5.1 1, 4, 5
5.2 1bd, 3
5.3 2
7.2 2ac, 4, 5, 15aceg

5.1

1. a) reflexive: Let A be a set, then consider the identity map, f:A->A defined by f(x) = x for all x in A. Then f is clearly a bijection from A to A, so A~A.

b) symmetric: Let A and B be sets, and suppose A~B. Then there exists a bijection f:A->B. Since f is a bijection, then f^{-1}:B->A is a bijection, so B~A.

c) transitive: Let A, B, C be sets and suppose A~B and B~C. Then there is a bijection f:A->B and a bijection g:B->C. Now the composite function gof: A->C is a bijection, so A~C.

Thus ~ is an equivalence relation on the class of sets.

4. f is one-to one: Suppose f(x) = f(y)
Then [(d-c)/(b-a)](x-a) + c = [(d-c)/(b-a)](y-a) + c
➔ [(d-c)/(b-a)](x-a) = [(d-c)/(b-a)](y-a)
➔ x-a = y-a
➔ x = y

f is onto: Let y be an element of (c,d)
Now choose x = (y-c)[(b-a)/(d-c)] + a. Plugging this in to f, we get f(x) = y. Thus, f is ont.

Therefore (a,b) ~ (c,d)

5. Finite sets: a, c, f, g, h, j, l, m

5.2

1. b. N~T+ by the bijection f: N-> T+ given by f(x) = 3x.
---f is one-to-one since f(x) = f(y) ➔ 3x = 3y ➔ x = y.
---f is onto since every y in T+ has the form y = 3k = f(k) for some k in N.

1 d. Let S = {7, 8, 9, . . .}. Then f(x) = x + 6 is a bijection from N to S, so N~S.
---f is one-to-one since f(x) = f(y) ➔ x+6= y+6 ➔ x = y
---f is onto since every y in S is mapped to by x = y – 6 in N. f(x) = (y-6)+6 = y.

3. a. FALSE, e.g. A = {2}
b. TRUE
c. FALSE, A~N means that A cannot be finite
d. TRUE

e. TRUE
f. FALSE, e.g A = {2}

5.3 2. We can create a bijection from N to $\{2^x/3^y \mid x, y, \text{ in } N\}$ by listing the ordered pairs (x,y):

(1,1) (1,2) (1,3) . . .
(2,1) (2,2) (2,3) . . .
. . .

Now create the map from N to these pairs (corresponding to x and y) by going down the diagonals of the grid. Clearly everything will get mapped to eventually and nothing gets hit twice, so this map is a bijection.

7.2

4. The following are the interior points of the given sets:

a. (-1,1)
b. (-1,1)
c. None
d. None
e. None
f. None
g. R – N
h. R – {1/3k | k in N} – {0}
i. $\bigcup_{n \text{ in } N} (n + 0.1, n + 0.2)$

5.
a. open
b. open
c. neither open nor closed
d. neither open nor closed
e. open
f. closed
g. open
h. open
i. closed
j. neither open nor closed

15.
a. not compact (since it is not bounded)
b. compact
c. compact
g. not compact (since it is not closed)