3.1 1a, 4, 5, 6bdf, 7bdf, 8bdf, 9c, 10efh, 11e, 13, 17

1a.

$A \times B = \{(1,a), (1,e), (1,k), (1,n), (1,r), (3,a), (3,e), (3,k), (3,n), (3,r), (5,a), (5,e), (5,k), (5,n), (5,r)\}$

$B \times A = \{(a,1), (a,3), (a,5), (e,1), (e,3), (e,5), (k,1), (k,3), (k,5), (n,1), (n,3), (n,5), (r,1), (r,3), (r,5)\}$

4. Let $A = \{a\}$, $B = \{b\}$, $C = \{c\}$, $D = \{d\}$

a) $(A \times B) \cup (C \times D) = \{(a,b), (c,d)\}$

$(A \cup C) \times (B \cup D) = \{(a,b), (c,b), (a,d), (c,d)\}$

b) $(C \times C) - (A \times B) = \{c,c\}$, $(C - A) \times (C - B) = \emptyset$

where $C = A = \{c\}$, $B = \{b\}$

c) $A \times (B \times C) = \{(a, (b,c)\}$, $(A \times B) \times C = \{((a,b), c)\}$

where $A = \{a\}$, $B = \{b\}$, $C = \{c\}$

5. a) $\text{dom}(T) = \{1,2,3\}$

b) $\text{rng}(T) = \{1, 2, 3, 5, 6\}$

c) $T^{-1} = \{(1,3), (3,2), (5,3), (2,2), (6,1), (6,2), (2,1)\}$

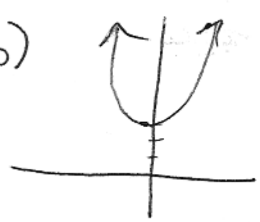
d) $(T^{-1})^{-1} = T = \{(3,1), (2,3), (3,5), (2,2), (1,6), (2,6), (1,2)\}$

6. b) $\text{Dom}(W) = \mathbb{R}$, $\text{Rng}(W) = [3, \infty)$

d) $\text{dom}(W) = (-\infty, 0) \cup (0, \infty)$, $\text{Rng}(W) = (0, \infty)$

f) $\text{dom}(W) = (-2, 2)$, $\text{Rng}(W) = \{3\}$

7. b)

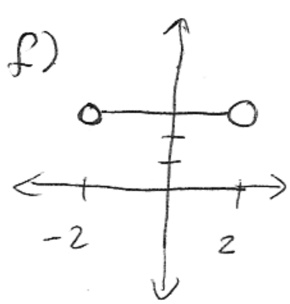


$y = x^2 + 3$

d)

$y = \frac{1}{x^2}$

f)



$|x| < 2$ and $y = 3$

8. $x = -5y + 2$

b. $x - 2 = -5y$ $\qquad R_2^{-1} = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = \frac{2-x}{5}\}$

$\frac{2-x}{5} = y$

d. $\pm\sqrt{x-2} = y$ $\qquad R_4^{-1} = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = \sqrt{x-2}$ or $y = -\sqrt{x-2}\}$

f. $R_6^{-1} = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y > x - 1\}$

$x < y + 1 \Rightarrow y > x - 1$

9c. $T \circ S = \{(2,1), (3,1), (3,4)\}$

10. e. $R_2 \circ R_4 = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = -5x^2 - 8\}$

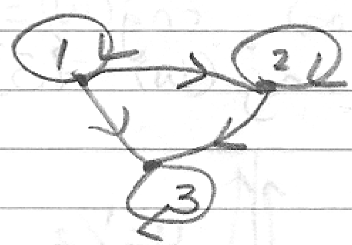
$y = -5(x^2+2) + 2 = -5x^2 - 10 + 2$

f. $R_4 \circ R_2 = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = 25x^2 - 20x + 6\}$

$y = (-5x+2)^2 + 2 = 25x^2 - 20x + 4 + 2$

h. $R_6 \circ R_2 = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y < -5x + 3\}$

$y < (-5x+2) + 1 \qquad y < -5x + 3$

11c. 

13. $R: A \to B \qquad S: B \to C$

a) $S \circ R = \{(a,c) \mid \exists\, b \in B$ such that $(a,b) \in R$ and $(b,c) \in S\}$

a) let $a \in \mathrm{Dom}(S \circ R)$, then $(a,c) \in S \circ R$ for some $c \in C$. and $(a,b) \in R$ and $(b,c) \in S$ for som $b \in B$. Thus $a \in \mathrm{dom}(R)$.

Thus $\mathrm{Dom}(S \circ R) \subseteq \mathrm{Dom}(R)$

b) $R = \{(a,b)\} \qquad S = \{(a,b)\} \qquad S \circ R = \emptyset$

c) $Rng(S \circ R) \subseteq Rng(S)$ is true.

$R = \{(a,b)\}$ $S = \{(a,b)\}$ $\Rightarrow S \circ R = \emptyset$

$Rng(S \circ R) = \emptyset$, $Rng(S) = \{b\}$

So, $Rng(S)$ may not be a subset of $Rng(S \circ R)$

d) $R = \{(a,b)\}$ $S = \{(a,b)\}$

$R \circ S = \emptyset = S \circ R$

17. Suppose $|A| = m$ and $|B| = n$

Then $A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$

Thus $|A \times B| = m \times n$ since we have m choices for a, n choices for b.

A relation from A to B is a subset of $A \times B$. Since $A \times B$ has mn elements, $A \times B$ has 2^{mn} subsets (as shown in class $\mathcal{P}(S)$ has $2^{|S|}$ elements).

Thus, there are 2^{mn} relations from A to B.

Math 301

3.2 1bfh, 2bcg, 4ac, 5,6d, 8
3.3 2abc, 4, 6ac, 7, 8b

3.2 1bfh

1b. $\leq$ on $\mathbb{N}$
a) reflexive since $x \leq x$ for all $x \in \mathbb{N}$
b) not symmetric e.g. $2 \leq 5$, but $5 \not\leq 2$
c) transitive since if $x \leq y$ and $y \leq z$, then $x \leq z$

f. $\neq$ on $\mathbb{N}$
a) not reflexive since $x = x$
b) Symmetric since if $x \neq y$ then $y \neq x$
c) not transitive ~~since~~ e.g. $1 \neq 3$, $3 \neq 1$, but $1 = 1$

h. $x+y=10$
a) not reflexive ~~since~~ e.g. $6+6 \neq 10$
b) symmetric since if $x+y=10$, then $y+x=10$
c) not transitive ~~since~~ e.g. $4+6=10$, $6+4=10$, $4+4 \neq 10$

2b. reflexive, not transitive, not symmetric $\{(1,1),(2,2),(3,3),(1,2),(2,3)\}$

c. not refl, symm, not transitive $\{(1,2),(2,1)\}$

g. not refl, Symm, transitive $\{(1,2),(2,1),(2,2),(1,1)\}$

4a. $0/R = \{0\}$, $4/R = \{4,-4\}$, $-72/R = \{72,-72\}$

reflexive on $\mathbb{R}$ i) R is reflexive on $\mathbb{Z}$ since $x^2 = x^2$ for all $x \in \mathbb{Z}$

symmetric ii) Suppose $x^2 = y^2$. Then $y^2 = x^2$. Thus, if xRy then yRx for all $y, x \in \mathbb{Z}$

transitive iii) Suppose $x^2 = y^2$ and $y^2 = z^2$. Then $x^2 = z^2$. Thus, if xRy and yRz then xRz for all $x,y,z \in \mathbb{Z}$

4c. $3/R = \{3, 1/3\}$, $-2/3/R = \{-2/3, -3/2\}$, $0/R = \{0\}$

reflexive on $\mathbb{R}$ i) R is reflexive on $\mathbb{R}$ since $x = x$ for all $x \in \mathbb{R}$

symmetric ii) Suppose xRy. Then $x=y$ or $xy=1$. So $y=x$ or $yx=$ [redacted]. Thus yRx.

transitive iii) Suppose xRy and yRz.
Then $x=y$ or $xy=1$ and ~~y~~$=z$ or $yz=1$
a) if $x=y$ and $y=z$, then $x=z$, so xRz
b) if $x=y$ and $yz=1$, then $xy=1$, so xRz
c) if $xy=1$ and $y=z$, then $xz=1$, so xRz
d) if $xy=1$ and $yz=1$, then $xy=yz$ with $y \neq 0$, so $x=z$. Thus xRz.

Thus, R is an equivalence relation on $\mathbb{R}$

5. i) reflexive b, c, d
ii) symmetric b, c
iii) transitive a, b, c

6d. $0/R = \{7k \mid k \in \mathbb{Z}\} = \{\ldots, -14, -7, 0, 7, 14, 21, \ldots\}$
$1/R = \{7k+1 \mid k \in \mathbb{Z}\} = \{\ldots, -13, -6, 1, 8, 15, \ldots\}$
$2/R = \{7k+2 \mid k \in \mathbb{Z}\} = \{\ldots, -12, -5, 2, 9, 16, \ldots\}$
$3/R = \{7k+3 \mid k \in \mathbb{Z}\} = \{\ldots, -11, -4, 3, 10, \ldots\}$
$4/R = \{7k+4 \mid k \in \mathbb{Z}\} = \{\ldots, -10, -3, 4, 11, \ldots\}$
$5/R = \{7k+5 \mid k \in \mathbb{Z}\} = \{\ldots, -9, -2, 5, 12, \ldots\}$
$6/R = \{7k+6 \mid k \in \mathbb{Z}\} = \{\ldots, -8, -1, 6, 13, \ldots\}$

8. a) $x R y$ iff $2 \mid (x+y)$
i) R is reflexive on $\mathbb{N}$ since $x + x = 2x$, so $2 \mid (x+x)$. Hence $x R x$ for all $x \in \mathbb{N}$.
ii) Let $x, y \in \mathbb{N}$ with $x R y$. Then $2 \mid (x+y)$. Thus $2 \mid (y+x)$. So $y R x$. Thus, R is symmetric.
iii) Let $x, y, z \in \mathbb{N}$ with $x R y$ and $y R z$. Then $2 \mid (x+y)$ and $2 \mid (y+z)$. So $2m = x+y$ and $2n = y+z$ for some $m, n \in \mathbb{N}$.
Now $x + z = 2m - y + 2n - y$
$= 2(m - y + n)$. Thus $2 \mid (x+z)$.
So, $x R z$. Thus, R is transitive.

b) e.g. $x = 2$. Then ~~x+2~~ $x + x = 4$ and $3 \nmid 4$ so $2 \not R 2$.
Thus S is not an equiv rel.

3.3 2abc, 4, 6ac, 7, 8b

2a. no. e.g. $\{1,2\} \neq \{2,3\}$, but $\{1,2\} \cap \{2,3\} \neq \emptyset$

b. no. $\bigcup_{X \in \mathcal{A}} X = \{1,2,3,4,5\} \neq A$

c. Yes.

4. $i/R = \{i, -i\}$ since $i^2 = -1$, $i(-i) = -1$, $(-i)^2 = -1$

$1/R = \{1, -1\}$, since $1^2 = 1$, $(-1)(1) = -1$, $(-1)^2 = 1$

$$\mathcal{A} = \{ i/R, 1/R \} = \{ \{i, -i\}, \{1, -1\} \}$$

6a. xRy iff $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor$

or xRy iff $2^k \leq x < 2^{k+1}$ and $2^k \leq y < 2^{k+1}$ for some $k \in \mathbb{N}$.

c. xRy iff x and y have the sign

Note: sign $(x) = +$ if $x \in (0, \infty)$

sign $(x) = -$ if $x \in (-\infty, 0)$

sign (0) None (neutral)

equivalently

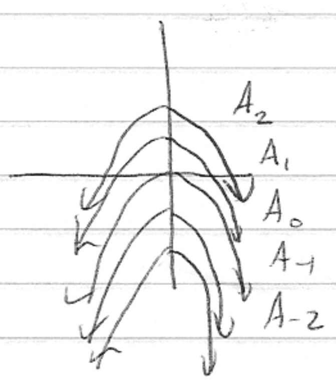
xRy iff $x = 0$ and $y = 0$

or $x < 0$ and $y < 0$ (x, y both negative)

or $x > 0$ and $x > 0$ (x, y both positive)

7. $A_a = \{(x,y) \in \mathbb{R} \times \mathbb{R} : y = a - x^2\}$

a)



(b) ~~xRy iff xparity~~ ~~(x,y) is on y = a − x²~~ ~~(all po~~

b) $\{A_a : a \in \mathbb{R}\} = \mathcal{A}$ is a partition of $\mathbb{R} \times \mathbb{R}$

i) Let $A_a \in \mathcal{A}$. $A_a \neq \emptyset$ since $(0, a) \in A_a$

ii) Suppose $A_a \cap A_b \neq \emptyset$.

Then $\exists (x,y) \in \mathbb{R} \times \mathbb{R}$ such that

$y = a - x^2$ and $y = b - x^2$

$\Rightarrow a - x^2 = b - x^2$

$\Rightarrow a = b$

$\Rightarrow A_a = A_b$

So, either $A_a \cap A_b = \emptyset$ or $A_a = A_b$

iii) Let $(x,y) \in \mathbb{R} \times \mathbb{R}$

Choose $a = y + x^2 \in \mathbb{R}$.

Thus $y = a - x^2$, so $(x,y) \in A_a$.

Therefore every point $(x,y) \in \mathbb{R} \times \mathbb{R}$ lies on a parabola of the form $y = a - x^2$. Thus $\bigcup_{a \in \mathbb{R}} A_a = \mathbb{R} \times \mathbb{R}$

By i), ii), iii)

$\mathcal{A} = \{A_a \mid a \in \mathbb{R}\}$ is a partition of $\mathbb{R} \times \mathbb{R}$

c) Two points (x,y) and (x_1, y_1) are related iff they lie on the same parabola $y = a - x^2$.

8b. $\{(1,1), (2,2), (5,5), (3,3), (3,4), (4,3), (4,4)\}$

4.1

1. b. Not a function. 1 has multiple images.
   c. Yes. Domain$(R_3) = \{1,2\}$, Codomain $= \{1,2\}$ (or larger)
   e. Not a function. $1 \le 1$, $1 \le 2$
   f. Not a function. 1 maps to 1 and -1
   h. Yes. Domain $(R_8) = \{1, 4, 9, 16, \ldots\}$, Codomain $(R_3) = \mathbb{N}$
   i. Yes. Domain $(R_9) = \{1,2,3,4,5\}$ Codomain $(R_3) = \{1,2,3\}$

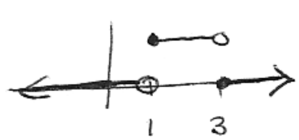
2. a. $\{(5,3),(5,4)\}$
   b. $\{(5,3)\}$
   c. $\{(5,3),(6,4),(7,5)\}$
   d. $\{(5,3),(6,3),(7,3)\}$

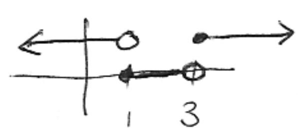
4. a. $f(5) = 24$
   b. 4 or -4 since $f(4) = 15 = f(-4)$
   c. $\{5,-5\}$
   d. $\sqrt{21}$ or $-\sqrt{21}$
   e. $f(-1) = 0$
   f. None, since $x^2 - 1 \neq -10$ for any $x \in \mathbb{R}$

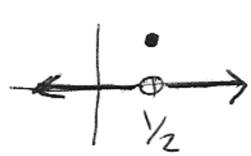
5a. $f(x) = \dfrac{(x-4)(x-3)}{x-3}$ domain$(f) = (-\infty,3) \cup (3,\infty)$
range$(f) = (-\infty,-1) \cup (-1,\infty)$

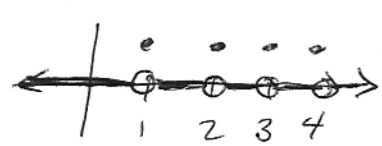
c. $f(x) = \dfrac{1}{\sqrt{x+\pi}}$ domain$(f) = (-\pi,\infty)$
range$(f) = (0,\infty)$

6a. $(1,1)$ and $(1,-1)$ are both in f
c. $(1,0)$ and $(1,2\pi)$ are both in f

9. a.

χ_A

b.

$\chi_{\tilde{A}}$

c.

$\chi_{\{1/2\}}$

d.

$\chi_{\mathbb{N}}$

4.1

14. a. $f(3) = \overline{3}$ c. 3, 9, −3, etc.

b. $f(6) = \overline{0}$ d. $\{\ldots -5, 1, 7, 13, \ldots\}$

4.2

1. f. $f \circ g = \{(1,2), (2,5), (4,5), (5,2)\}$

$g \circ f = \{(1,7), (3,4), (4,3), (5,3)\}$

g. $(f \circ g)(x) = \dfrac{x^2+1}{x^2+2}$, $(g \circ f)(x) = \left(\dfrac{x+1}{x+2}\right)^2 + 1$

2. f. dom $(f \circ g) = \{1, 2, 4, 5\}$, ran $(f \circ g) = \{2, 5\}$

dom $(g \circ f) = \{1, 3, 4, 5\}$ ran $(g \circ f) = \{3, 4, 7\}$

g. dom $(f \circ g) = (-\infty, \infty)$ ran $(f \circ g) = [\frac{1}{2}, 1)$

dom $(g \circ f) = (-\infty, -2) \cup (-2, \infty)$

ran $(g \circ f) = [1, \infty)$

3 d. f^{-1} is not a function since, for example, $f(0) = 0 = f(\pi)$. If we restrict f to $[-\pi/2, \pi/2]$, then $f^{-1}(x) = \sin^{-1} x$ is a function.

i. $x = \dfrac{-y}{3y-4} \Rightarrow 3xy - 4x = -y$ $y(3x+1) = 4x$

$\Rightarrow 3xy + y = 4x \Rightarrow y = \dfrac{4x}{3x+1}$

$f^{-1}(x) = \dfrac{4x}{3x+1}$ is a function.

13. a. Let $x \in \mathbb{R}$. Then $(f_1 + f_2)(x) = f_1(x) + f_2(x) \in \mathbb{R}$ which is unique since $f_1(x)$ and $f_2(x)$ are unique. Similarly, $(f_1 f_2)(x) = f_1(x) \cdot f_2(x) \in \mathbb{R}$ and this value is unique since $f_1(x)$ and $f_2(x)$ are unique. Thus $f_1 + f_2$, $f_1 \cdot f_2$ are functions on $\mathbb{R}$.

b. $(f+g)(x) = (2x+5) + (6-7x) = -5x + 11$

$(fg)(x) = (2x+5)(6-7x) = -14x^2 - 23x + 30$

$(f+h)(x) = (2x+5) + (3x^2 - 7x + 2) = 3x^2 - 5x + 7$

$(gh)(x) = (6-7x)(3x^2 - 7x + 2) = -21x^3 + 67x^2 - 56x + 12$

Math 301 Homework

4.3 1beh, 2beh, 3, 4, 6, 9d, 13, 15, 16

1beh

1b. $f: \mathbb{Z} \to \mathbb{Z}$, $f(x) = -x + 1000$

f is onto: ~~since if we~~ Take $y \in \mathbb{Z}$ and let $x = -y + 1000$, we get:

$f(x) = -(-y+1000) + 1000 = y$.

1e. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \sqrt{x^2+5}$ is not onto

for example, choose $y = 0$

$0 = \sqrt{x^2+5} \Rightarrow x^2+5 = 0$

$\Rightarrow x^2 = -5$

Which is not true for any $x \in \mathbb{R}$.

1h. $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $f(x,y) = x - y$

f is onto: Let $z \in \mathbb{R}$ and choose $(x,y) = (z,0)$.

Then $f((x,y)) = f(z,0) = z - 0 = z$.

Thus f is onto.

2beh.

2b. f is one-to-one:

Suppose $f(x_1) = f(x_2)$

then $-x_1 + 1000 = -x_2 + 1000$

$\Rightarrow -x_1 = -x_2$

$\Rightarrow x_1 = x_2$

Thus, if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

2e. f is not one-to-one. e.g. let $x = -2$, $y = 2$

$f(x) = \sqrt{4+5} = 9 = f(y)$, but $x \neq y$

2h. f is not one-to-one. e.g. $f(2,0) = 2 = f(3,1)$

3, 4, 6, 9d, 13, 15, 16

3. a, $B = \{1\}$ $f = \{(1,1),(2,1),(3,1),(4,1)\}$

b. $B = \{1,2,3,4,5\}$ $f = \{(1,1),(2,2),(3,3),(4,4)\}$

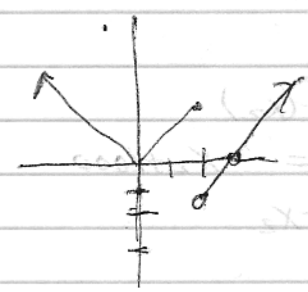
c. $B = \{1,2,3,4\}$ $f = \{(1,1),(2,2),(3,3),(4,4)\}$

d. $B = \{1,2\}$ $f = \{(1,1),(2,1),(3,1),(4,1)\}$

4. Let $z \in C$. Then $\exists\, y \in B$ with $g(y) = c$ since $g: B \to C$ is onto. Since $f: A \to B$ is also onto, then $\exists\, x \in A$ with $f(x) = y$.
~~thus~~ Now $(g \circ f)(x) = g(f(x)) = g(y) = z$.
So, $g \circ f$ is onto.

6. Suppose $g \circ f: A \to C$ is one-to-one
Now suppose $f(x_1) = f(x_2)$
Then $g(f(x_1)) = g(f(x_2))$
$\Rightarrow (g \circ f)(x_1) = (g \circ f)(x_2)$
$\Rightarrow x_1 = x_2$ (Since $g \circ f$ is one-to-one
Thus $f: A \to B$ is one-to-one.

9d.



Not one-to-one since
$f(3) = f(0) = 0$, for example.

Not onto since, for example,
If we choose $y = -3$
$|x| \neq -3$ for any $x \in (-\infty, 2]$
Since $|x| \geq 0$. And
$x - 3 > -1$ when $x > 2$, So
$x - 3 \neq -3$ for any $x \in (0, \infty)$

13. Recall that f is increasing on $\mathbb{R}$ if $f(x_2) > f(x_1)$ whenever $x_2 > x_1$.

Suppose $x_1 \neq x_2$. Then without loss of generality, assume $x_1 < x_2$.
Since f is increasing, then $f(x_1) < f(x_2)$.
Thus $f(x_1) \neq f(x_2)$.
Thus $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$, so f is one-to-one.

15. a) 1, 1, 2, 3, 5, 8, ...
1, 1, 1, 3, 3, 3, 5, 5, 5, ...

b) 1, 2, 3, 4, 5, 6, ...
2, 1, 4, 3, 6, 5, ...

c) 1, 3, 5, 7, 9, ...
2, 4, 6, 8, 10, ...

d) 1, 1, 2, 3, 4, 5, 6, 7, ...
1, 1, 1, 2, 3, 4, 5, 6, 7, ...

16. a) $n \cdot (n-1)(n-2) \cdots (n-m+1) = P(n, m)$
b) $m! = n! = P(n,n)$
c) None
d) None
e) $m! = n! = P(n,n)$
f) ~~(m+n)~~ $n \cdot \binom{n+1}{2} \cdot (n-1)!$ ← map the rest as a 1-1 corresp.
~~choose~~ leave choose element to get hit twice; choose two elements to hit it
g) $n! = m!$ (one-to-one $\Leftrightarrow$ onto when $|m| = |n|$)

Which are functions on A? $f: A \to A$ only R_3, R_8

Let $A = \{a,\ b,\ c\}$. Answer questions for the following relations on A.

$R_1 = \{(a,b)\}$
$R_3 = \{(a,b),\ (b,c),\ (c,a)\}$
$R_5 = \{(a,a),\ (b,b),\ (a,b),\ (c,c)\}$
$R_7 = \{(a,b),\ (b,c),\ (a,c)\}$
$R_9 = \emptyset$

$R_2 = \{(a,a),\ (a,c),\ (c,\ a)\}$
$R_4 = \{(a,a),\ (a,c),\ (c,\ c)\}$
$R_6 = \{(a,a),\ (b,b),\ (c,\ c),\ (b,c),(c,b)\}$
$R_8 = \{(a,a),\ (b,b),\ (c,\ c)\}$
$R_{10} = A \times A$
$R_{11} = \{(a,a)\}$

a) Which of the above relations are reflexive? R_5, R_8, R_6, R_{10}

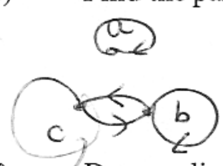
b) Which of the above relations are symmetric? $R_2, R_6, R_8, R_9, R_{10}, R_{11}$

c) Which of the above relations are transitive? $R_1, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, R_{11}$

d) Which of the above relations are equivalence relations.

R_6, R_8, R_{10}

e) Find the partition of A for each equivalence relation.



$A = \{\{a\}, \{b,c\}\}$

for R_6

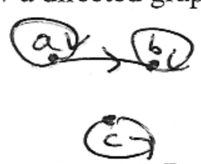
$A = \{\{a\}\ \{b\}, \{c\}\}$

for R_8

for R_{10}

$\{\{a,b,c\}\}$

f) Draw a directed graph for R_5.



g) Find the relation $R_3 \circ R_4$. $R_3 \circ R_4 = \{(a,b), (a,a), (c,a)\}$

h) Find the relation R_7^{-1} $\{(b,a), (c,b), (c,a)\}$

Let R be the relation defined by xRy iff $y < x + 2$. Does R form an equivalence relation on $\mathbb{Z}$?

✓ ① Reflexive: $x < x+2$ for all $x \in \mathbb{Z}$ ✓

No ② Symmetric: No e.g. $2 < 5+2$ but $5 \not< 2+2$
So $2R5$ but $5 \not R 2$

No! ③ Transitive: Suppose $y < x+2$ and $z < y+2$, is $z < x+2$?
No: $2 < 1+2$ and $3 < 2+2$ but $3 \not< 1+2$

Let R be the relation defined by xRy iff 3 divides $y - x$. Does R form an equivalence relation on $\mathbb{Z}$?

① Reflexive: $3 \mid (x-x)$ for all $x \in \mathbb{Z}$ since $3 \cdot 0 = x-x$

② Symm: Supp $3 \mid (y-x)$ then $3k = y-x$ for some $k \in \mathbb{Z}$.
Now $3(-k) = x-y$, so $3 \mid (x-y)$
Thus, if $3 \mid (y-x)$ then $3 \mid (x-y)$

③ Trans: supp $3 \mid (y-x)$ and $3 \mid (z-y)$ then $z-x = z-y+y-x = 3k_2 + 3k_1 = 3k$.