

# Chapter 2

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## Electronics I - Semiconductors

Fall 2017

# Charged Particles

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- The operation of all electronic devices is based on controlling the flow of charged particles
- There are two type of charge in solids
  - Electrons
  - Holes
- There are two mechanism through which charge can be **transported** in a material
  - **Drift** (motion of charge caused by an electric field)
  - **Diffusion** (motion resulting from a non-uniform charge distribution)

# Electronic Structure of the elements

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- Atom's chemical activity depends on the electrons in the outermost shells (orbitals). These electrons are called VALENCE electrons.
- In extremely pure elements, such as silicon, the atoms arrange themselves in regular patterns called CRYSTALS. The valence electrons determine the exact shape (= LATTICE structure)

# Electronic orbitals in silicon

Element	Atomic Number	Configuration
Si	14	$(1s)^2 (2s)^2 (2p)^6 (3s)^2 (3p)^2$

s, p, d designate orbital shape  
s holds up to 2 electrons  
p holds up to 6 electrons  
d holds up to 10 electrons

$(1s)^2$  (filled)



$(2s)^2$  (filled)



$(2p)^6$  (filled)



$(3sp)^4$



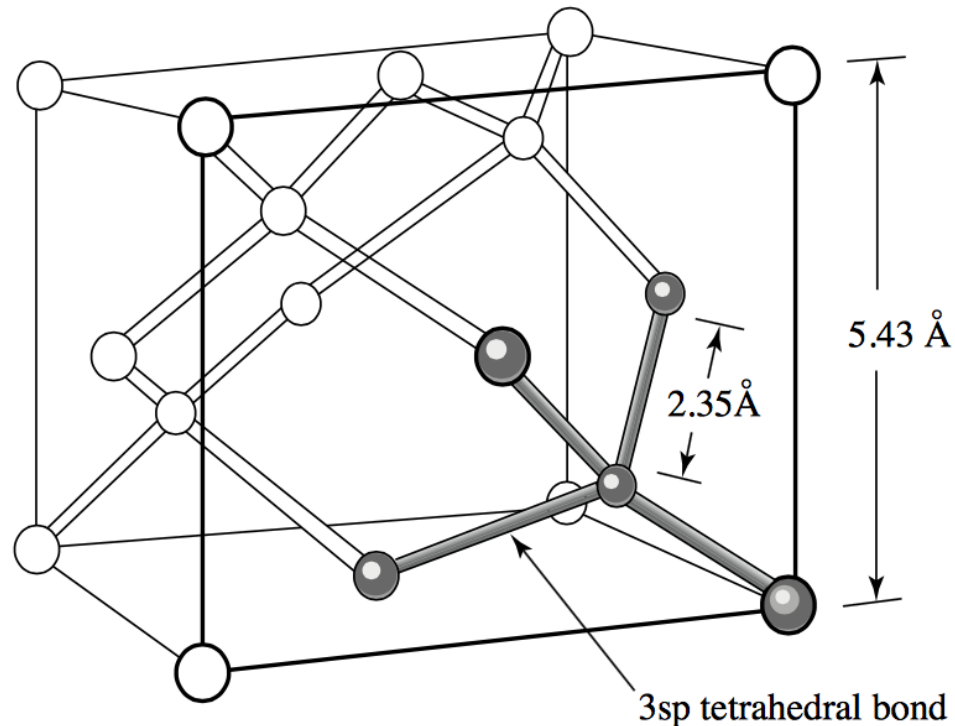
4 tetrahedral hybrid orbitals, half-filled with 4 valence electrons

electronic orbitals  
in silicon

Source: Howe & Sodini

# Silicon Crystal Lattice

Source: Howe & Sodini



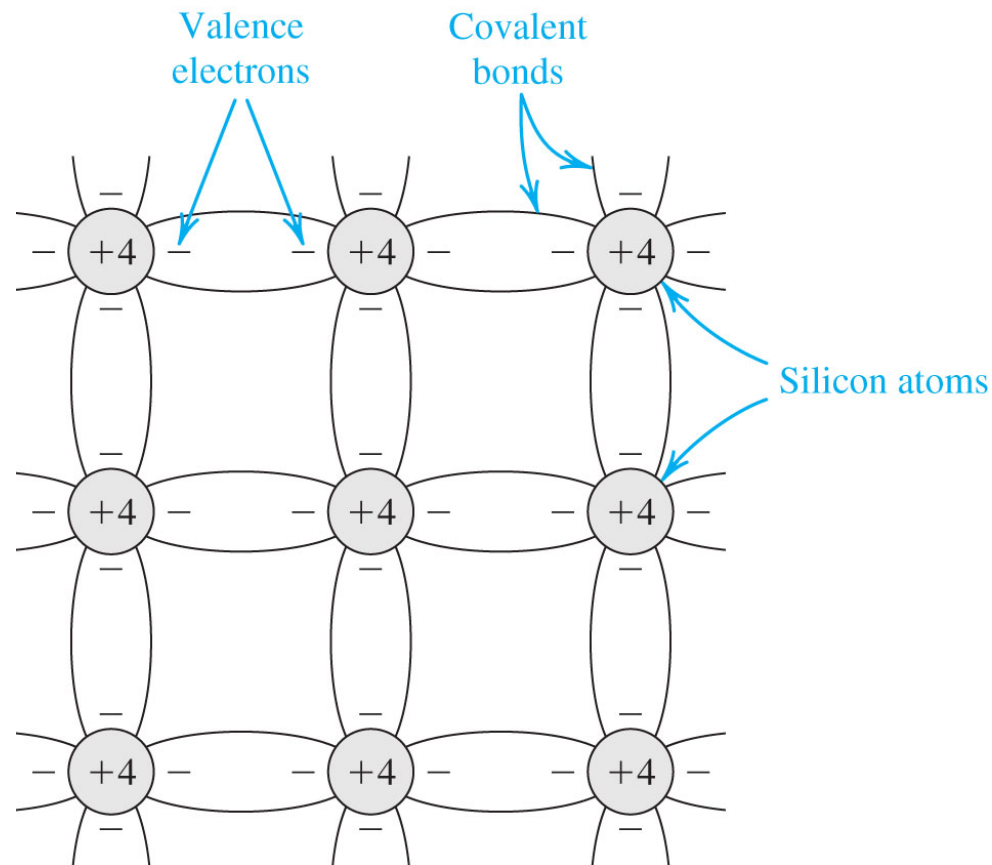
concentration (atoms/cm<sup>3</sup>)  
of atoms in silicon

$$N_{Si} = 8 \cdot n_{cell} = 8 \cdot \frac{1}{a_o^3} = \frac{8}{(5.43 \times 10^{-8} \text{ cm})^3} = 5.00 \times 10^{22} \text{ cm}^{-3} \quad (2.1)$$

where  $n_{cell}$  is the number of unit cells per cm<sup>3</sup>. Since each atom has four tetrahedral bonds and contributes one electron to each bond, the concentration of valence electrons is  $4 N_{Si} = 2 \times 10^{23} \text{ cm}^{-3}$ .

# 2D-Representation of Silicon Crystal

Source:  
Sedra & Smith

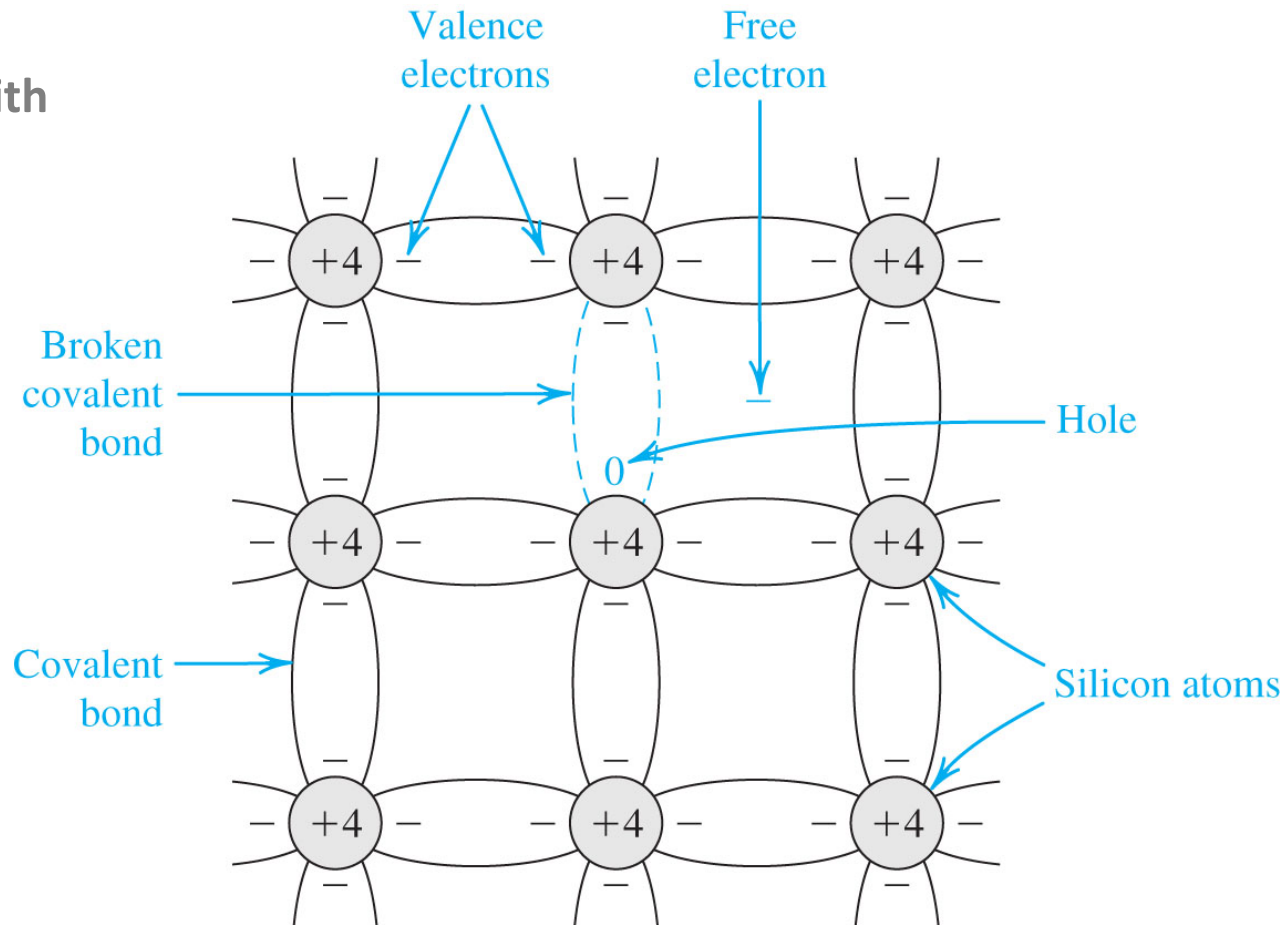


**Figure 3.1** Two-dimensional representation of the silicon crystal. The circles represent the inner core of silicon atoms, with +4 indicating its positive charge of  $+4q$ , which is neutralized by the charge of the four valence electrons. Observe how the covalent bonds are formed by sharing of the valence electrons.

At 0 K, all bonds are intact and no free electrons are available for current conduction.

# Free electrons and holes

Source:  
Sedra & Smith



**Figure 3.2** At room temperature, some of the covalent bonds are broken. Each broken bond gives rise to a free electron and a hole, both of which become available for current conduction.

# Periodic Table

**Table 1.2** Abbreviated Periodic Chart of the Elements.

II	III	IV	V	VI
4 Be	5 B	6 C	7 N	8 O
12 Mg	13 Al	14 Si	15 P	16 S
30 Zn	31 Ga	32 Ge	33 As	34 Se
48 Cd	49 In	50 Sn	51 Sb	52 Te
80 Hg	81 Tl	82 Pb	83 Bi	84 Po



# Energy Band Structure

- Band gap energy ( $E_g$ ) is the minimum energy to dislodge an electron from its covalent bond.
- For Silicon at room temp. ( $T=300\text{ }^\circ\text{K}$ )  $E_g = 1.12\text{ eV} = 1.792 \times 10^{-19}\text{ Joule}$

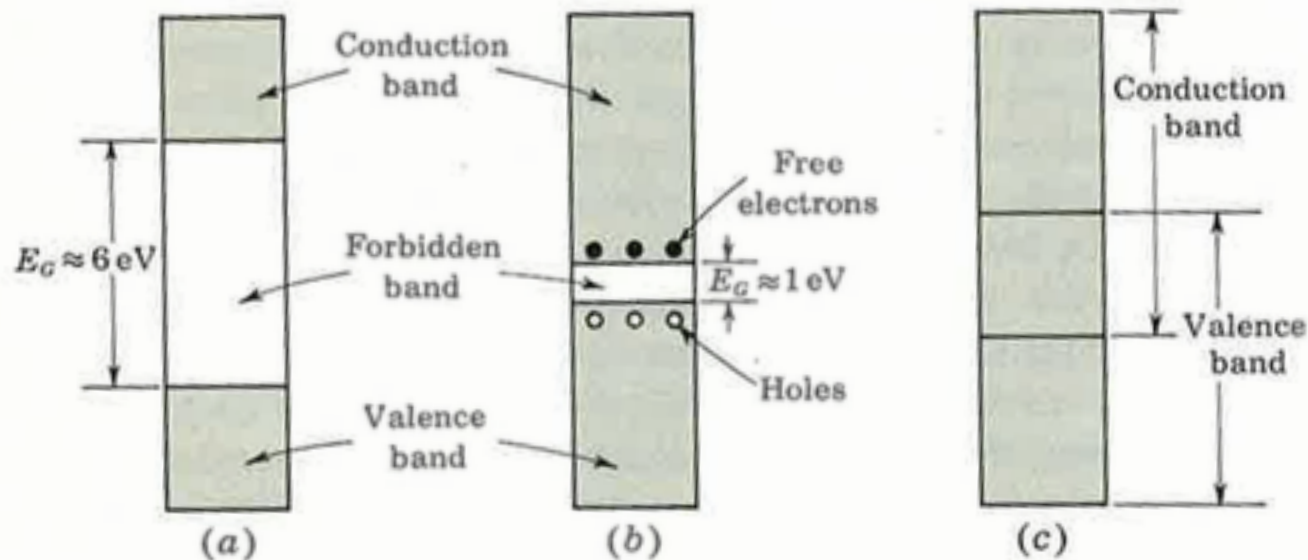


Fig. 1-4 Energy-band structure of (a) an insulator, (b) a semi-conductor, and (c) a metal.

# Concentration of free electrons

- The concentration of electrons (and holes) in pure silicon at room temperature is approximately:

$$n_i(T = 300 K) \cong 1 \times 10^{10} \text{ cm}^{-3}$$

- As temperature increases, the intrinsic concentration  $n_i$  approximately doubles every 10°C rise over room temperature (source: Howe & Sodini)

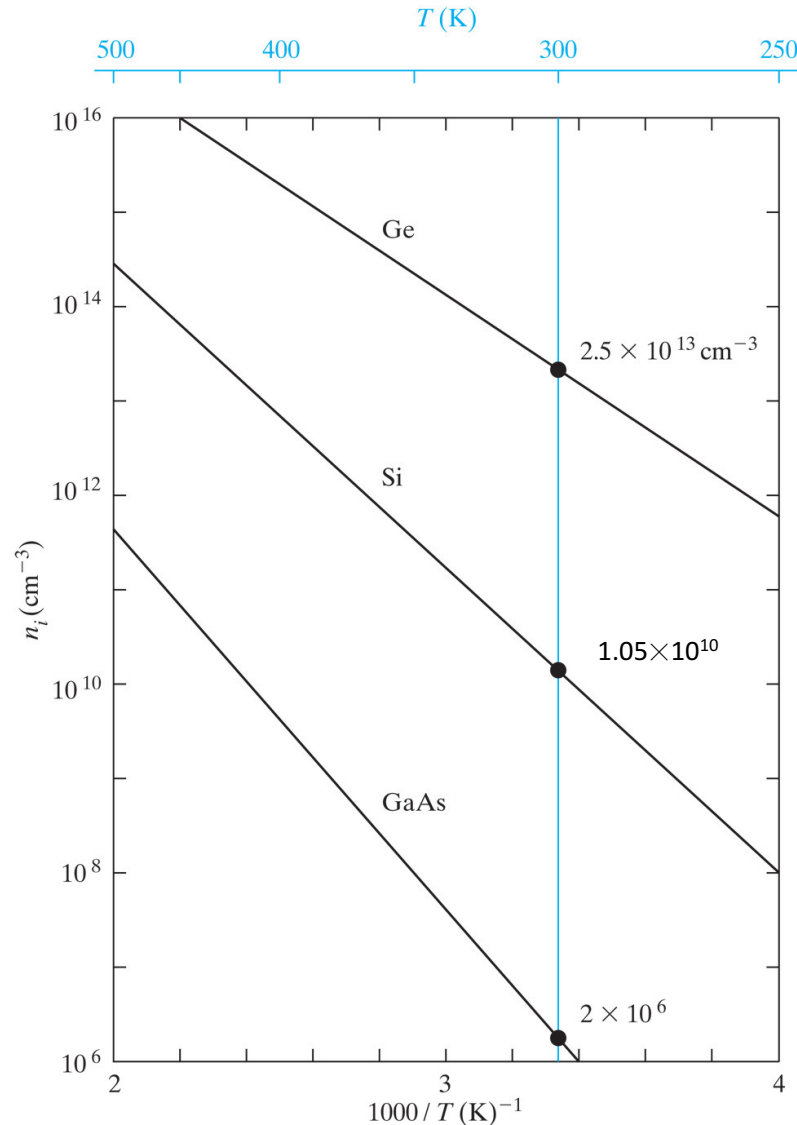
$$n_i(T) \cong n_i(T_{300}) \times 2^{\frac{T - T_{300}}{10}} \text{ cm}^{-3}$$

- Given that the number of bonds is  $2 \times 10^{23} \text{ cm}^{-3}$ , at room temperature only an extremely small fraction of the bonds are broken (1 in  $20 \times 10^{12}$  bonds, that is 1 in  $5 \times 10^{12}$  atoms)
- **Too few: we need more !!!**

# Intrinsic carrier concentration as a function of temperature

source: Streetman

For an intrinsic semiconductor:  
 $n = p = n_i$



$300 \text{ }^\circ\text{K} = 27 \text{ }^\circ\text{C}$

$1000/300 \approx 3.33$

# Intrinsic carrier concentration

$E_G$  also depends on  $T$



$$n_i = \sqrt{A_C A_V} T^{3/2} e^{-E_G/(2KT)} \cong 5 \times 10^{15} T^{3/2} e^{-E_G/(2KT)} \text{ (cm}^{-3}\text{)}$$

The constants  $A_C$  and  $A_V$  can be derived from the effective density of the states in conduction band  $N_C$  (cm<sup>-3</sup>) and valence band  $N_V$  (cm<sup>-3</sup>).

$$N_C = A_C T^{3/2}$$

$$N_V = A_V T^{3/2}$$

For silicon at  $T = 300\text{K}$  (source Pierret):  $N_C = 3.22 \times 10^{19} \text{ cm}^{-3}$  and  $N_V = 1.83 \times 10^{19} \text{ cm}^{-3}$

$$n_i(T = 300 \text{ K}) \cong 1 \times 10^{10} \text{ cm}^{-3}$$

Boltzmann constant =  $K = 8.617 \times 10^{-5} \text{ eV/K} = 1.38 \times 10^{-23} \text{ J/}^\circ\text{K}$

Energy Gap for silicon at room temperature = 1.12 eV

# Energy Band Gap

- The energy band gap  $E_g$  is affected by temperature according to the following Varshni equation:

$$E_g(T) = E_g(0) - \frac{\alpha_E T^2}{T + \beta_E} \quad (\text{eV})$$

- where  $E_g(0)$  is the band gap energy at absolute zero and  $\alpha_E$  and  $\beta_E$  are material specific constants

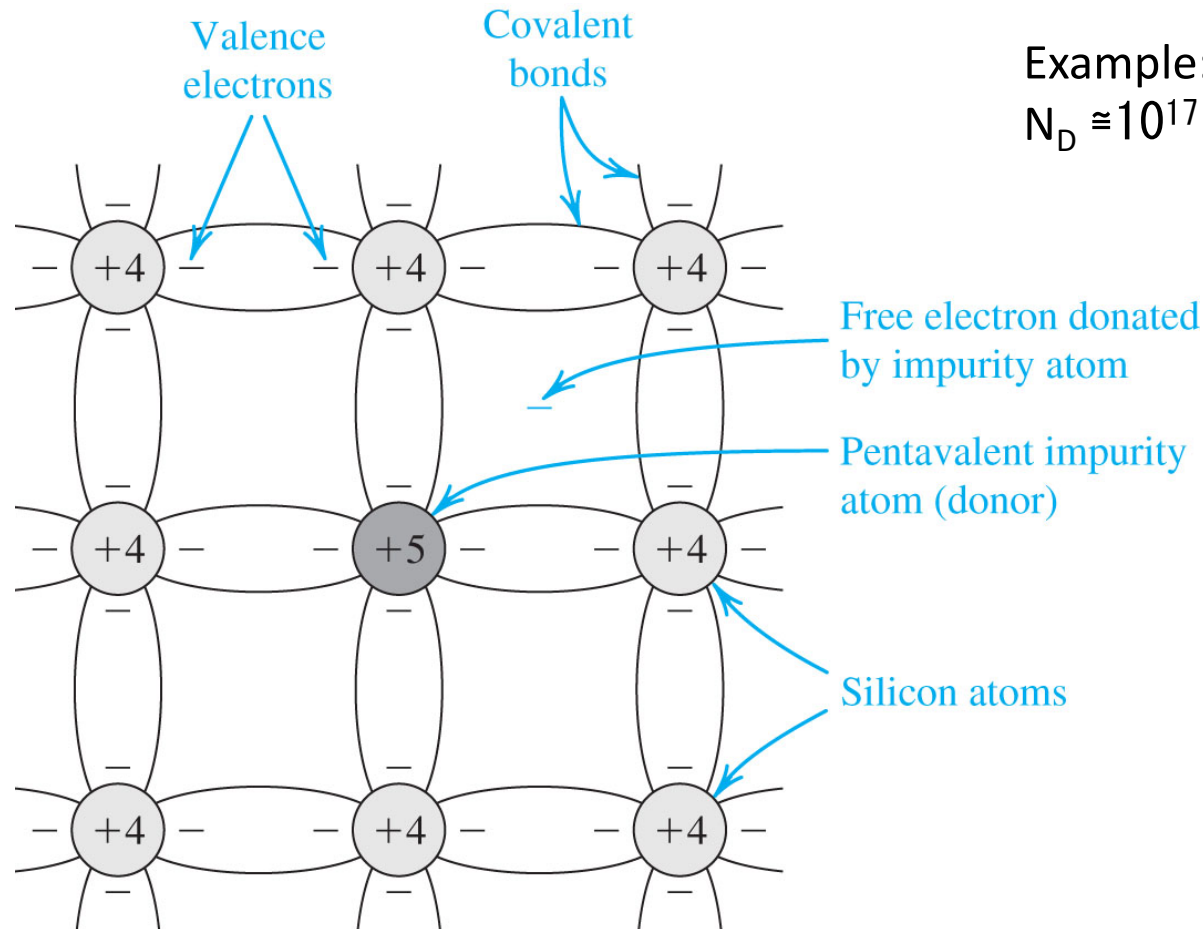
Material	$E_g(0)$ (eV)	$\alpha_E$ (eV/K)	$\beta_E$ (K)
GaAs	1.519	$5.41 \cdot 10^{-4}$	204
Si	1.170	$4.73 \cdot 10^{-4}$	636
Ge	0.7437	$4.77 \cdot 10^{-4}$	235

# Extrinsic Semiconductors (1)

- Doping with donor impurities (N-type semiconductor)

source:  
Sedra & Smith

Example:  
 $N_D \approx 10^{17} \text{ cm}^{-3}$

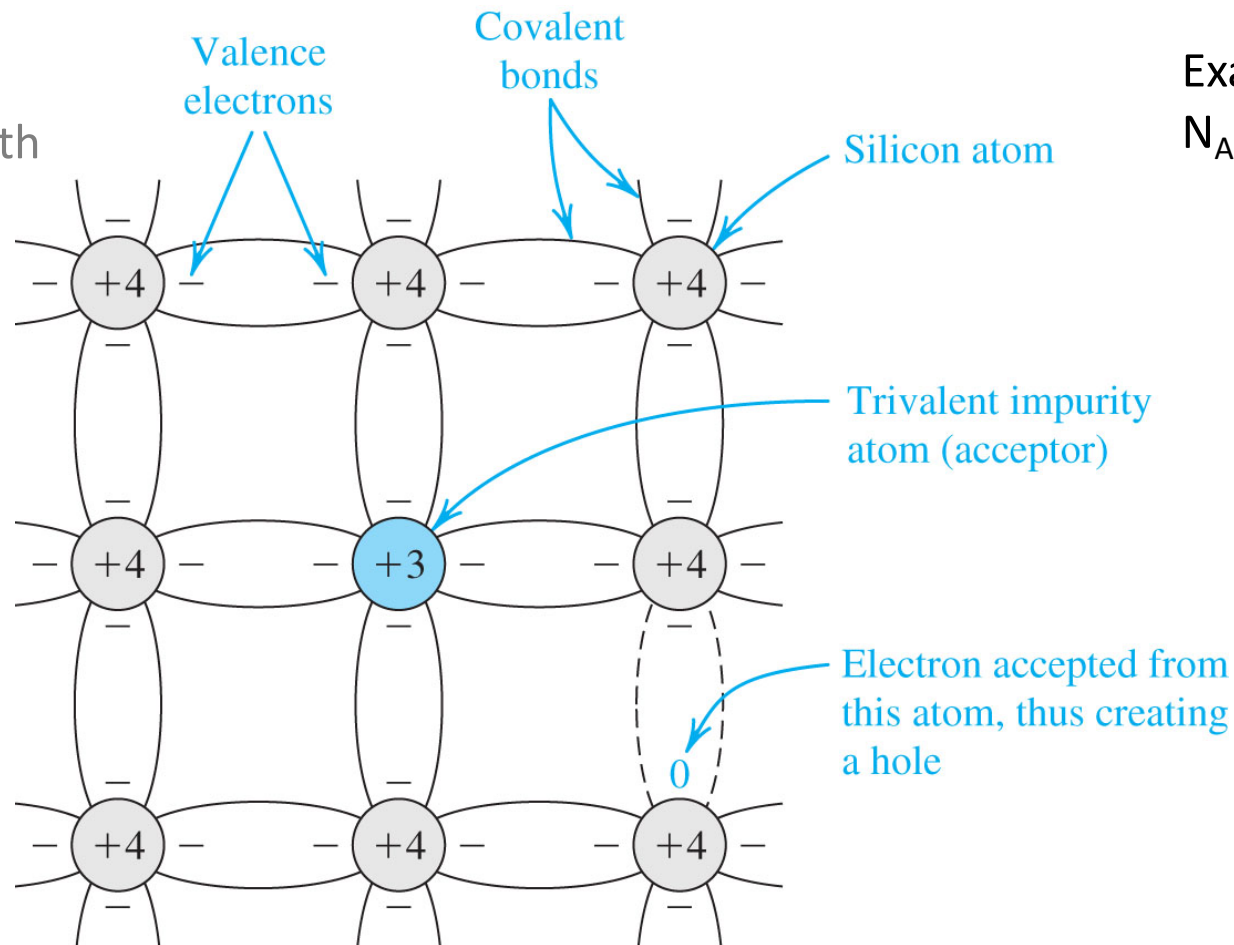


**Figure 3.3** A silicon crystal doped by a pentavalent element. Each dopant atom donates a free electron and is thus called a donor. The doped semiconductor becomes *n* type.

# Extrinsic Semiconductors (2)

- Doping with acceptor impurities (P-type semiconductor)

source:  
Sedra & Smith



Example:  
 $N_A \approx 10^{19} \text{ cm}^{-3}$

**Figure 3.4** A silicon crystal doped with boron, a trivalent impurity. Each dopant atom gives rise to a hole, and the semiconductor becomes *p* type.

# N-type semiconductor

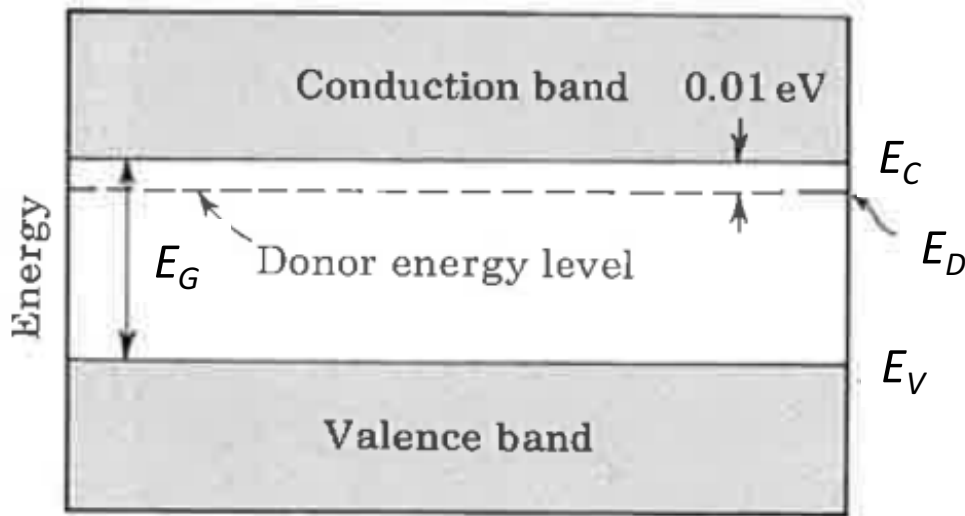
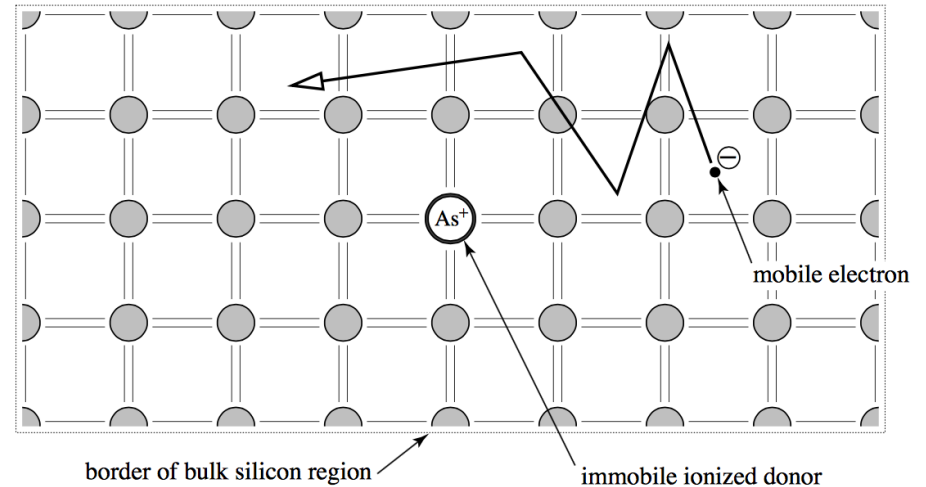


Fig. 2-7 Energy-band diagram of *n*-type semiconductor.

source: Millman & Halkias



source: Howe & Sodini

$$q = 1.6 \times 10^{-19} \text{ Cb}$$

$$\rho = \text{charge density [Cb / cm}^3] = 0 = \underbrace{(-qn)}_{\text{electrons}} + \underbrace{(qp)}_{\text{holes}} + \underbrace{qN_D}_{\text{donors}}$$



# P-type semiconductor (1)

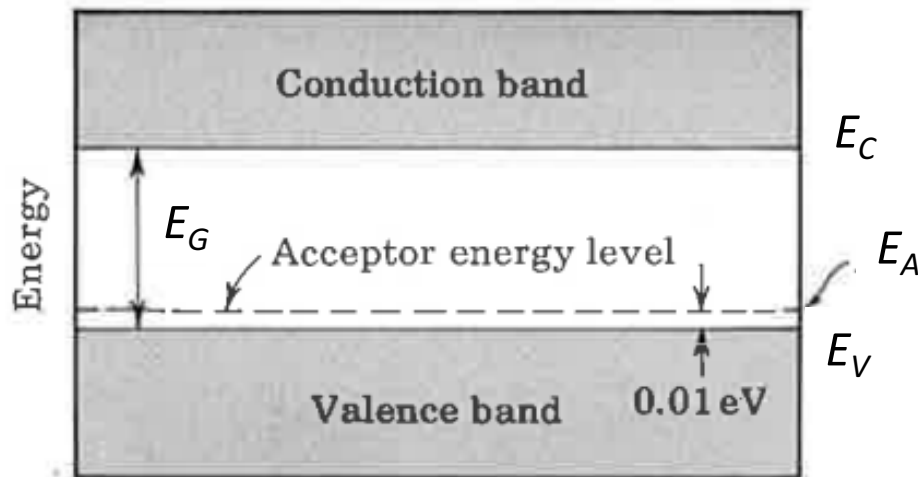
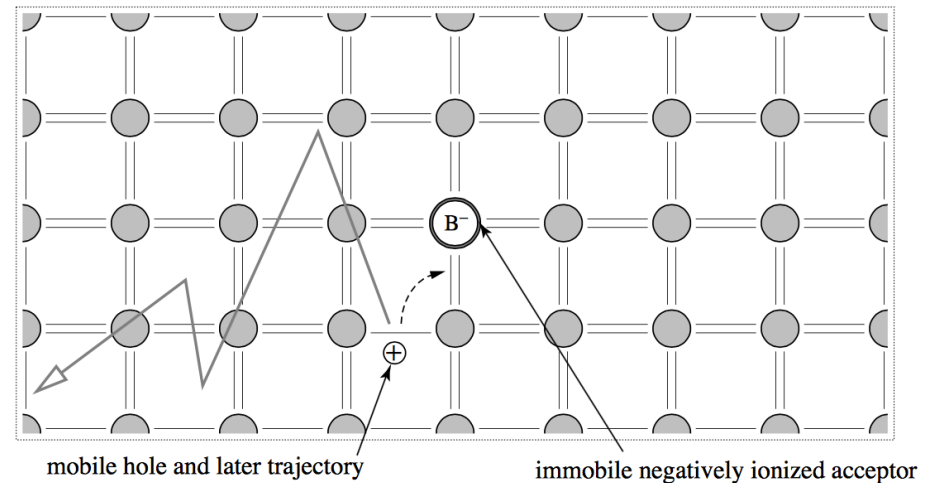


Fig. 2-9 Energy-band diagram of *p*-type semiconductor.

source: Millman & Halkias



source: Howe & Sodini

$$q = 1.6 \times 10^{-19} \text{ Cb}$$

$$\rho = \text{charge density [Cb / cm}^3] = 0 = \underbrace{(-qn)}_{\text{electrons}} + \underbrace{(qp)}_{\text{holes}} - \underbrace{qN_A}_{\text{acceptors}}$$

# P-type semiconductor (2)

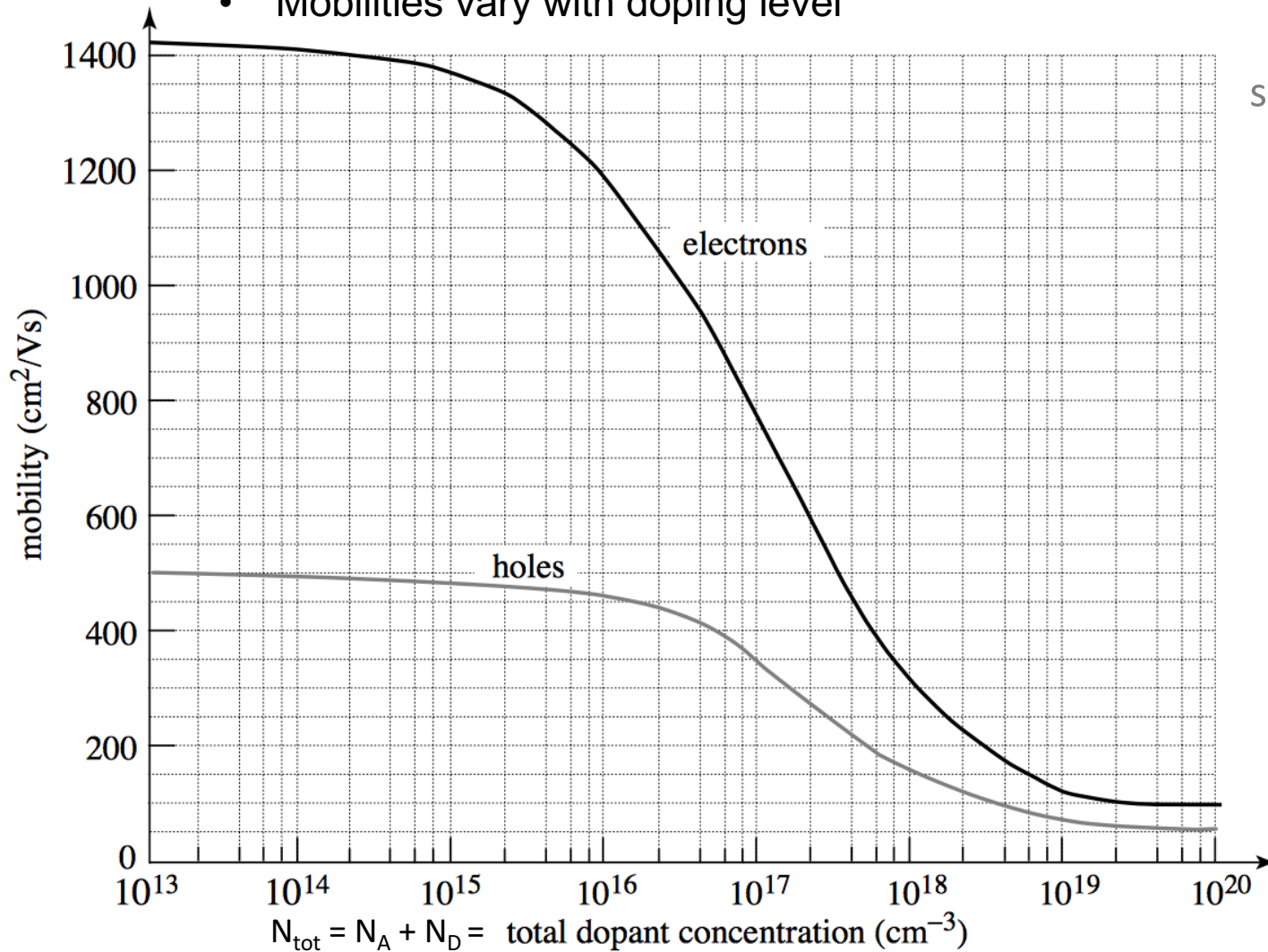
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- Holes can be filled by absorbing free electrons, therefore there is an “effective” flow of holes
- Holes are slower than free electrons (due to the probability of a hole to be filled)
- The effective mass of holes is larger than the effective mass of the free electrons:

$$m^*_h > m^*_e$$

# Mobility of free electrons and holes

- Electron and Hole mobilities for silicon at 300 K
- Mobilities vary with doping level



source: Howe & Sodini

For intrinsic silicon  
at T=300 K:

$$\begin{aligned}\mu_p &\approx 480 \text{ cm}^2/(\text{V}\cdot\text{s}) \\ \mu_n &\approx 1350 \text{ cm}^2/(\text{V}\cdot\text{s}) \\ &\approx 2.8 \times \mu_p\end{aligned}$$

$$\mu \propto T^{-3/2}$$

# Mass Action Law

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- The mass-action law is valid for both intrinsic (pure) and extrinsic (doped) semiconductors

$$n_i^2 = n \cdot p$$

- If  $n \uparrow$  then  $p \downarrow$   $\longrightarrow$  A larger number of free electrons causes the recombination rate of free electrons with holes to increase

# Doping with donors (n-type)

- Charge neutrality:

$$\rho = 0 = q(p - n + N_D)$$

- Using mass-action law:

$$\frac{n_i^2}{n} - n + N_D = 0 \xleftrightarrow{\text{flip sides}} -\frac{n_i^2}{n} + n - N_D = 0 \xleftrightarrow{\text{multiply both sides by } n} n^2 - N_D \cdot n - n_i^2 = 0$$

$$n = \frac{N_D \pm \sqrt{N_D^2 - 4n_i^2}}{2} \approx N_D$$

doping with  $N_D \gg n_i$

Free electrons  
are Majority  
Carriers

$$p \approx \frac{n_i^2}{N_D}$$

Holes are  
Minority  
Carriers

# Doping with acceptors (p-type)

- Charge neutrality:

$$\rho = 0 = q(p - n + N_D)$$

- Doping with  $N_A \gg n_i$

$$p \approx N_A$$

Holes  
are Majority  
Carriers

$$n \approx \frac{n_i^2}{N_A}$$

Free electrons  
are Minority  
Carriers

# Doping with both donors and acceptors

- Charge neutrality:

$$\rho = 0 = q(p - n + N_D - N_A)$$

- Assuming that  $|N_D - N_A| \gg n_i$   
(nearly always true)

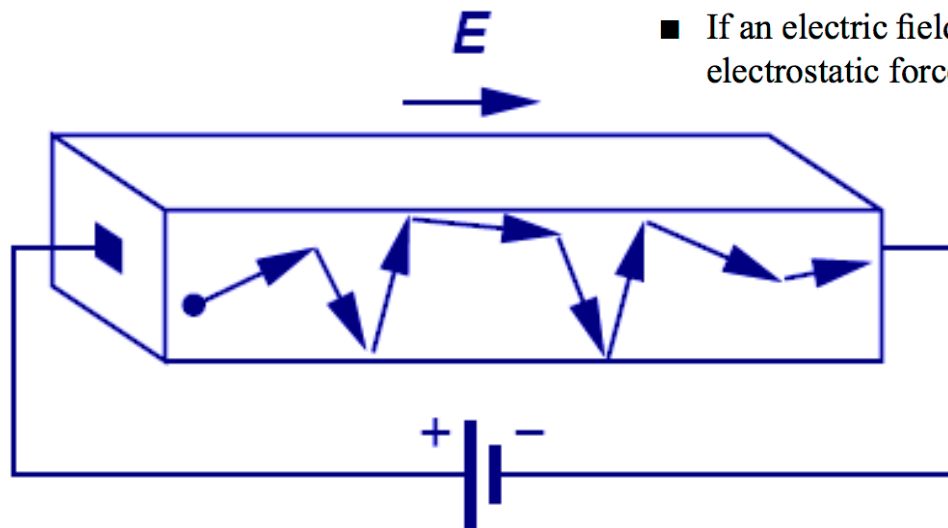
- For  $N_D > N_A$

$$n \simeq N_D - N_A \quad \text{and} \quad p \simeq \frac{n_i^2}{N_D - N_A}$$

- For  $N_A > N_D$

$$p \simeq N_A - N_D \quad \text{and} \quad n \simeq \frac{n_i^2}{N_A - N_D}$$

# First Carriers Transport Mechanism: Drift



- If an electric field is applied to silicon, the holes and the electrons “feel” an electrostatic force  $F_e = (+q \text{ or } -q)E$ .

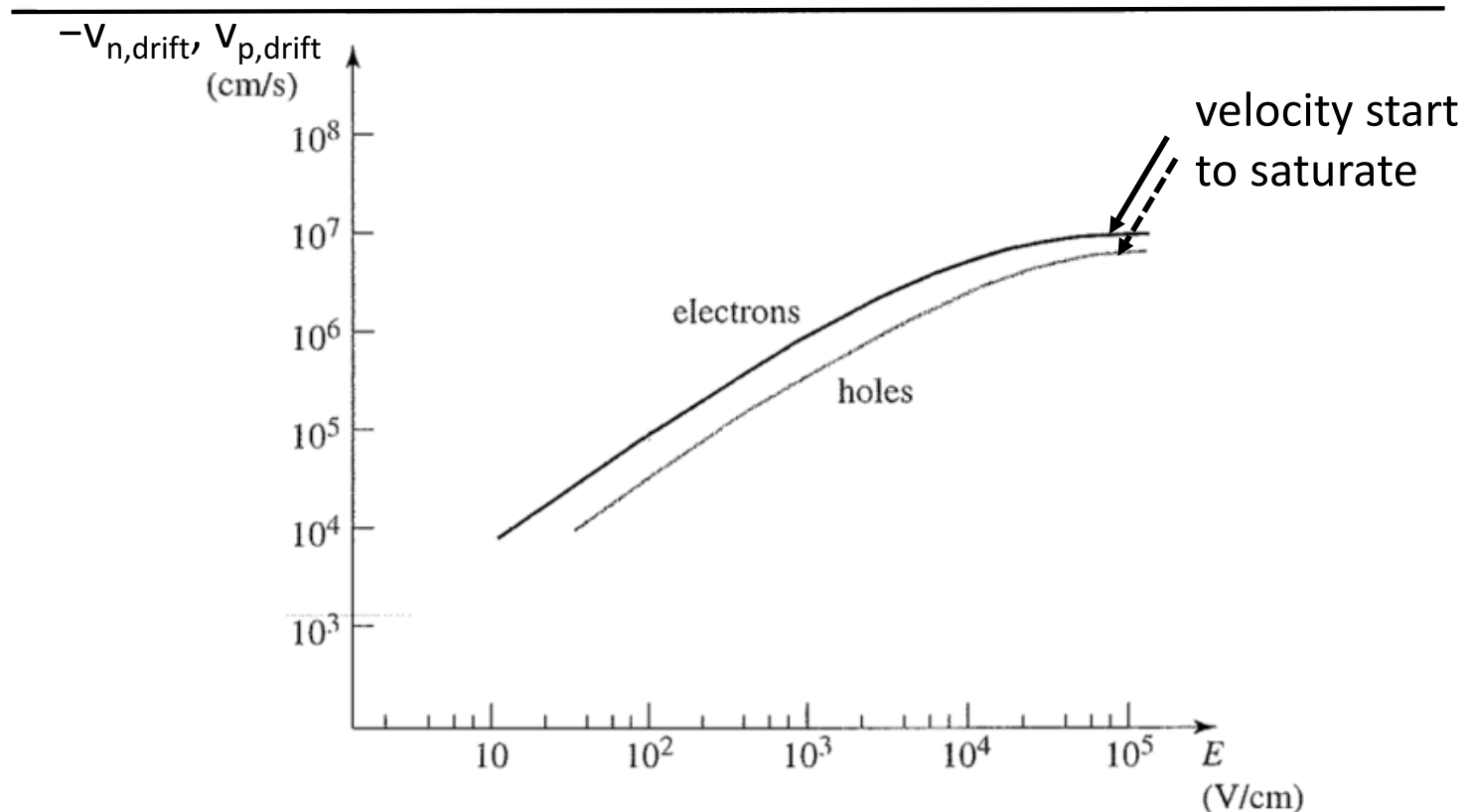
$$\vec{v}_p^{drift} = \mu_p \vec{E}$$

$$\vec{v}_n^{drift} = -\mu_n \vec{E}$$

- The process in which charged particles move because of an electric field is called drift.
- Charged particles will move at a velocity that is proportional to the electric field (this is true as long as the field doesn't become too large)

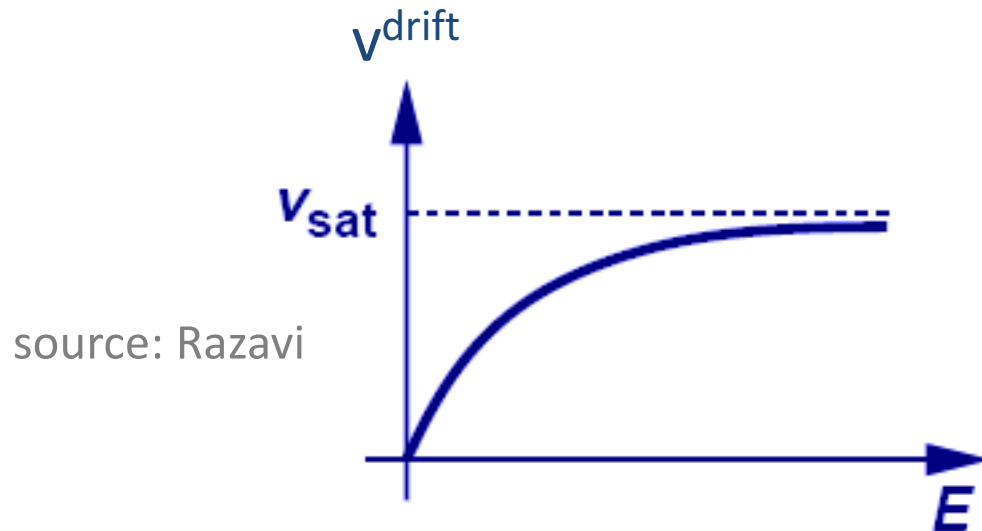


# Drift velocity in silicon



► **Figure 2.9** Log-log plot of the drift velocity as a function of electric field for electrons and holes, showing the linear region where the velocity is proportional to the field and velocity saturation at high fields. After R. S. Muller and T. I. Kamins, *Device Electronics for Integrated Circuits*, 2<sup>nd</sup> ed., Wiley, 1986.

# Saturation of the drift velocity



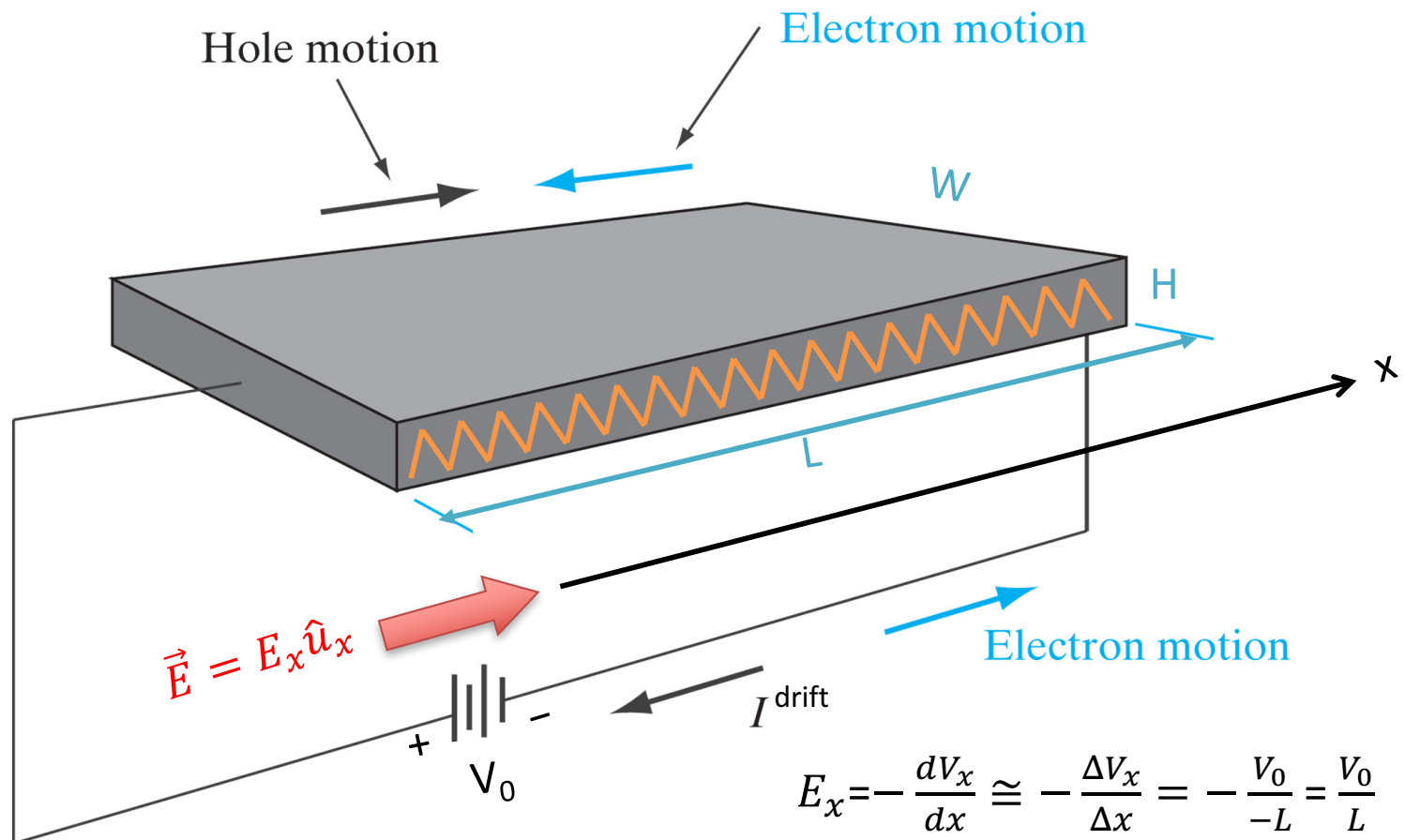
source Gray & Meyer:

$$v^{\text{drift}}(E) \simeq \frac{\mu E}{1 + \frac{E}{E_C}} = \frac{\mu E}{1 + \frac{\mu E}{v_{\text{sat}}}}$$

- Eventually the drift velocity saturates: there are too many collisions among carriers and between carriers and lattice
- $v_{\text{sat}}$  for silicon is  $\approx 10^7 \text{ cm/s} = 10^5 \text{ m/s}$

# Drift Current

source:  
Streetman



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# Drift current and current density

- **electric current:** amount of charge that flows through a reference plane per unit time

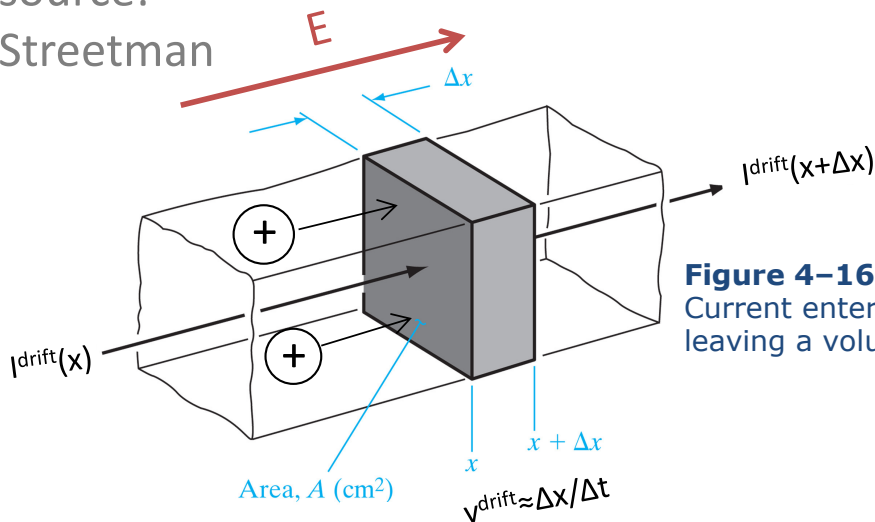
$$I^{drift} = \frac{\Delta Q}{\Delta t} = \frac{\Delta x \times W \times H \times n \times q}{\Delta t} = v^{drift} \times \underbrace{W \times H}_{\text{cross-section Area [cm}^2\text{]}} \times \underbrace{n \times q}_{\text{charge per unit volume (aka charge density) [Cb/cm}^3\text{]}} \left[ \frac{\text{Cb}}{\text{s}} = \text{A} \right]$$

charge per unit volume [Cb/cm<sup>3</sup>]

v<sup>drift</sup> ≈ Δx / Δt

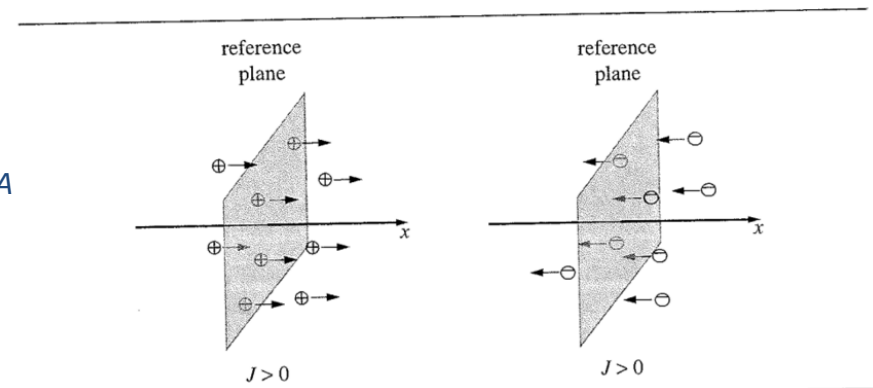
volume per unit time [cm<sup>3</sup>/s]

source:  
Streetman



**Figure 4-16**  
Current entering and leaving a volume Δx•A

$$J^{drift} = \frac{I^{drift}}{W \times H} \left[ \text{A} / \text{m}^2 \right] \quad \leftarrow \text{Current Density}$$



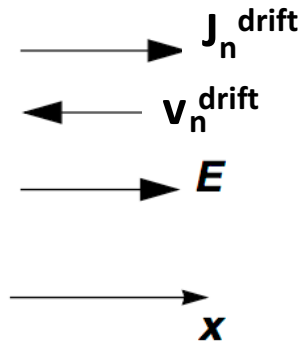
► **Figure 2.10** Sign convention for current density  $J$ . Both cases represent positive values of  $J$  since there is a net transport of positive charge in the  $+x$  direction.

source: Howe & Sodini

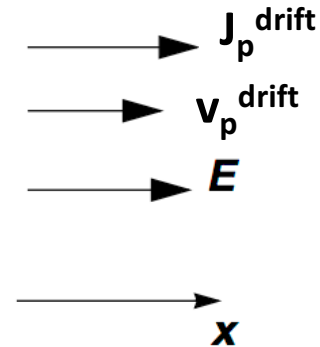
# Drift Current Density

Source:  
Howe & Sodini

**electron drift  
current density**



**hole drift  
current density**



$$q_h = -q_e \equiv q = 1.6 \times 10^{-19} \text{ Cb}$$

$$J^{drift} = J_n^{drift} + J_p^{drift} = v_n^{drift} q_e n + v_p^{drift} q_p p = -\mu_n E q_e n + \mu_p E q_h n =$$

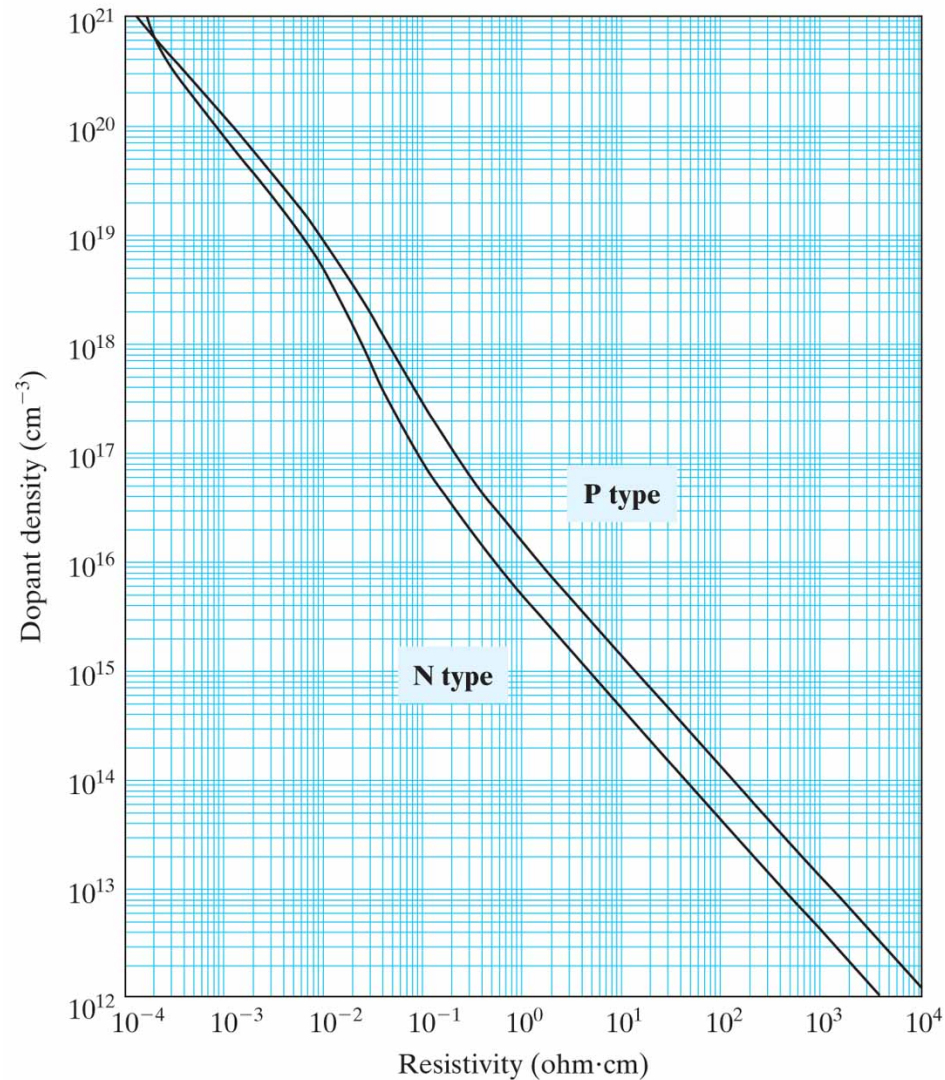
$$= \mu_n E q n + \mu_p E q p = \underbrace{(\mu_n q n + \mu_p q p)}_{= \sigma = 1/\rho} E$$

$= \sigma = 1/\rho$   
conductivity  $[\Omega\text{m}]^{-1}$

$$J^{drift} = \sigma E \leftarrow \text{Ohm Law}$$

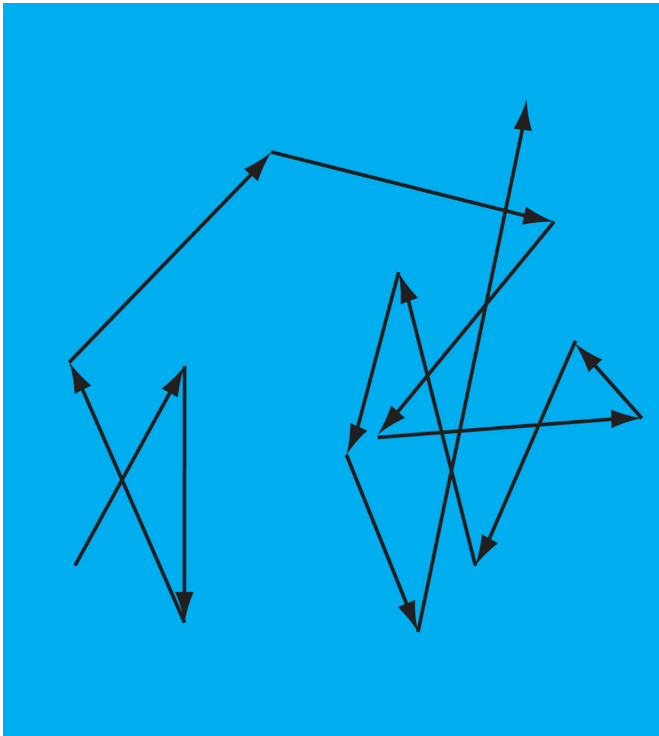
# Conversion between resistivity and dopant density of silicon at room temperature

source: Hu



# Second Carriers Transport Mechanism: **Diffusion**

- The thermal motion of an electron or a hole changes direction frequently by scattering off imperfections in the semiconductor crystal



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Random thermal motion of an electron or hole in a solid.

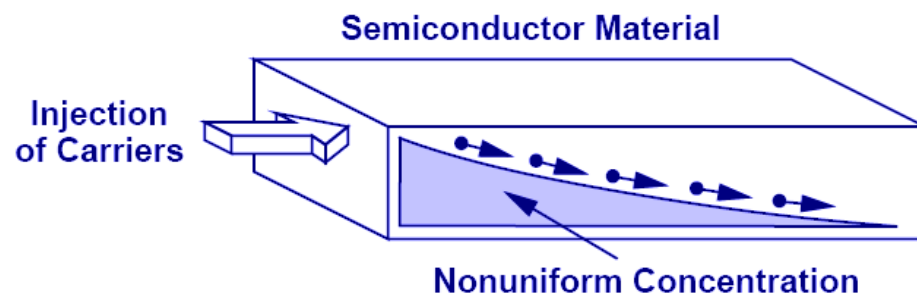
source:  
Streetman

- In a material where the concentration of particles is uniform the random motion balances out and no net movement result (drunk sail-man walk → Brownian walks)



# Diffusion Current

- If there is a difference (gradient) in concentration between two parts of a material, statistically there will be more particles crossing from the side with higher concentration to the side with lower concentration than vice versa
- Therefore we expect a net flux of particles



Source: Razavi

$$q_p = -q_e \triangleq q$$

$$I_n^{diff} \propto Aq_e \frac{dn}{dx} = -Aq \frac{dn}{dx}$$

$$I_p^{diff} \propto Aq_p \frac{dp}{dx} = Aq \frac{dp}{dx}$$

The more non uniform  
is the concentration the  
more is the current

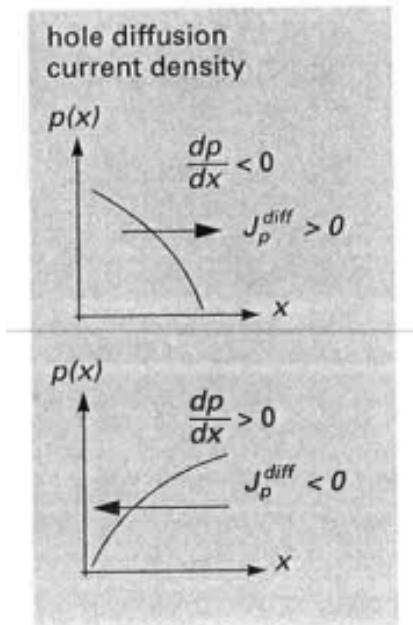
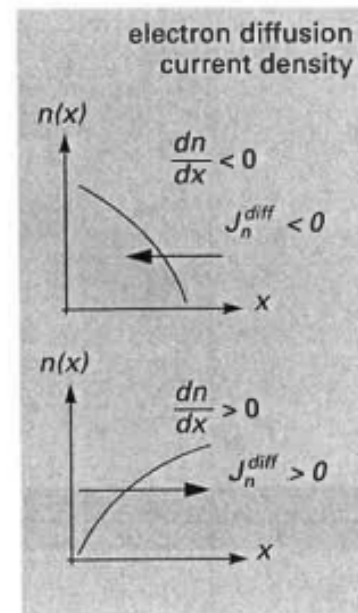


# Electron and hole diffusion current

- Assuming the charge concentration decreases with increasing  $x$  it means that  $dn/dx$  and  $dp/dx$  are negative quantities so to conform with conventions we have to put a – sign in front of the proportionality constant  $D$

$$I_n^{diff} = -D_n A q_e \frac{dn}{dx} = D_n A q \frac{dn}{dx}$$

$$I_p^{diff} = -D_p A q_p \frac{dp}{dx} = -D_p A q \frac{dp}{dx}$$



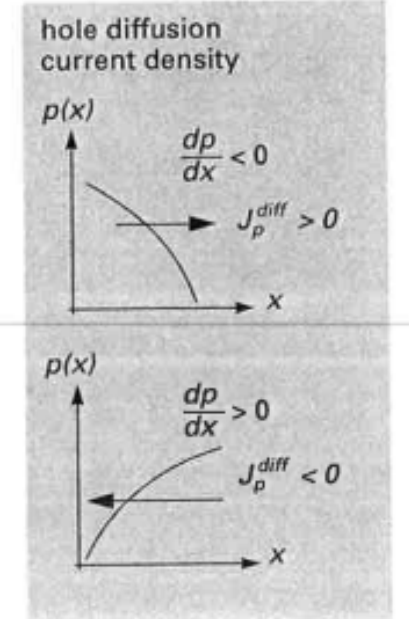
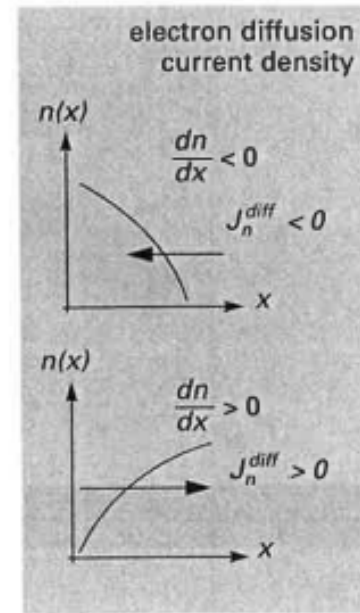
Source: Howe & Sodini

# Diffusion current densities

$$J_{diff} = J_p^{diff} + J_n^{diff}$$

$$J_n^{diff} = -D_n q_e \frac{dn}{dx} = D_n q \frac{dn}{dx}$$

$$J_p^{diff} = -D_p q_p \frac{dp}{dx} = -D_p q \frac{dp}{dx}$$



Source: Howe & Sodini

# Einstein's Relation

- Since both  $\mu$  and  $D$  are manifestation of thermal random motion (i.e. are due to statistical thermodynamics phenomena) they are not independent

$$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{KT}{q} \quad \leftarrow \quad \text{Einstein's Relation}$$

$K$  = Boltzmann Constant =  $1.38 \times 10^{-23} \text{ J/}^\circ\text{K} = 8.62 \times 10^{-5} \text{ eV/}^\circ\text{K}$

$T$  = temperature in  $^\circ\text{K}$

$q$  = charge of proton =  $1.602 \times 10^{-19} \text{ Cb}$

$$V_T \triangleq \frac{KT}{q} \quad \leftarrow \quad \text{Thermal Voltage}$$

At room temperature  
 $V_T \approx 25.9 \text{ mV}$

# Total current density

- The electron and hole total current density is:

$$J = J_p + J_n = J_p^{drift} + J_p^{diff} + J_n^{drift} + J_n^{diff}$$

$$J_p = J_p^{drift} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx}$$

$$J_n = J_n^{drift} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J = qp\mu_p E - qD_p \frac{dp}{dx} + qn\mu_n E + qD_n \frac{dn}{dx}$$