Practice roblems for final exam

1. A certain vault requires that an entry code be 8 characters. If the first 4 characters must be letters (**repeated letters are allowed**) and the last 4 characters are numeric digits (**repeated digits are not allowed**), how many possible entry codes are there?

2. A music library has 200 songs. How many 5 song playlists can be constructed in which the order of the songs matters?

3. It is known that on Valentine's Day 30% of all couples exchange gifts, 20% dine out and 18% exchange gifts **and** dine out. If a couple is randomly selected, calculate the following probabilities (Hint: sketch a Venn diagram)

   (a) Find P(they do not exchange gifts)

   (b) Find P(they exchange gifts or they dine out)

   (c) Find P(they dine out given that they exchange gifts)

   (d) Find P(they exchange gifts given that they do not dine out)

   (e) Are the events "Dining out" and "Exchanging gifts" **independent** or **dependent** of one another? To receive credit for you answer, you must explain your choice.

4. A box contains 18 marbles (4 green, 14 red) and we randomly select two marbles from the box, selecting one at a time without replacement.

   (a) Find the probability that the first one is red and the second one is green.

   (b) Find the probability that one of the selected marbles is red and the other is green.

5. A bag contains 30 M&M candies (8 yellow, 5 green, 7 red and 10 blue). We randomly select 6 of the M&Ms all at once.

   (a) How many possible outcomes are there?

   (b) Find the probability that in our selection, all 6 are blue.

   (c) Find the probability that in our selection all 6 are green.

   (d) Find the probability that in our selection none are red.

   (e) Find the probability that in our selection, 2 are yellow, 2 are blue and 2 are green.

6. A computer store sells three different brands of computers. 50% of the computers sold are brand A, 30% are brand B and 20% are brand C. Of the brand A computers 2% are returned for repair, 4% of brand B are returned for repair and 3% of the brand C are returned for repair.

   (a) Sketch and label a tree diagram that you could use to describe the associated conditional probabilities.

   (b) Find the probability that a randomly selected computer is brand C **and** not returned for repair.

   (c) Find the probability that a randomly selected computer is not returned for repair.

(d) Given that a computer is returned for repair, what is the probability that it is brand B?.

7. A boiler has five identical relief valves. The probability that any particular valve will open on demand is .95. Assuming independent operation of the valves, calculate:

   (a) P(all valves open)

   (b) P(at least one valve opens)

8. A sample of 7 beans is randomly selected and planted in the ground. The number of days it takes each bean to germinate is recorded. The **sample** data values are: 9, 7, 8, 8, 6, 9, 9.

   (a) Calculate the median of this sample data.

   (b) Calculate the mean of this sample data.

   (c) Calculate the **sample** variance of this sample data.

   (d) Carefully sketch a dot plot for this data.

9. Consider the probability mass function for the discrete r.v. X:

x	-2	0	1	2
$p(x)$	.3	.4	.1	

   (a) Construct the associated cumulative distribution of X.

   (b) Find the expected value of X.

   (c) Find the variance of X.

10. It is believed that 20% of all college students this year will get the H1N1 flu virus. In an experiment we randomly select 15 students and we let the random variable X be the number of students (among the selected 15) that get the virus.

   (a) What is the probability that exactly 6 students get the H1N1 virus?

   (b) What is the probability that *at least four* of the students get the H1N1 virus?

   (c) Of these 15 students, what is the expected number students getting the H1N1 virus?

   (d) Find the **standard deviation** of X.

11. A machine produces 12,000 memory chips in a day. The probability that a single memory chip is defective is .0003. What is the probablity that in a given day's production of 12,000 memory chips, exactly 4 are defective.

12. On average, **three** fish pass through the McNary Dam fish ladder every second. Suppose the number of fish passing through the ladder is a Poisson random variable. Find the probability that during a 5 second period

   (a) exactly 10 fish pass through the ladder

   (b) at least 10 fish pass through the ladder.

   (c) What is the expected number of fish passing through the ladder in 5 seconds.

(d) If X is the random variable that describes the number of fish passing through the ladder in a 5 second period, find the standard deviation of X.

13. The continuous random variable X has the following probability function:

$$f(x) = \begin{cases} ke^{-3x} & ,0 < x \\ 0 & ,otherwise \end{cases}$$

(a) Find the value of k that makes f a probability density function.

(b) Find the associated cumulative distribution function, $F(x)$.

(c) Find $P(X \geq 2)$

(d) Find $P(X = 3)$

14. If Z is a continuous random variable with a standard normal distribution find the following

(a) $P(Z \leq -1.23)$

(b) $P(|Z| < .22)$

(c) Find the value c such that $P(-c \leq Z \leq c) = .90$

15. The eggs from Old McDonald's farm have weights that are normally distributed with mean $\mu = 63$ gm. and standard deviation $\sigma = 2.75$ gm.

(a) If we randomly select an egg from this farm, what is the probability that it weighs more than 65 gm.?

(b) Calculate the 90^{th} percentile for the weights of these eggs.

16. Let X be a random variable such that $E[X] = 20$ and $V[X] = 5$. If Y is a random variable defined by $Y = 3X + 2$,

(a) Calculate $E[Y]$

(b) Calculate $V[Y]$

17. Let X be a random variable with mean, $\mu = 26$ and variance $\sigma^2 = 4$. If samples of size 9 are taken from this distribution and the statistic $\overline{X}$ is calculated,

(a) what is the mean of $\overline{X}$?

(b) what is the standard deviation of $\overline{X}$?

18. Concrete blocks made by ACME Block have a compression strength that is normally distributed with a mean of 5220 psi and standard deviation 40 psi.

(a) Calculate the probability that the sample mean $\overline{X}$ is less than 5209 psi if a random sample of size $n = 25$ is taken.

(b) If the sample size from part 18a is changed from 25 to 36, how does $P\left(\overline{X} < 5209\right)$ change? Does it increase, decrease or does it stay the same? Explain your answer.

19. Let X be a descrete random variable with the following distribution.

x	1	2
$p(x)$	.6	.4

   (a) Construct the distribution (pmf) for the sample mean $\overline{X}$, with sample size $n = 3$.
   (b) Find $P\left(\overline{X} \leq 1.5\right)$
   (c) Show that $E\left(\overline{X}\right) = \mu_X$

20. In a sample of 14 ACME brand automotive batteries, the sample mean lifetime is 5.8 years. Assuming that battery lifetimes are normally distributed with standard deviation 0.4 years,

   (a) construct a 99% confidence interval of the true mean lifetime of all ACME automotive batteries.
   (b) construct a 99% **lower confidence bound** for the true mean lifetime of ACME automotive batteries.

21. If the sugar content of apples is normally distributed with standard deviation $\sigma = 4.3$ grams, **how large a sample do we need** in order to construct a 90% confidence interval for the true average sugar content of the apples, within 1 gram.

22. A sample of 50 surgical patients had a mean hospital stay of 5.1 days with a standard deviation of 0.4 days. Find a 95% confidence interval for the true mean hospital stay.

23. We want to estimate the true proportion of Spokane residents who are in support of a new swimming center. We want our 95% confidence interval to estimate this within 4% . How large a sample should we take?

24. Let p represent the proportion of university faculty with Ph.D. degrees. In a random sample of 40 faculty members we find that 25 have Ph.D.s. Find an 80% confidence interval for p.

25. Consider a continuous random variable X whose distribution is given by the following pmf:

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & otherwise \end{cases}$$

   (a) Calculate the value of k.
   (b) Calculate $E[X]$, the expected value of X.

26. Consider a continuous random variable X whose distribution is given by the following pmf:

$$f(x) = \begin{cases} \frac{2}{x^3} & 1 \leq x \\ 0 & x < 1 \end{cases}$$

   (a) Construct the cumulative distribution function, $F(x)$.
   (b) Calculate the 75^{th} percentile for this distribution.

27. In a sample of 14 ACME brand automotive batteries, the sample mean lifetime is 5.8 years and the sample standard deviation of 0.4 years. Assume that battery lifetimes are normally distributed.

   (a) Construct a 99% confidence interval of the true mean lifetime of all ACME automotive batteries.

   (b) Construct a 95% confidence upper bound of the true mean lifetime of all ACME automotive batteries.

28. A sample of nine customers purchase gasoline at a local gas station and the purchased gallons are as follows:

8.2	4.5	26.6	13.1	10	14.8	18.6	9.3	11.2

with sample variance $s^2 = 41.576$. If we know that the gallons purchased is normally distributed, **find an 80% confidence interval** for the population standard deviation, σ.

29. Determine the 95th percentile of a χ^2 chi-square distribution with 12 degrees of freedom.

30. A cereal company claims that they have 250 raisins in each 24 oz. box of cereal. Two hundred boxes were randomly selected and the sample mean was found to be 247 raisins, with a sample standard deviation of 15.3 raisins. Test at $\alpha = 0.01$ the null hypothesis $H_0 : \mu = 250$ versus the alternative hypothesis $H_a : \mu < 250$.

   (a) Give the critical value or values:

   (b) Compute the value of the test statistic:

   (c) State your conclusion in terms of the null hypothesis:

31. In Yakima County it is generally believed that 63% of the voting population consider themselves Republican. However, a research group believes that proportion to be higher. They sample 120 voters, of which, 81 say they are Republican. At significance level $\alpha = 0.1$ test the null hypothesis $H_0 : p = 0.63$ against the alternative hypothesis $H_a : p > 0.63$

   (a) Give the critical value or values:

   (b) Compute the value of the test statistic:

   (c) State your conclusion in terms of the null hypothesis:

32. A machine is used to fill soda pop bottles. It fills bottles in a **normally distributed** manner with a standard deviation $\sigma = 0.4$ ounces and with a supposed mean of 16 oz. A random sample of 36 bottles has a sample mean 16.15. We test the null hypothesis $H_0 : \mu = 16$ against the alternative $H_a : \mu \neq 16$ with a level of significance $\alpha = 0.02$.

   (a) Give the critical value or values:

   (b) Compute the value of the test statistic:

   (c) State you conclusion in terms of the null hypothesis.

33. An airline says that it books an average of 75 people on each plane trip. A sample of 9 trips showed a mean of 80 and a standard deviation of 7. At $\alpha = 0.01$ test the null hypothesis $H_0 : \mu = 75$ versus the alternative hypothesis $H_a : \mu > 75$. It is assumed that the number of people on each plane is normally distributed.

(a) Give the critical value or values:

(b) Compute the value of the test statistic:

(c) State your conclusion in terms of the null hypothesis:

34. A mathematics instructor thinks that the mean of scores on the final exam in Calc III is greater for the Fall semester than for the Spring semester. She samples students from each of these two groups and obtains the following data. Based on the this data, test the null hypothesis $H_0 : \mu_1 - \mu_2 = 0$ against the alternative hypothesis $H_a : \mu_1 - \mu_2 > 0$ at $\alpha = 0.05$.

	Fall Semester (X)	Spring Semester (Y)
Sample Size	45	50
Mean Score	156.5	142.7
Sample **Variance**	146.1	206.2

(a) Give the critical value or values:

(b) Compute the test statistic:

(c) State your conclusion in terms of the null hypothesis:

35. Honda has introduced new engineering in its automobile engines in 2007 models with the hope of increasing fuel efficiency. To see if the program is working they sample the fuel economies of Honda Accords produced in 2006 and a sample of fuel economies of Honda Accords produced in 2007. Based on the following sample data, test the null hypothesis, $H_o : \mu_1 - \mu_2 = 0$, against the alternative hypothesis $H_a : \mu_1 - \mu_2 < 0$ at $\alpha = .05$. You may assume that the populations are normally distributed, however, $\sigma_!^2 \neq \sigma_2^2$.

Year	2006 Models	2007 Models
Mean m.p.g	$\overline{X}_1 = 23.4$	$\overline{X}_2 = 29.3$
Sample size	$n_1 = 5$	$n_2 = 7$
Standard deviation	$s_1 = 0.8$	$s_2 = 2.3$

(a) Give the critical value or values:

(b) Compute the test statistic:

(c) State your conclusion in terms of the null hypothesis:

36. We are testing a null hypothesis $H_0 : \mu_1 - \mu_2 = 0$ against an alternative hypothesis $H_a : \mu_1 - \mu_2 < 0$. We take a sample of 50 from each of two populations and our test statistic has a value of $z = -2.69$. Find the P-value associated with this test statistic.

37. In a test specified by the null hypothesis $H_0 : \mu = 30$, and alternative hypothesis $H_a : \mu < 30$ where the sample size is 50 and the critical value is -2.41, what is the probability of a Type I error?

38. A researcher suggests that male nurses earn more than female nurses. A survey of 16 male nurses and 20 female nurses reports the following data. Is there enough evidence to support the claim that male nurses earn more than female nurses? Use $\alpha = 0.05$. Assuming that

both populations are normally distributed and $\sigma_1^2 = \sigma_2^2$, use the following sample information:

Male	Female
$\overline{X}_1 = \$23,800$	$\overline{X}_2 = \$23,750$
$n_1 = 16$	$n_2 = 20$
$s_1 = \$300$	$s_2 = \$250$