*Transfer function of a LTI system*

The behavior of an LTI system is fully determined by the location of its poles (roots of the denominator) and its zeros (roots of the numerator). Poles affect both natural and transient response. Zeros affect only transient response.

$$\mathrm{T(s)} = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + \cdots + a_m s^m}{b_0 + b_1 s + \cdots + b_n s^n} = \frac{a_m}{b_n}\frac{(s - s_{z1}) \dots (s - s_{zm})}{(s - s_{p1}) \dots (s - s_{pn})} =$$

$$= \frac{a_0}{b_0}\frac{\left(1 - \frac{s}{s_{z1}}\right) \dots \left(1 - \frac{s}{s_{zm}}\right)}{\left(1 - \frac{s}{s_{p1}}\right) \dots \left(1 - \frac{s}{s_{pn}}\right)} = \frac{a_0}{b_0}\frac{\left(1 + \frac{s}{\omega_{z1}}\right) \dots \left(1 + \frac{s}{\omega_{zm}}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right) \dots \left(1 + \frac{s}{\omega_{pn}}\right)}$$

If s=jω (sinusoidal analysis):

$$\mathrm{T(j\omega)} = \frac{a_0}{b_0}\frac{\left(1 + \frac{j\omega}{\omega_{z1}}\right) \dots \left(1 + \frac{j\omega}{\omega_{zm}}\right)}{\left(1 + \frac{j\omega}{\omega_{p1}}\right) \dots \left(1 + \frac{j\omega}{\omega_{pn}}\right)} = A_{DC}\frac{\left(1 + \frac{j\omega}{\omega_{z1}}\right) \dots \left(1 + \frac{j\omega}{\omega_{zm}}\right)}{\left(1 + \frac{j\omega}{\omega_{p1}}\right) \dots \left(1 + \frac{j\omega}{\omega_{pn}}\right)}$$

$$\mathrm{T}(j\omega) = T_R(\omega) + jT_I(\omega) = Me^{j\Phi} = Mcos\Phi + jMsin\Phi$$

$$M(\omega) = |T(j\omega| = \sqrt{T_R^2(\omega) + T_I^2(\omega)}$$

$$\Phi(\omega) = \langle T(j\omega) = \tan^{-1}\frac{T_I(\omega)}{T_R(\omega)}$$

Since:

$$\mathrm{T(j\omega)} = A_{DC}\frac{\left(1 + \frac{j\omega}{\omega_{z1}}\right) \dots \left(1 + \frac{j\omega}{\omega_{zm}}\right)}{\left(1 + \frac{j\omega}{\omega_{p1}}\right) \dots \left(1 + \frac{j\omega}{\omega_{pn}}\right)}$$

$$\mathrm{M(\omega)} = A_{DC}\sqrt{\frac{\left[1 + \left(\frac{\omega}{\omega_{z1}}\right)^2\right] \dots \left[1 + \left(\frac{\omega}{\omega_{zm}}\right)^2\right]}{\left[1 + \left(\frac{\omega}{\omega_{p1}}\right)^2\right] \dots \left[1 + \left(\frac{\omega}{\omega_{pn}}\right)^2\right]}}$$

$$\Phi(\omega) = \tan^{-1}\left(\frac{\omega}{\omega_{z1}}\right) + \cdots + \tan^{-1}\left(\frac{\omega}{\omega_{zm}}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{p1}}\right) - \cdots - \tan^{-1}\left(\frac{\omega}{\omega_{pn}}\right)$$

*Bode Rules*

When all poles and zeros are real it is possible to identify a set of rules to construct the Bode plots by inspection.

EE303's additional assumption: all poles are real (no feedback) and in the LHP (stable system)

1) Identify all pole frequencies $\omega pi$ (breakpoint frequencies that appear at the denominator of the TF), and all zero frequencies $\omega zi$ (breakpoint frequencies that appear at the numerator of the TF) and list them in increasing order
2) As $\omega$ passes each pole frequency $\omega pi$ the slope of the magnitude ( $|T(j\omega)|$ ) decreases by 20dB/dec. (20 dB means a tenfold change)
3) As $\omega$ passes each zero frequency $\omega zi$ the slope of the magnitude ( $|T(j\omega)|$ ) increases by 20dB/dec.
4) As $\omega$ crosses a pole frequency $\omega=\omega pi$ the phase ( $\Phi=\angle T(j\omega)$ ) contributes a shift of −45 degrees. Over the interval of frequencies $0.1 \times \omega pi \le \omega \le 10 \times \omega pi$ the phase changes linearly by −90 degrees.
5) As $\omega$ crosses a zero frequency $\omega=\omega zi$ the phase ( $\Phi=\angle T(j\omega)$ ) contributes a shift of +45 degrees for a LHP zero and −45 degrees for a RHP zero. Over the interval of frequencies $0.1 \times \omega zi \le \omega \le 10 \times \omega zi$ the phase changes linearly by +90 degrees (LHP zero) or −90 degrees (RHP zero).

*Useful results to remember about asymptotic Bode plots:*

- At $\omega pi$ the asymptotic plot of the magnitude is 3 dB above the effective plot
- At $0.5 \times \omega pi$ and $2 \times \omega pi$ the asymptotic plot of the magnitude is 1 dB above the effective plot
- 1 dB corresponds to a ratio of 1.1 (that is a 10% error)
- −3 dB corresponds to a ratio of $1/\sqrt{2} \approx 0.707$
- At $0.1 \times \omega pi$ the asymptotic plot of the phase is $5.7^\circ$ below the effective plot
- At $10 \times \omega pi$ the asymptotic plot of the phase is $5.7^\circ$ above the effective plot