2.1 1bd, 2bd, 3bdfhj, 4bdfhjl, 6be, 8abc
9bdfhjl, 10b, 12, 16

1\. b $\{x \mid x \in \mathbb{Z} \wedge x^2 < 17\}$
or $\{x \in \mathbb{Z} \mid x^2 < 17\}$
d. $\{x \in \mathbb{R} \mid -1 < x \leq 9\}$

2\. b. $\{0, 1, -1, 2, -2, 3, -3, 4, -4\}$
d. cannot be listed.

3\. b. F e.g. $2/3 \notin \mathbb{Z}$
d. F e.g. $\sqrt{2} \notin \mathbb{Q}$
f. F e.g. $\sqrt{2} \notin \mathbb{Q}$
h. F
j. T

4\. b. T
d. T
f. T
h. F
j. F
l. T

6\. b. $\mathcal{P}(X) = \{\emptyset, \{S\}, \{\{S\}\}, \{S, \{S\}\}\}$
e. $\mathcal{P}(X) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, X\}$

8a. A must have 6 elements since $2^6 = 64$ e.g. $A = \{1, 2, 3, 4, 5, 6\}$
b. $A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ and $\mathcal{P}(B) \subseteq \mathcal{P}(A)$ iff $B \subseteq A$.
So $A \subseteq B$ and $B \subseteq A$. Thus $A = B$. e.g. $A = B = \{1, 2\}$
c. Not possible $\emptyset \in \mathcal{P}(A)$ for any set A, But $\emptyset$ has no elements

9\. b. T
d. T
f. T
h. F
j. T
l. F

10b. $A = \{x \mid P(x)\}$ $B = \{x \mid Q(x)\}$
Prove $(\forall x)(P(x) \Leftrightarrow Q(x))$, then $A = B$
Proof: Suppose $(\forall x)(P(x) \Leftrightarrow Q(x))$.
Then $x \in A \Leftrightarrow P(x)$
$\Leftrightarrow Q(x)$
$\Leftrightarrow x \in B$.
$\therefore A = B$.
Thus if $(\forall x)(P(x) \Leftrightarrow Q(x))$, then $A = B$

$\boxed{\text{OR}}$ Suppose $x \in A$. Then $P(x)$ is true and $(\forall x)(P(x) \Leftrightarrow Q(x))$, so $Q(x)$ is true. Thus, $x \in B$. Therefore $A \subseteq B$. Similarly, suppose $x \in B$. Then $Q(x)$ is true. Since $(\forall x)(P(x) \Leftrightarrow Q(x))$ then $P(x)$ is true, so $x \in A$. Thus $B \subseteq A$. Therefore: $A = B$.

12. Let A, B, and C be sets.
Suppose $A \subseteq B$ and $B \subseteq C$.
Let $x \in A$. Since $A \subseteq B$,
Then $x \in B$.
Since $B \subseteq C$, then $x \in C$.
Therefore $A \subseteq C$ (Since $x \in A \Rightarrow x \in C$)
Thus if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

16. $X = \{x \in \mathbb{Z} \mid |x| \leq 3\}$ $Y = \{-3, -2, -1, 0, 1, 2, 3\}$
Let $a \in X$. Then $a \in \mathbb{Z}$ with $|a| \leq 3$
Thus $-3 \leq a \leq 3$ and $a \in \mathbb{Z}$.
Hence $a = -3$ or $a = -2$ or $a = -1$ or $a = 0$
or $a = 1$ or $a = 2$ or $a = 3$.
Thus $a \in Y$.
Therefore $X \subseteq Y$
Let $b \in Y$. Then $b = -3, -2, -1, 0, 1, 2,$ or 3
Thus b is an integer with $-3 \leq b \leq 3$
So, $b \in \mathbb{Z}$ and $|b| \leq 3$.
Thus, $b \in X$
Therefore $Y \subseteq X$
Since $X \subseteq Y$ and $Y \subseteq X$, $X = Y$.

OR
$$
\begin{aligned}
X &= \{x \in \mathbb{Z} \mid |x| \leq 3\} \\
&= \{x \in \mathbb{Z} \mid -3 \leq x \leq 3\} \\
&= \{x \mid x = -3 \lor x = -2 \lor x = -1 \lor x = 0 \lor x = 1 \lor x = 2 \lor x = 3\} \\
&= \{-3, -2, -1, 0, 1, 2, 3\} \\
&= Y
\end{aligned}
$$

2.2 1.bdf, 2.djl, 3.aeil, 6., 7., 8.fil, 9.g, 10.b, 14.bdf
2.3 1.bdfhjkl, 2.bdfhjkl, 12., 13.

2.2 1.b. $A \cap B = \emptyset$

d. $A - (B - C) = A - \{0, 6, 8\} = \{1, 3, 5, 7, 9\} = A$

f. $A \cap (C \cap D) = A \cap \{1, 2, 5, 7, 8\} = \cancel{\emptyset}\ \{1, 5, 7\}$

2.d. $P - N = P$

j. $U - P = N \cup \{0\}$

l. $\tilde{\emptyset} = U = \mathbb{Z}$

3.a. $A \cup B = [3, 8) \cup [2, 6] = [2, 8)$

e. $A \cap B = [3, 8) \cap [2, 6] = [3, 6]$

i. $B - D = [2, 6] - (5, \infty) = [2, 5]$

l. $\tilde{D} = (-\infty, 5]$

6. E and D   P and N

7. C and D

8. f. Prove $A \cup A = A$

proof: $x \in A \cup A$ iff $x \in A$ or $x \in A$
iff $x \in A$ (since $p \vee p \Rightarrow p$)
Thus $A \cup A = A$

i. Prove $A - \emptyset = A$

proof: $A - \emptyset = \{x \mid x \in A \wedge x \notin \emptyset\}$ (definition of $A - B$)
$= \{x \mid x \in A \wedge T\}$ (since $x \notin \emptyset$ is always true)
$= \{x \mid x \in A\}$ (since $P \wedge T \Leftrightarrow P$)
$= A$
Thus, $A - \emptyset = A$

l. Prove $A \cap (B \cap C) = (A \cap B) \cap C$

proof: $x \in A \cap (B \cap C)$
iff $x \in A \wedge (x \in B \cap C)$ (defn. of $\cap$)
iff $x \in A \wedge (x \in B \wedge x \in C)$ (defn. of $\cap$)
iff $(x \in A \wedge x \in B) \wedge x \in C$ (assoc. prop of $\wedge$)
iff $(x \in A \cap B) \wedge x \in C$ (defn of $\cap$)
iff $x \in (A \cap B) \cap C$
Therefore $A \cap (B \cap C) = (A \cap B) \cap C$

r. Prove "If $A \subseteq B$, then $A \cap C \subseteq B \cap C$"

Proof: Suppose $A \subseteq B$. Then $(\forall x)(x \in A \Rightarrow x \in B)$
Let $x \in A \cap C$. Then $x \in A$ and $x \in C$.
Since $A \subseteq B$ and $x \in A$, then $x \in B$.
So, $x \in B$ and $x \in C$. Hence, $x \in B \cap C$, so $A \cap C \subseteq B \cap C$
Thus, if $A \subseteq B$, then $A \cap B \subseteq B \cap C$

9g, 10b, 14bdf

9g. Prove $\widetilde{A \cap B} = \tilde{A} \cup \tilde{B}$

Proof: $x \in \widetilde{A \cap B}$

iff $x \notin A \cap B$ (defn of $\sim$)

iff $\sim(x \in A \cap B)$

iff $\sim(x \in A \wedge x \in B)$ (defn. of $\cap$)

iff $\sim(x \in A) \vee \sim(x \in B)$ (deMorgan's law for $\wedge$)

iff $x \notin A \vee x \notin B$

iff $x \in \tilde{A} \vee x \in \tilde{B}$ (defn of $\sim$)

iff $x \in \tilde{A} \cup \tilde{B}$ (defn of $\cup$)

Thus $\widetilde{A \cap B} = \tilde{A} \cup \tilde{B}$

10b. Prove that if $A \subseteq B \cup C$ and $A \cap B = \emptyset$, then $A \subseteq C$.

proof: Suppose $A \subseteq B \cup C$ and $A \cap B = \emptyset$.

Let $x \in A$. Then $x \in B \cup C$ since $A \subseteq B \cup C$.

So, $x \in B$ or $x \in C$ (definition of $\cup$).

But, since $A \cap B = \emptyset$, then $x \in A$ implies $x \notin B$. Thus $x \notin B$. Thus, $x \in C$.

Therefore $A \subseteq C$.

Therefore if $A \subseteq B \cup C$ and $A \cap B = \emptyset$, then $A \subseteq C$.

14 b. Counter-example: $A = \{1, 2\}$ $B = \{1, 3\}$ $C = \{1\}$

$A \cap C = \{1\} = B \cap C$ so $A \cap C \subseteq B \cap C$

However $A \not\subseteq B$.

d. Counter-example: $A = \{1, 2\}$, $B = \{1, 3\}$

$\mathcal{P}(A - B) = \mathcal{P}(\{2\}) = \{\emptyset, \{2\}\}$

$\mathcal{P}(A) - \mathcal{P}(B) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\} - \{\emptyset, \{1\}, \{3\}, \{1,3\}\} = \{\{2\}, \{1,2\}\}$

$\mathcal{P}(A) - \mathcal{P}(B) \not\subseteq \mathcal{P}(A - B)$

f. counter-example: $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{1\}$

$(A - B) - C = \{2\} - \{1\} = \{2\}$

$A - (B - C) = \{1, 2\} - \{3\} = \{1, 2\}$ Thus, $A - (B - C) \neq (A - B) - C$

2.3. 1. b d f h j k l

b. $\bigcup_{A \in \mathcal{A}} A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ $\bigcap_{A \in \mathcal{A}} A = \emptyset$

d. $\bigcup_{B_n \in \mathcal{B}} B_n = \{2, 3, \dots, \} = \mathbb{N} - \{1\}$ $\bigcap_{B_n \in \mathcal{B}} B_n = \emptyset$

2.3 f h j k l

f. $\bigcap_{n=1}^{10} A_i = \emptyset$ $\bigcup_{n=1}^{10} A_i = \{1, 2, 3, \ldots, 18, 19\}$

h. $\bigcap_{r \in \mathbb{R}^+} [-\pi, r) = [-\pi, 0]$ $\bigcup_{r \in \mathbb{R}^+} [-\pi, r) = [-\pi, \infty)$

j. $\bigcap_{n=1}^{\infty} \{\ldots -2n, -n, 0, n, 2n, \ldots\} = \{0\}$ $\bigcup_{n=1}^{\infty} M_n = \mathbb{Z}$

k. $\bigcap_{n=3}^{\infty} [\frac{1}{n}, 2+\frac{1}{n}) = [1/3, 2]$ $\bigcup_{n=3}^{\infty} [1/n, 2+1/n) = (0, 2\frac{1}{3})$

l. $\bigcap_{n \in \mathbb{Z}} [n, n+1) = \emptyset$ $\bigcup_{n \in \mathbb{Z}} [n, n+1) = (-\infty, \infty)$

2. b d f h j k l

b. Yes, d. No. e.g. $3 \in B_1$, $3 \notin B_2$

f. No. e.g. $3 \in A_2$, $3 \in A_3$

h. No. all intervals contain $-\pi$

j. No. $0 \in$ every set

k. No. $2 \in$ every set

l. Yes.

12. a) $\mathcal{A} = \{A_n \mid n \in X\}$

where $A_n = \{1, 2, \ldots, n\}$

Then $\bigcap_{n=1}^{20} A_n = \{1\}$ and $\bigcup_{n=1}^{20} A_n = \{1, \ldots, 20\} = X$

b) $\mathcal{B} = \{\{1\}, \{2, 3, 4\}, \{5\}, \{6, \ldots, 20\}\}$

$\bigcup_{B \in \mathcal{B}} B = X$ and $B \cap C = \emptyset$ for all $B, C \in \mathcal{B}$ with $B \neq C$

c) $\mathcal{C} = \{\{1\}, \{2\}, \ldots, \{20\}\}$

$\bigcup_{C \in \mathcal{C}} C = X$ and $C \cap D = \emptyset$ for all $C, D \in \mathcal{C}$ with $D \neq C$

13. e.g. $A_\alpha = (0, 1/\alpha)$ where $\alpha \in (1, \infty) = \Delta$

$\bigcap_{\alpha \in \Delta} A_\alpha = \emptyset$ But, for $\alpha, \beta \in \Delta$

$(0, 1/\alpha) \cap (0, 1/\beta) = (0, 1/\gamma) \neq \emptyset$

where $\gamma = \max\{\alpha, \beta\}$

Homework 2.4 4cdfg, 6abde, 8bcjl, 9b, 10
2.5 1, 2

4c. $\frac{97!}{96!} = \frac{97 \cdot 96!}{96!} = 97$

f. $\frac{(n+3)!}{(n+1)!} = (n+3)(n+2)$

4d. $\frac{8!}{3!5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 1\, 5!} = 56$

g. $(n^2+3n+2)(n!)$
$(n+2)(n+1)(n!)$
$= (n+2)!$

6a. $\{n \mid n = 5k \text{ for some } k \in \mathbb{N}\} = \{5, 10, 15, 20, \ldots\}$
① $5 \in S$
② If $k \in S$, then $k+5 \in S$

6b. $\{n \mid n \in \mathbb{N} \wedge n > 10\}$
① $11 \in S$
② If $k \in S$, then $k+1 \in S$.

6d. $\{a, a+d, a+2d, \ldots\}$
① $a \in S$
② if $k \in S$, then $k+d \in S$.

6e. $\{a, ar, ar^2, \ldots\}$
① $a \in S$
② If $k \in S$, then $kr \in S$

8b. $3 + 11 + 19 + \ldots + (8n-5) = 4n^2 - n$

Proof:

① Basis Step: When $n=1$, the left side is $8(1) - 5 = 3$
the right side is $4(1)^2 - 1 = 3$
Thus, the equation holds when $n=1$

② Inductive Step: Suppose $3 + 11 + 19 + \cdots + (8k-5) = 4k^2 - k$ for some $k \in \mathbb{N}$.
Now $3 + 11 + 19 + \cdots + (8k-5) + 8k+3$
$= 4k^2 - k + 8k + 3$ (By the inductive hypothesis)
$= 4k^2 + 8k + 4 - 4 - k + 3$
$= 4(k+1)^2 - k - 1$
$= 4(k+1)^2 - (k+1)$
Thus, if the formula holds when $n=k$, it also holds when $n=k+1$

③ By parts ①, ② and PMI, $3 + 11 + \cdots + (8n-5) = n^2 - n$ for all $n \in \mathbb{N}$

8c. Prove $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2 \quad \forall n \in \mathbb{N}$.

① Basis Step: When $n=1$ $\sum_{i=1}^{1} 2^i = 2^1 = 2$

and $2^{1+1} - 2 = 4 - 2 = 2$

Thus, the equation holds when $n=1$.

② Inductive Step: Suppose $\sum_{i=1}^{k} 2^i = 2^{k+1} - 2$ for some $k \in \mathbb{N}$.

Now $\sum_{i=1}^{k+1} 2^i = \left(\sum_{i=1}^{k} 2^i\right) + 2^{k+1}$

$= (2^{k+1} - 2) + 2^{k+1}$ (By the inductive hypothesis)

$= 2(2^{k+1}) - 2$

$= 2^{k+2} - 2$

Thus the formula holds for $n = k+1$ when ~~it~~ it holds for $n=k$.

③ By ①, ② and PMI $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2 \quad \forall n \in \mathbb{N}$.

8j. Prove $n^3 + 5n + 6$ is divisible by 3 $\forall n \in \mathbb{N}$.

① Basis Step: when $n=1$, $n^3 + 5n + 6 = 1^3 + 5(1) + 6 = 12$
$= 3 \cdot 4$

Thus $3 \mid (n^3 + 5n + 6)$ when $n=1$.

② Inductive Step: Suppose $3 \mid (k^3 + 5k + 6)$ for some $k \in \mathbb{N}$.

~~Now~~ Then $k^3 + 5k + 6 = 3t$ for some $t \in \mathbb{Z}$.

Now $(k+1)^3 + 5(k+1) + 6$

$= k^3 + 3k^2 + 3k + 1 + 5k + 5 + 6$

$= k^3 + 5k + 6 + 3k^2 + 3k + 6$

$= 3t + 3k^2 + 3k + 6$

$= 3(t + k^2 + k + 2)$

Since $t + k^2 + k + 2$ is an integer, then

$3 \mid [(k+1)^3 + 5(k+1) + 6]$.

Thus, if the statement is true when $n=k$, it is also true when $n = k+1$.

③ Now by ① and ② and PMI

$3 \mid (n^3 + 5n + 6)$ for all $n \in \mathbb{N}$.

2.5 1, 2 Note: Alternate Proof of 1 on next page

1. Prove $\forall n \geq 4$ $n = 2x + 5y$ for some $x, y \in \mathbb{Z}$.

① Basis Step: When $n=4$, we have $4 = 2(2)+5(0)$ $(x=2, y=0)$ so we can write $4 = 2x+5y$

② Inductive Step: Suppose $k = 2x+5y$ where $x, y \in \mathbb{Z}$ and $k \geq 4$

Now
$$k+1 = 2x+5y+1$$
$$= 2(x-2) + 4 + 5y + 1$$
$$= 2(x-2) + 5(y+1) \quad x-2, y+1 \in \mathbb{Z}$$

Thus, if k can be expressed as a linear combination of 2 and 5, so can $k+1$

③ Thus by ①, ② and PMI $n = 2x+5y$ for some $x, y \in \mathbb{Z}$ where $n \geq 4$.

2. ~~②. Basis~~ Prove. If $a_1 = 2, a_2 = 4$ and $a_{n+2} = 5a_{n+1} - 6a_n$ for $n \geq 1$. Then $a_n = 2^n$ $\forall n \in \mathbb{N}$.

① Basis cases: when $n=1$ $a_1 = 2$ and $2^1 = 2$

$n=2$ $a_1 = 4$ and $2^2 = 4$

Thus $a_n = 2^n$ for $n = 1, 2$.

② ~~Suppose~~ Inductive Step: Suppose $a_n = 2^n$ for $n = 1, 2, \ldots, k+1$

Now
$$a_{k+2} = 5a_{k+1} - 6a_k$$
$$= 5(2^{k+1}) - 6(2^k) \quad \text{(By inductive hypothesis)}$$
$$= 5(2^{k+1}) - 3(2^{k+1})$$
$$= (5-3)(2^{k+1})$$
$$= 2(2^{k+1})$$
$$= 2^{k+2}$$

Thus $a_{k+2} = 2^{k+2}$ when $a_{k+1} = 2^{k+1}$ and $a_k = 2^k$.

③ Now By ①, ② and PMI, $a^n = 2^n$ $\forall n \in \mathbb{N}$.

8l, 9b, 10   2.5 1,2

8l. Prove $3^n \geq 1 + 2^n \quad \forall n \in \mathbb{N}$.

① Basis Step: $3^1 = 3 = 1+2^1$, so $3^n \geq 1+2^n$ when $n=1$

② Inductive step: Suppose $3^k \geq 1+2^k$ for some $k \in \mathbb{N}$.

Now $3^{k+1} = 3(3^k) = 3^k + 2\cdot 3^k$

$\geq 1 + 2\cdot 2^k$ (since $3^k > 2^k$ and $3^k > 1$)

$= 1 + 2^{k+1}$

Thus, if $3^k \geq 1+2^k$

then $3^{k+1} \geq 1+2^{k+1}$

③ By ① and ② and PMI, $3^n \geq 1+2^n$ for all $n \in \mathbb{N}$.

9b. Prove $2^n > n^2$ for all $n > 4$

① Basis step: ~~suppose~~ $n=5$

Then $2^5 = 32$ and $5^2 = 25$

Since $32 > 25$, $2^n > n^2$ when $n=5$

② Inductive Step: suppose $2^k > k^2$ for some $k > 4$

now $2^{k+1} = 2(2^k) > 2(k^2)$ (Inductive Hypothesis

$= k^2 + k^2$

$> k^2 + 4k$ since $k > 4$

$= k^2 + 2k + 2k$

$> k^2 + 2k + 1$ since $2k > 1$ when $k > 4$

$= (k+1)^2$

Thus, if $2^k > k^2$, then $2^{k+1} > (k+1)^2$ where $k > 4$

③ By ① ② and PMI $2^n > n^2 \quad \forall n > 4$.

10. Not necessarily.

e.g. Consider the statement "n is odd" = $P(n)$

① $P(1)$ is True since 1 is odd

② If $P(n)$ is True, then $P(n+2)$ is true $\forall n \in \mathbb{N}$.

But $P(n)$ is only true for $n = 1, 3, 5, \ldots$

1. Proof of $\forall n \geq 4$ we can write $n = 2x + 5y$ for some $x, y \in \mathbb{Z}$.

① Basis step(s): $n = 4 = 2(2) + 5(0)$ $(x=2, y=0)$

$n = 5 = 2(0) + 5(1)$ $(x=0, y=1)$

② Inductive step(s): Suppose we can write $n = 2x + 5y$ for $n = 4, 5, \ldots, k$ $(k \geq 5)$

Now $k+1 = k-1 + 2$

$= (2x + 5y) + 2$ since $k-1 = 2x + 5y$ for some $x, y \in \mathbb{Z}$

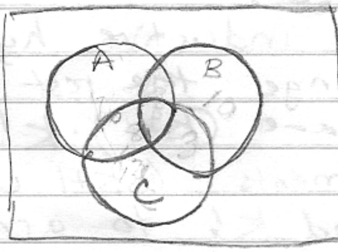
$= 2(x+1) + 5y$

Thus $k+1$ can be expressed as a linear combination of ~~integers~~ 2 and 5 when $4, 5, \ldots, k$ can be expressed as a linear combination of 2 and 5.

③ By ① and ② and PMI we have $\forall n \geq 4$, $n = 2x + 5y$ for some $x, y \in \mathbb{Z}$

2.5 1, 2
2.6 2, 3, 6, 8, 10, 11, 14, 16bc, 17

2.6
2.



$|A \cup B| = |A| + |B| - |A \cap B|$
$37 = 24 + 21 - |A \cap B|$

a) $8 = |A \cap B|$

b) $|A - B| = |A| - |A \cap B| = 24 - 8 = 16$

c) $|B - A| = |B| - |A \cap B| = 21 - 8 = 13$

d) $|B \cup C| = |B| + |C - B| = 21 + 12 = 33$

$|B \cup C| = |C| + |B - C| = 33$
$= |C| + 10 = 33$

e) $|C| = 23$

f) $|A \cup C| = |A| + |C| - |A \cap C| = 24 + 23 - 11 = 36$

3. $1, 2, \ldots, 999{,}999$ how many are squares or cubes

$S = \{n \in \mathbb{N} \mid n < 1000{,}000$ and $n = k^2$ for some $k \in \mathbb{N}$
$C = \{n \in \mathbb{N} \mid n < 1{,}000{,}000$ and $n = k^3$ for some $k \in \mathbb{N}$
$S \cap C = \{n \in \mathbb{N} \mid n < 1{,}000{,}000$ and $n = k^6$ for some $k \in \mathbb{N}\}$
$S = \{1, 4, 9, 16, \ldots, k^2, \ldots, 999^2\}$
$C = \{1, 8, 27, \ldots, k^3, \ldots, 99^3\}$
$S \cap C = \{1, 64, \ldots, k^6, \ldots, 9^6\}$
$|S| = 999$, $|C| = 99$
$|S \cap C| = 9$

$|S \cup C| = |S| + |C| - |S \cap C|$
$= 999 + 99 - 9 = 1089$

6. a) $10 \cdot 9 = \boxed{90}$
b) $3 \cdot 2 \cdot 4 = \boxed{24}$
c) $3 \cdot 4 \cdot 5 = \boxed{60}$

8. The number of permutations of a set of n elements is $n!$
Proof: Basis ① (P(1) true) If S has 1 element, s_1, then there is 1 way to arrange it. Since $1! = 1$ then the theorem holds when $n = 1$.
Inductive Step ② ($P(k) \to P(k+1)$) Suppose there are $k!$ permutations of a set of size k for some $k \geq 1$. Now take set S with $k+1$ elements. $S = \{s_1, s_2, \ldots, s_{k+1}\}$.
To arrange the elements of S, we first select the first element of the arrangement. There are $k+1$ ways to do this. Now remove this element, e, from S.

17\. a. $(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

b. $(a+2b)^4 = a^4 + a^3(2b)(4) + a^2(2b)^2(6) + a(2b)^3(4) + (2b)^4$
$= a^4 + 8a^3b + 24a^2b^2 + 32ab^3 + 16b^4$

c. $C(13,10)$ d. $C(12,2)\cdot(2)^{10}$

$S-\{e\}$ has k elements. By the inductive hypothesis, there are $k!$ ways to arrange the rest of the elements. Thus, there are $(k+1)\cdot k! = (k+1)!$ ways to arrange the elements ($k+1$ ways to choose the first one and $k!$ to arrange the rest). Hence, the theorem holds for $k+1$ when it holds for k.

Now by ①, ② and PMI, there are $n!$ permutations of a set with n elements.

11\. a) $\boxed{3!\,4!}$ (arrange girls, arrange boys)

b) G _ _ _ _ _ G _ _ _ _ _ G _ _ _ _ _ G _ _ _ _ _ G

$\boxed{5\cdot 3!\,4!}$

place girls, arrange girls, arrange boys

c) B _ _ _ _ B _ _ _ _ B _ _ _ _ B

$\boxed{4\cdot 3!\,4!}$ (place boys, arrange girls, arrange boys)

d) BGBGBGB $\boxed{4!\,3!}$ (arrange boys, arrange girls)

14\. a) $C(11+8,3) = \boxed{C(19,3)}$

b) $\boxed{C(11,2)\cdot C(8,1)}$ (Choose 2 boys, one girl)

c) $\boxed{C(11,1)\cdot C(8,2)}$ (Choose 1 boy, 2 girls)

16\. b) $\binom{n-1}{r}+\binom{n-1}{r-1} = \frac{(n-1)!}{(n-1-r)!\,r!} + \frac{(n-1)!}{(n-1-r+1)!\,(r-1)!}$

$= \frac{(n-1)!\,(n-r)}{(n-1-r)!\,(n-r)\,r!} + \frac{(n-1)!\,r}{(n-r)!\,(r-1)!\,r}$

$= \frac{(n-1)!\,(n-r) + (n-1)!\,r}{(n-r)!\,r!} = \frac{(n-1)!\,n}{(n-r)!\,r!} = \frac{n!}{(n-r)!\,r!} = \binom{n}{r}$

c) $\binom{n}{r}$ is the number of subsets of size r taken from a set of size n.

add these to get all subsets of size r (those without e and those with e)

$\binom{n-1}{r}$ is the number of subsets of size r taken from a set of size n that exclude element $e\in S$.

$\binom{n-1}{r-1}$ is the number of subsets of size r taken from a set of size n that include element $e\in S$

Since $\binom{n}{r}$ and $\binom{n-1}{r}+\binom{n-1}{r-1}$ both count all subsets of size r taken from a set of size n, $\binom{n}{r} = \binom{n-1}{r}+\binom{n-1}{r-1}$