11.1 Functions of Several Variables

Be familiar with functions of two variables and the graphs of functions of two variables. Given a function of two variables you should be able to correctly choose a graph of that function from several graphs. Be able to sketch a level curve of a function of two variables.

11.2 Limits and Continuity

Given a function of two variables be able to show that the limit $\lim_{(x,y)\to(a,b)} f(x,y)$ does not exist. Note: if (a,b) is in the domain of f and f is a polynomial, rational, or trigonometric function then f is continuous at (a,b) and we know that the limit $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$.

11.3 Partial Derivatives

Given a function of several variables, be able to calculute any of its partial derivatives, second partial derivatives and any of its mixed partial derivatives.

11.4 Tangent Planes and Linear Approximations

Given a function of two variables, be able to construct an equation of a tangent plane at a given point. Recall that an equation of a tangent plane can be constructed as

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

where (x_0, y_0) is a given point and $z_0 = f(x_0, y_0)$. An alternate form is:

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

where the given point in the domain is represented by (a,b).

11.5 The Chain Rule.

Given a function of several variables (e.g. $z = f(x_1, x_2, x_3)$ and the variables x_i are in turn written as functions of one or more variables (e.g. $x_3 = g(t_1, t_2)$ be able to calculate any of the associated partial derivatives $\frac{\partial z}{\partial t_i}$ by using the generalized chain rule. Note: Cases 1 and 2 mentioned in this section are covered by the general version of the chain rule mentioned on page 630. It is also very helpful to construct the associated tree diagrams.

In this section we discussed implicit differentiation and if you have an equation of multiple variables you should be prepared to calculate the derivative of any one of theres variables with respect to any of the others. Recall we rewrote the equation in the form

$$F(x,y,z) = 0$$

then we were able to find

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \quad \frac{\partial x}{\partial y} = -\frac{F_y}{F_x} \quad etc.$$

11.6 Directional Derivatives and the Gradient Vector

Given a function of several variables, be able to calculate the value of the directional derivative $D_{\mathbf{u}}$ for a unit vector $\mathbf{u}$. Be familiar with the definition of the gradient vector ∇f and its properties.

11.7 Maximum and Minimum Values

Given a function of two variables be able to find critical points and use the second derivative test to determine if the the function has a relative maximum or minimum at the critical points. Also, if you are given a closed and bounded region for the function, be able to absolute maximum and minimum values of the function on the region.