) in Washington D.C. Hosted by the International Society for Technology in Education (ISTE), NECC is the largest educational technology exhibit in the United States. The feedback from mathematics educators and information technology personnel verified there is a need for technology access aimed at engaging students in mathematics education. The challenge is particularly large in communities facing budgeting shortfalls, with the greatest need frequently found in the communities least able to afford commercial solutions. Conference attendees offered anecdotal comments regarding their specific purpose at the conference. They sought free interactive technologies that could be brought back to their school districts. Some professionals were seeking solutions to target particular students, but many were just looking for any remedy that would promote inclusion of technology in the classroom.

The examples presented here are centered on improving the teaching and learning of two- and three-dimensional graphs using polar and spherical coordinates. The use of the ‘animate’ command found within the free math add-in is demonstrated and developed as a tool to aid discovery-style lessons. These sample examples would be conducive to whole class teaching, particularly if presented on Interactive Whiteboards (IWB).

1. **Microsoft free math add-in**

Set against this background, the use of the free add-in for *Microsoft* *Word 2007* is a practical solution for schools without the budget-base that would allow for access to *Mathematica*, *Maple*, or other fee-based computer algebra systems (CAS). Teachers seeking introductory explanatory materials or extensive worksheets examples should see: http://web02.gonzaga.edu/faculty/nord/links.htm

1. **Examples of polar graphs in two-dimensions**

Consider the graphs of the roses generated by:

.

The graphics package found within the math add-in takes the command syntax:

.

The content sensitive ‘right click’ generates a mathematical operations menu that contains, *Simplify.* *Animate* appears as an option in the pop-up Microsoft Math Graph Controls dialogue box. *Animate* can be used to generate a movie of different roses as *n* changes, or *n* can be directly controlled by changing the value of the upper limit on the right of the animate control panel that allows input for a fixed value as shown in Figure 4.

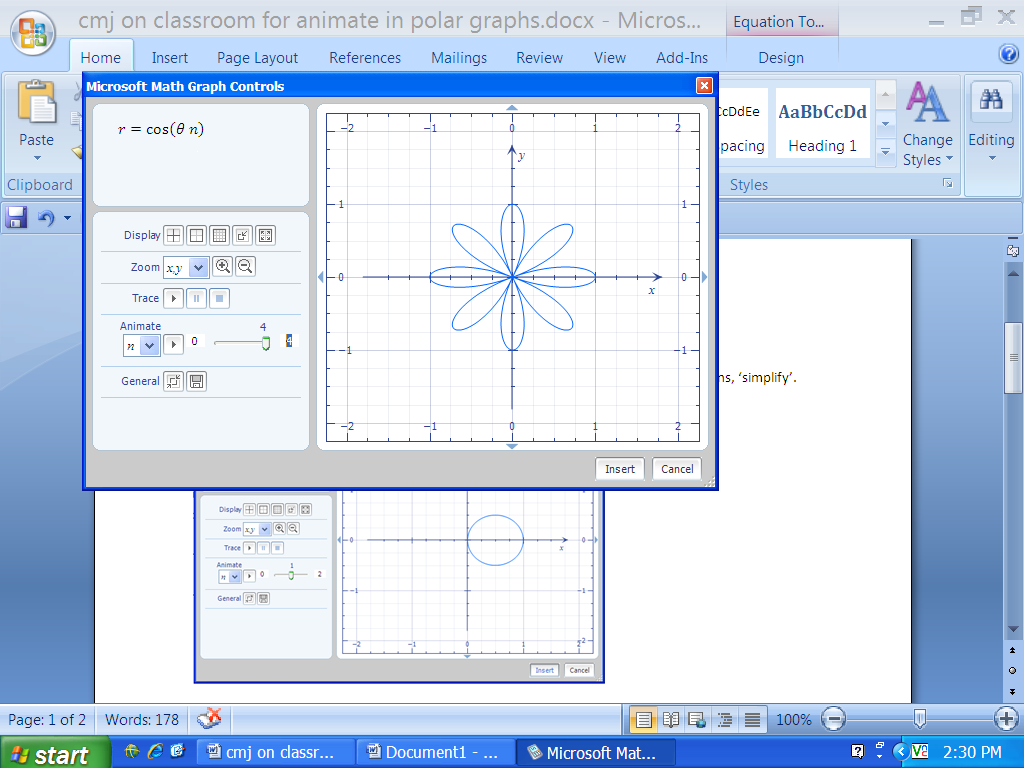


Figure 4: *a rose where n is fixed for the frame*

The students can interact with the upper limit to generate real-time examples and quickly seize upon the theorem for roses. The number of petals is *n* if *n* is odd and *2n* if *n* is even where or [6]. The *plotPolar* command can be absent, and an example using is given in Figure 5 using the *Plot in 2D* option from the pull-down screen. The input can merely be followed by a right click. The input does not require multifaceted syntax. The animate command will appear by default with the introduction of the variable, *n.*

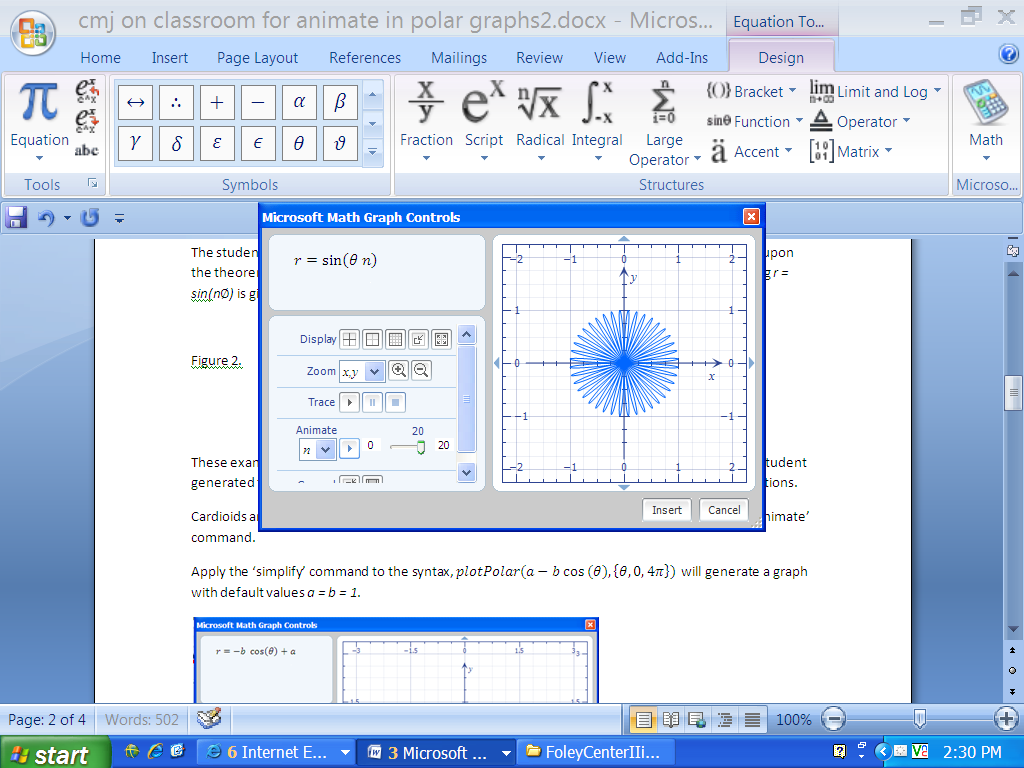


Figure 5: *creating a movie with the right arrow key*

Cardioids and limaçons can easily be treated with the same student interest generated by the *Animate* command. Apply the *Simplify* option from the command, and generate a graph with default values *a = b = 1*. For example, *b=1,* when animating on *a*. It is possible to introduce more than one variable for utilization of the animate feature.

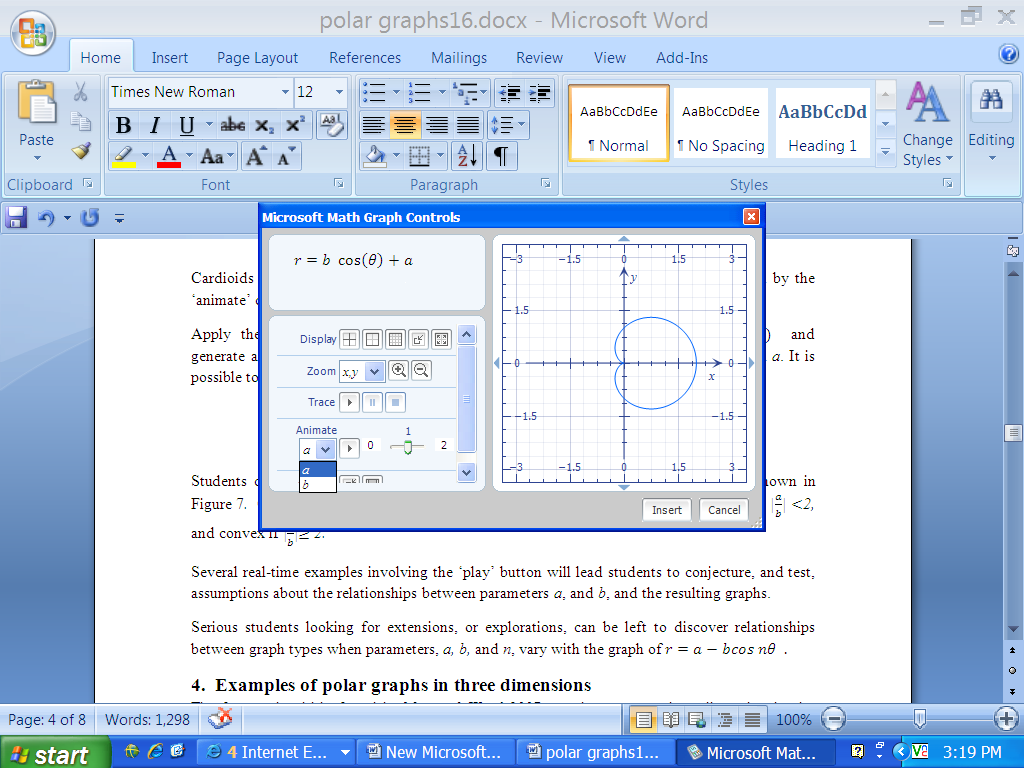


Figure 7: *cardioid*

Students can discover that is the graph of a cardioid if *|a| =|b|* as shown in Figure 7*.* Otherwise, the graph is a limaçon that has two loops if *|a| <|b|,* dimpled if *1 < || 2,* and convex if *|| 2* [6].

Several real-time examples involving the ‘play’ button will lead students to conjecture and test assumptions about the relationships between parameters *a*, and *b*, and the resulting graphs. Serious students looking for extensions, or explorations, can be left to discover relationships between graph types when parameters, *a, b,* and *n*, vary with the equation of The speed of computers enables students to produce many examples when exploring mathematical problems; this supports their observation of patterns and the building and justifying of generalizations [5].

1. **Examples of polar graphs in three-dimensions**

The free math add-in found in *Microsoft Word 2007* can also generate three-dimensional polar images, using spherical coordinates. An example of the syntax for three-dimensional polar graphs is,

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The image is shown in Figure 8.

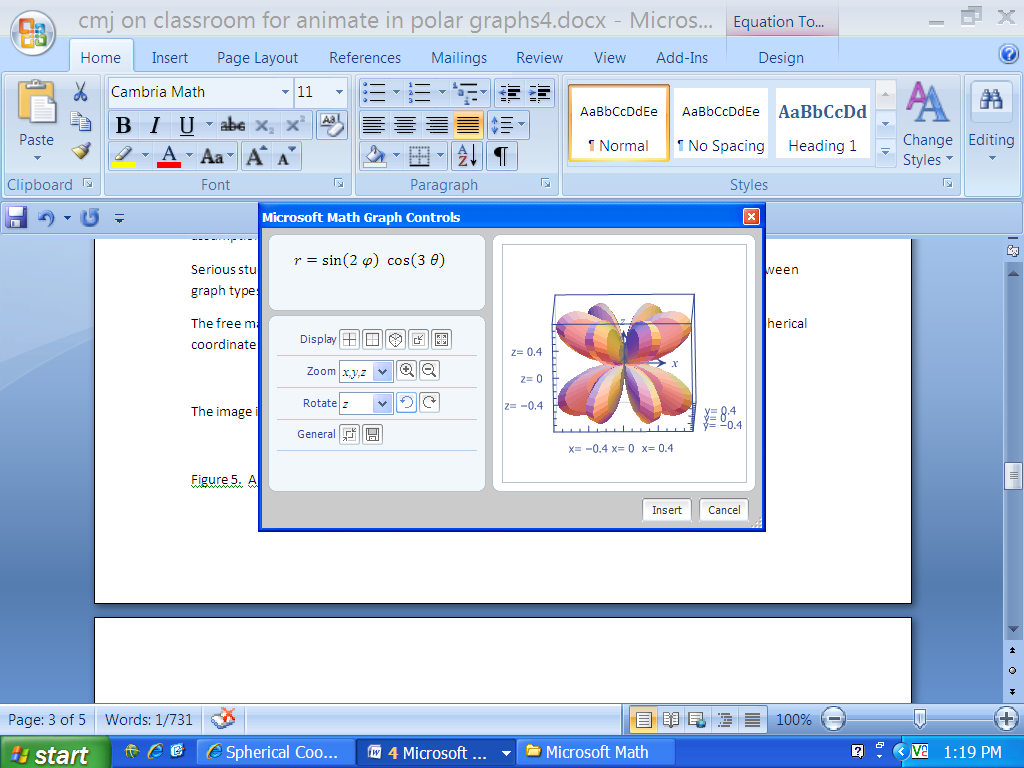
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Figure 8: *a three-dimensional image*

The National Council of Teachers of Mathematics *Curriculum Standards* (1989) encourages the use graphing utilities to investigate informally the surfaces generated by functions of two variables. “Such investigations not only contribute to further development of important visualization skills but also foreshadow more advanced work with functions.”

An application of the animate command to three-dimensional polar graphs could entail using  
or. Similarly, the option of omitting the *plotPolar3D* syntax is permitted. The equation *r = f(* can be inserted and the use of the command *Plot in 3D* can be applied from the menu.

The graph generated by the math *Word* command, is the spherical coordinates three-dimensional analog to the spiral of Archimedes as seen in Figure 9 [6]. It might be very difficult to draw an example resembling a chambered nautilus on the chalkboard of this quality. Students should use the *Rotate* option to revolve the surface around the *x, y,* or *z axis.* In addition, the *Zoom* feature allows for further interactive investigation. Having students use computers to manipulate pictures dynamically, encourages them to visualize the geometry as they generate their own mental images [5].

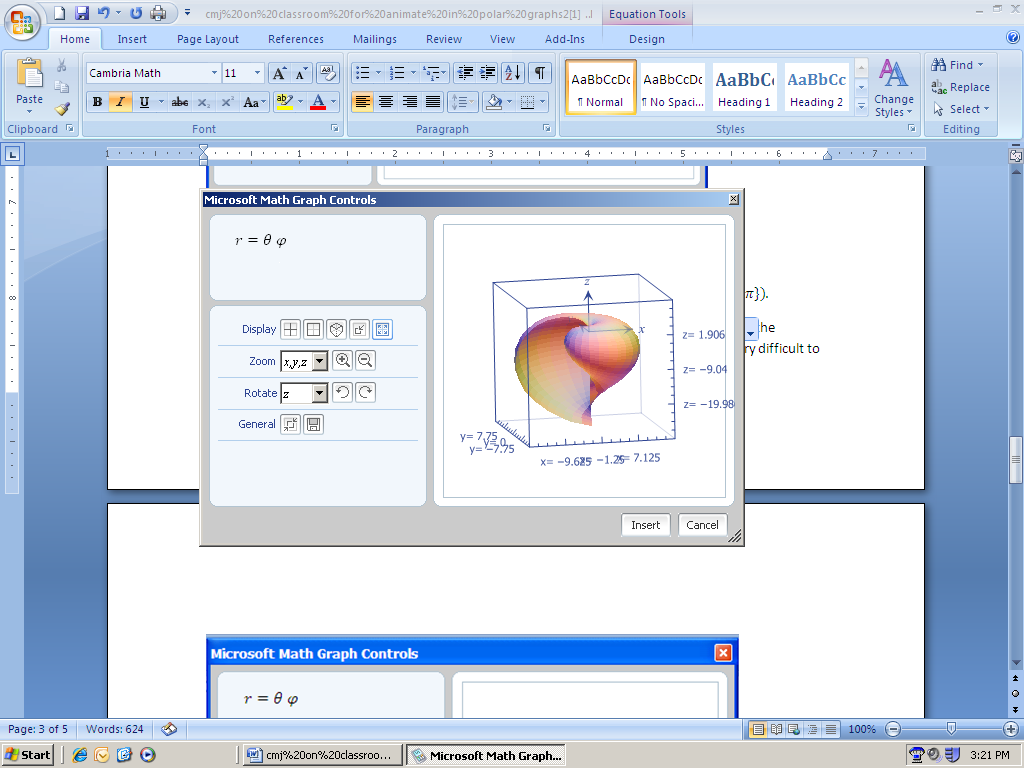


Figure 9: *image of a shell that resembles a chambered nautilus*

1. **Selected graphing commands**

Table 1 contains a selection of graphing commands designed to demonstrate the versatility and power of the software. Flexible input allows for alternative syntax. For example, followed with a right click and the selection of *Simplify* will produce the graph of a parabola. Alternatively, the input of the expression, or the equation, followed with a right-click and the selection of *Plot in 2D* all yield the identical graph.

Table 1: *other features*

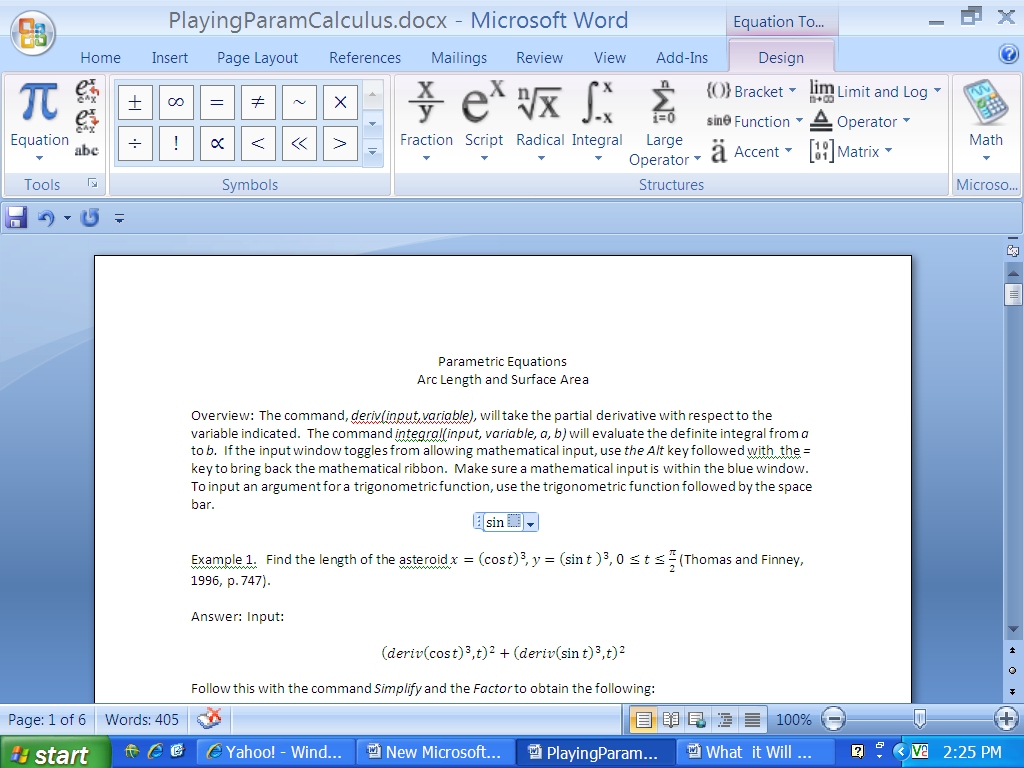
|  |  |  |
| --- | --- | --- |
| ***Command*** | ***Example*** | ***Input*** |
| *plot* |  | *Input function, f(x).* |
| *plot3D* |  | *Input where, z=f(x, y).* |
| *plotCylDataSet3D* |  | *Data point is {* |
| *plotCylParamLine3D* |  | *Insert* |
| *plotCylR3D* |  | *Input z=f(r,* |
| *plotDataSet* |  | *Input point, {x, y}.* |
| *plotDataSet3D* |  | *Input point, {x, y, z}.* |
| *plotEq* |  | *Input f(x, y) = c.* |
| *plotEq3D* |  | *Input f(x, y, z)=c.* |
| *plotIneq* |  | *Input inequality in x and y.* |
| *plotParam* |  | *Input (f(t), g(t)) where x=f(t) and y=g(t).* |
| *plotParam3D* |  | *Input (f(t, s), g(t, s),*  *h(t, s)) where x=f(t, s) and*  *y=g(t, s) and z=h(t, s).* |
| *plotParamLine3D* |  | *Input ( f(t), g(t), h(t)) where x=f(t) and y=g(t)and z=h(t).* |
| *plotPolarDataSet* |  | *Input point {* |
| *plotPolarDataSet3D* |  | *Input a point, {* |

To execute these commands using the drop-down menu, apply *Calculate* or *Simplify*.

1. **Parametric Equations Arc Length and Surface Area**

Instructors do not need to stop with graphing. Further explorations involving arc length and surface area are possible with the free Add-In.

Specific syntax is necessary for these applications. The command, *deriv(input,variable),* will take the partial derivative with respect to the variable indicated. The command *integral(input, variable, a, b)* will evaluate the definite integral from *a* to *b.* If the input window toggles from allowing mathematical input, use *the Alt* key followed with the *=* key to bring back the mathematical ribbon. Make sure a mathematical input is within the blue window. To input an argument for a trigonometric function, use the trigonometric function followed by the space bar. A blue square will appear for the argument. No parentheses are needed. To finish the argument, place the cursor outside of the blue square.



Example 1. Find the length of the asteroid , , [10, p. 747].

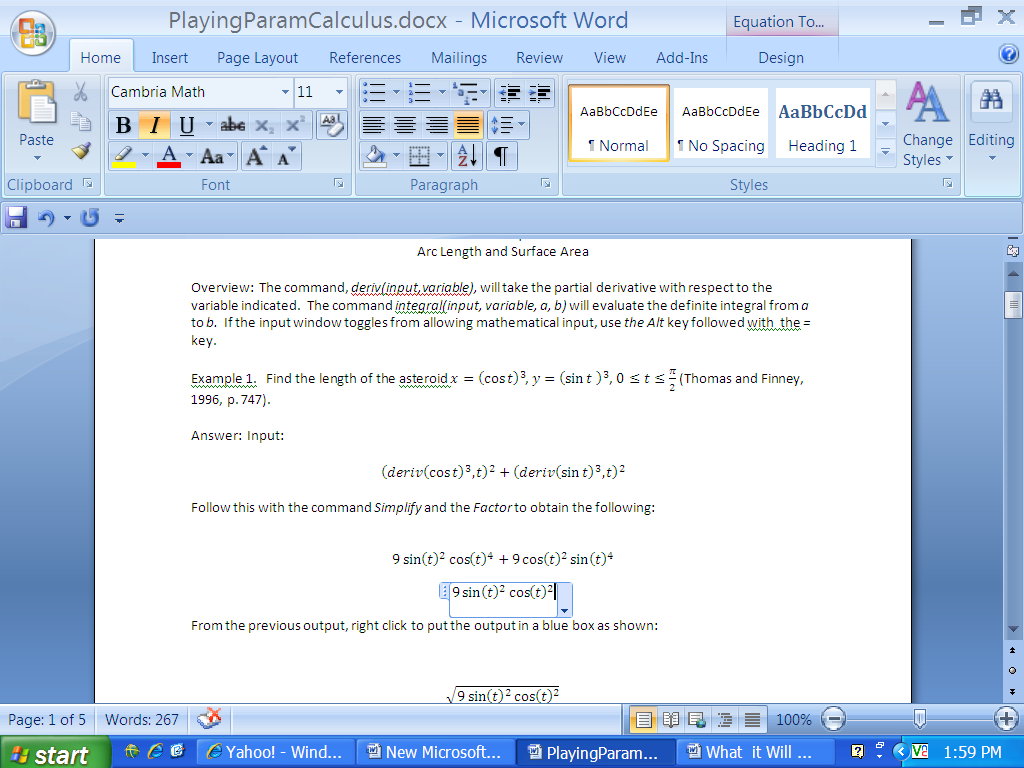
The arc length is computed by evaluating the following integral:

The free Math Add-In will not evaluate this integral directly. Users need to break-up the problem according to the order of operations. The requirement of breaking-up the problem into steps makes the CAS less desirable as an efficient mathematical tool, but it is good pedagogy.

The first step in the process of computing the arc length is to use this input to calculate the sum of the squares of the derivatives.

Follow this with the commands *Simplify* and *Factor* to obtain the following:

From the previous output, right click to put the output in a blue box as shown:



Use the *Alt* followed with an *=* to bring the math add-in back. Highlight and press the *square root* option in the menu ribbon. You will obtain:

The *Simplify* command will yield:

A math object cannot include paragraph marks or break characters. We will alter the output to the following:

Cut and paste the output into a new equation and set up the definite integral.

Press *Simplify* to obtain the result of *3/2.*

In conclusion, the set-up of the integral from the first output will not yield the result. The *Simplify* option appears, but does not execute. To use the free add-in, a break-down of the problem requires simplification first. The concise input below executes in *Mathematica*, but does not generate a result in the free Math Add-In.

Example 2. Find the length of the curves where , , [10, p. 749].

In order to compute the arc length in this example, execute the following commands in order.

Use the options *Factor*, *Expand,* and *Simplify* to obtain the following:

Cut and paste the last input into a new equation window. Highlight and execute the square root option from the ribbon.

The *Simplify* option gives the answer:

Use the result to set-up an integral.

The *Simplify* command yields the answer *21/2.*

Again, note that the set-up of the problem using a definite integral is too intricate for the free Add-In to evaluate in a single line.

*Simplify* yields a more pleasing looking set-up. Unfortunately, the simplification under the radical is needed before the introduction of an integral command. The output is:

Example 3. Find the surface area generated by revolving the curve about the *y*-axis. The curve is , , [10, p. 749].

Consider:

*Simplify* and *Factor* gives the outputs:

Cut and paste the previous output and multiply by *x.*

Unfortunately, the output is not simplified.

Some college algebra by the students will consider the input:

The integral will be evaluated after pressing *Simplify*. Note that the *u* substitution is done to give the surface area as *14/9.*

)

The math add-in would have considered an indefinite integral. Consider the following as an input:

The option of *Integrate on t* appears from the drop-down menu to give the answer using a *u* substitution of:

**7. Conclusion**

ISTE’s public policy objectives are based on a core principle that technology is an essential element of teaching, learning, and instructional design in effective 21st-century learning systems. Furthermore, fluency with technology must be embedded in the learning process and is a condition for attaining necessary skills to promote academic achievement and workforce preparedness [1].

The *Microsoft Math Add-in* for *Microsoft Office Word 2007* makes it easy to create graphs, perform calculations, and solve for variables with equations created in *Word* [2]. The user-friendly interface allows for modeling and solving of complex problems with minimal syntax instruction. This is an affordable and accessible computation tool that schools can quickly and widely adopt and one that students will embrace.

Teachers spend much of their time working problems from textbooks that were specifically created before sophisticated calculators were widely available. These problems were designed to be manipulated and solvable by hand. The free Add-in available for *Word* will handle such problems. *Mathematica* is a far more powerful tool and includes numeric approximation tools. Many surface area and arc length problems do not have closed-form solutions. *Mathematica* and *Maple* can generate digital approximation to the answer, *Word* has no such feature. *Word* is a fine solution for a cash-strapped school trying to incorporate more technology. It is not an appropriate solution for designing bridges.

1. **References**
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