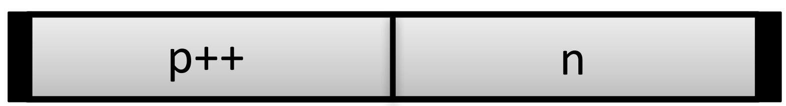
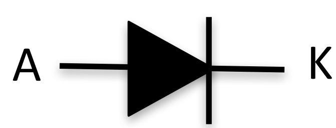
Chapter 3

Electronics I - PN Junction



$N_A >> N_D$



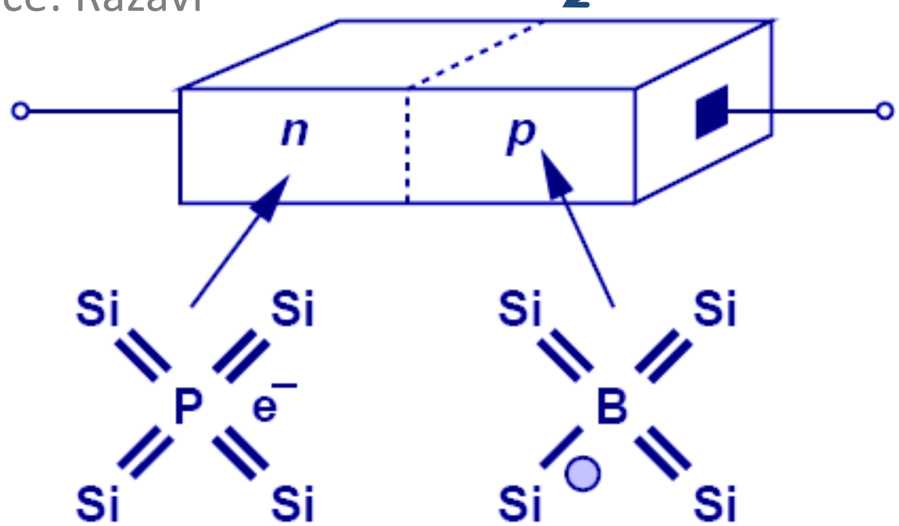
Fall 2017

PN Junction (Diode)

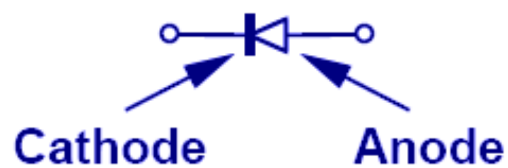
- When N-type and P-type dopants are introduced side-by-side in a semiconductor, a PN junction or a diode is formed.

JUNCTION

source: Razavi



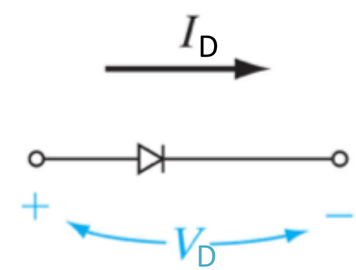
(a)

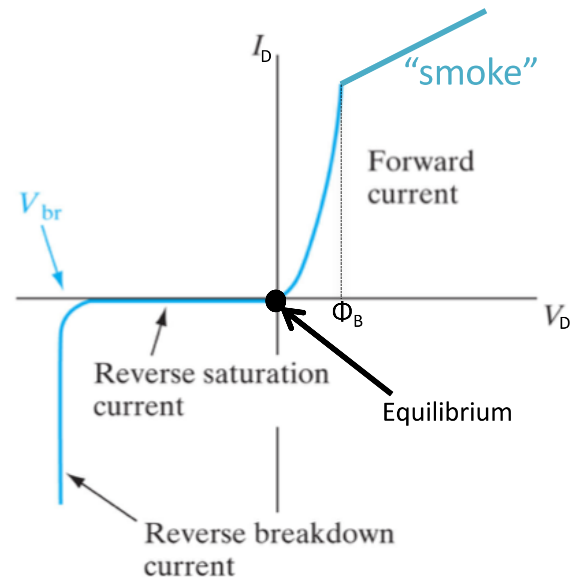


(b)

Diode I/V characteristics

- Our goal is to find out the current-voltage characteristics of the device





- Diode operation regions:
  - Equilibrium = open circuit terminals = no bias
  - Forward bias
  - Reverse bias
  - (Breakdown)

PN Junction in Equilibrium (no Bias applied)

- Because each side of the junction contains an excess of holes or electrons compared to the other side, there exists a large concentration gradient. Therefore, a **diffusion current** flows across the junction from each side.

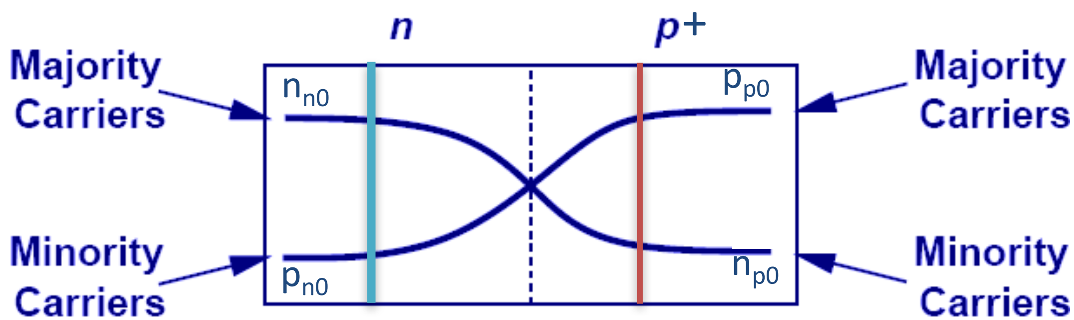
$$\frac{N_D < N_A}{}$$

$n_{n0} \approx N_D$

$p_{n0} \approx n_i^2/N_D$

$p_{p0} \approx N_A$

$n_{p0} \approx n_i^2/N_A$



n_n : Concentration of electrons on n side

p_n : Concentration of holes on n side

p_p : Concentration of holes on p side

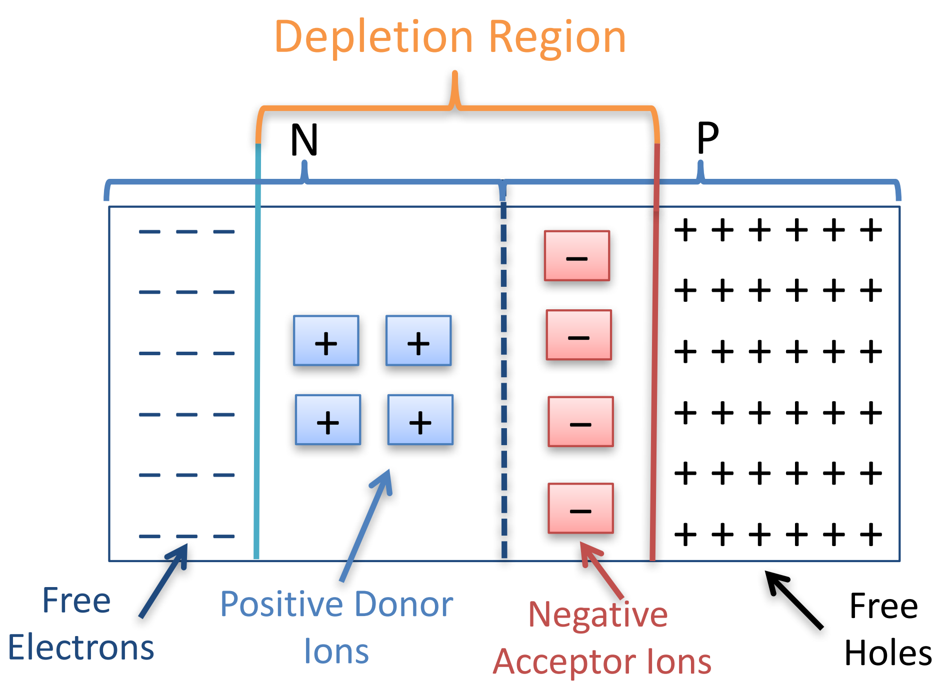
n_p : Concentration of electrons on p side

source: Razavi

PN junction in Equilibrium (no bias applied)

- Diffusion across the junction causes a lot of electrons and holes to recombine leaving a depletion region in proximity of the junction
- The fixed ions left behind create an electric field
- The electric field opposes the diffusion of holes in the N region and free electrons in the P region
- Once the electric field is strong enough to stop the diffusion currents the junction reaches equilibrium
  - A “barrier” opposing diffusion forms

$\underline{N_D < N_A}$



Depletion approximation

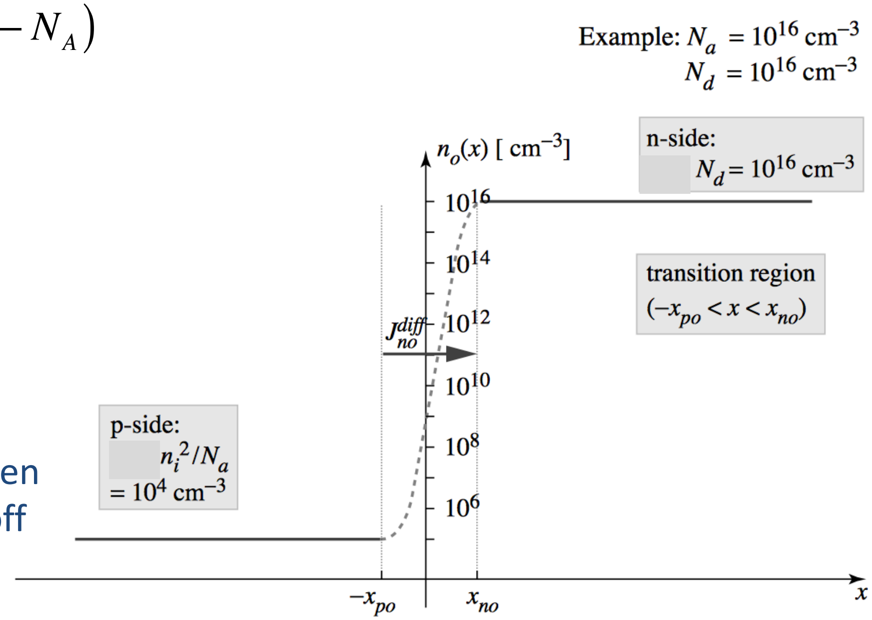
- What happens inside the PN junction in equilibrium is not quite as black and white as depicted

$$\rho(x) = q\left(p_0(x) - n_0(x) + N_D - N_A\right)$$

- On the N-side of the transition region ($0 < x < x_{n0}$), since there are no acceptors ($N_A = 0$) and the hole concentration is negligible ($p_0 \approx 0$)

$$\rho(x) = q\left(p_0(x) - n_0(x) + N_D - N_A\right) \approx q\left(-n_0(x) + N_D\right)$$

- Since $n_0(x) < N_D$ on the N-side of the transition (and even more so on the P-side of the transition) there is a roll-off in electron concentration



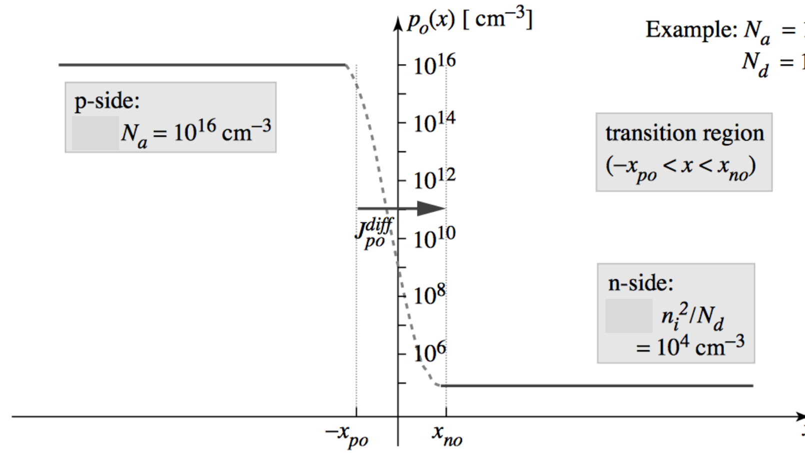
source: Howe & Sodini

Depletion approximation

- On the P-side of the transition region ($-x_p < x < 0$), since there are no donors ($N_D = 0$) and the electrons concentration is negligible ($n_0 \approx 0$)

$$\rho(x) = q\left(p_0(x) - n_0(x) + N_D - N_A\right) \approx q\left(p_0(x) - N_A\right)$$

- Since $p_0(x) < N_A$ on the P-side of the transition region (and even more so on the N-side of the transition) there is a roll-off in hole concentration



source: Howe & Sodini

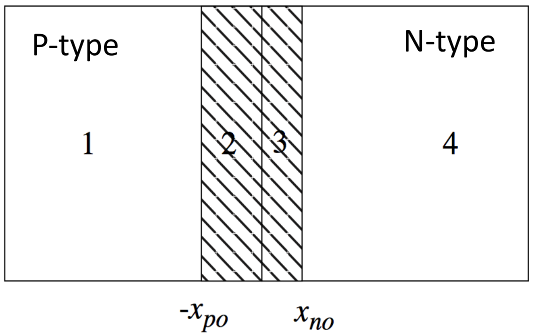
- The **depletion approximation** assumes that the mobile carriers (n and p) are small in number compared to the donor and acceptor ion concentrations (N_D and N_A) in the depletion region (SCR), and the device is charge-neutral elsewhere (QNR)

- Depletion approximation
  - $N_A >> p_p$ hence $\rho \approx -qN_A$ for $-x_p \le x \le 0$
  - $N_D >> n_n$ hence $\rho \approx qN_D$ for $0 \le x \le x_n$
  - The charge density is negligible in the bulk regions; that is $\rho \approx 0$ for $x > x_n$ and $x < -x_p$

That is, there are no free mobile charges in depletion region

source: Sodini

One more time …



NOTE: for the example in this picture NA < ND

- Bulk silicon is *NEUTRAL*, to a good approximation

  >> region 1 is *bulk*:

  $$\rho = q(N_d + p_o - N_a - n_o) \cong 0 \quad \text{-->} \quad p_o \cong N_a$$

  >> region 4 is *bulk*:

  $$\rho = q(N_d + p_o - N_a - n_o) \cong 0 \quad \text{-->} \quad n_o \cong N_d$$

- Near the junction, the silicon is *DEPLETED* of mobile carriers:

  >> region 2 is *depleted*:

  (P-side)

  $$\rho = q(\overset{=0}{\cancel{N_d}} + p_o - N_a - \overset{\approx 0 \text{ (minority carriers)}}{\cancel{n_o}}) \cong -qN_a$$

  $N_a >> p_0$ (depletion approximation)

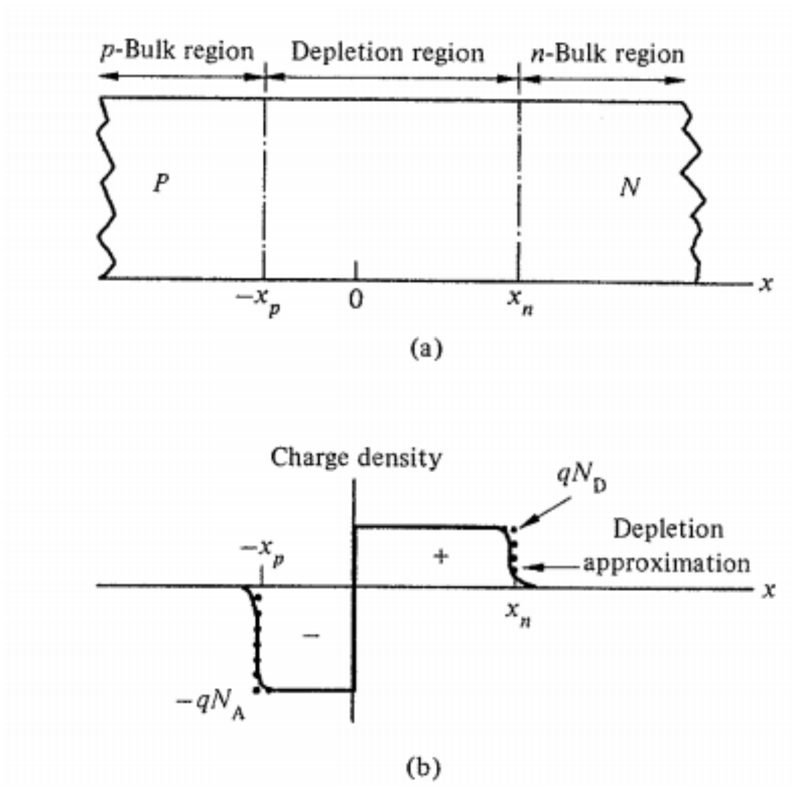
  >> region 3 is *depleted*:

  (N-side)

  $$\rho = q(N_d + \underset{\approx 0 \text{ (minority carriers)}}{\cancel{p_o}} - \overset{=0}{\cancel{N_a}} - n_o) \cong qN_d$$

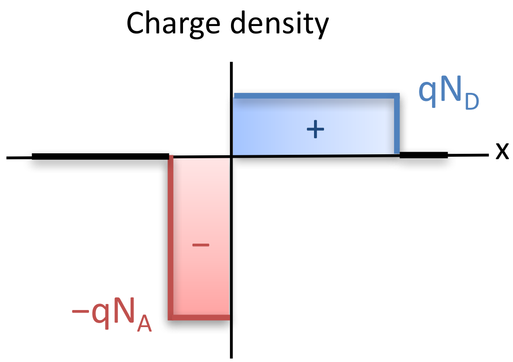
  $N_d >> n_0$ (depletion approximation)

Depletion approximation

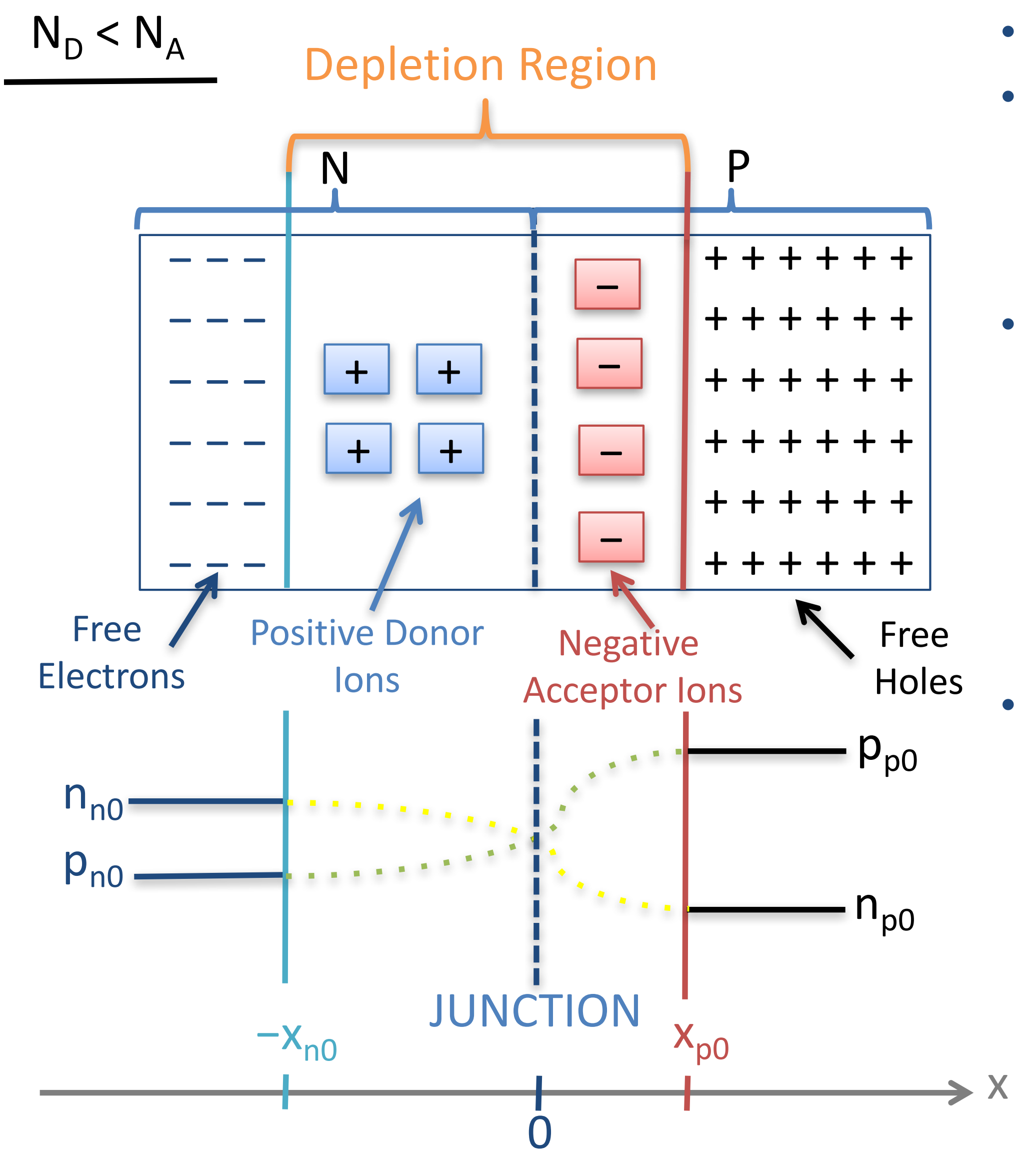


source: G.W. Neudeck

Depletion approximation means we assume the charge density in each transition region to be uniform



PN junction in Equilibrium (no bias applied)



- Since the circuit is open no currents exists
- Therefore in equilibrium the drift currents resulting from the electric field generated by the fixed ions in the depletion region must exactly cancel the diffusion currents
- This must hold for both types of charge carriers: $J_n = J_p = 0$

$$J_p = 0 = pq\mu_p E_x - qD_p \frac{dp}{dx}$$

$$J_n = 0 = nq\mu_n E_x + qD_n \frac{dn}{dx}$$

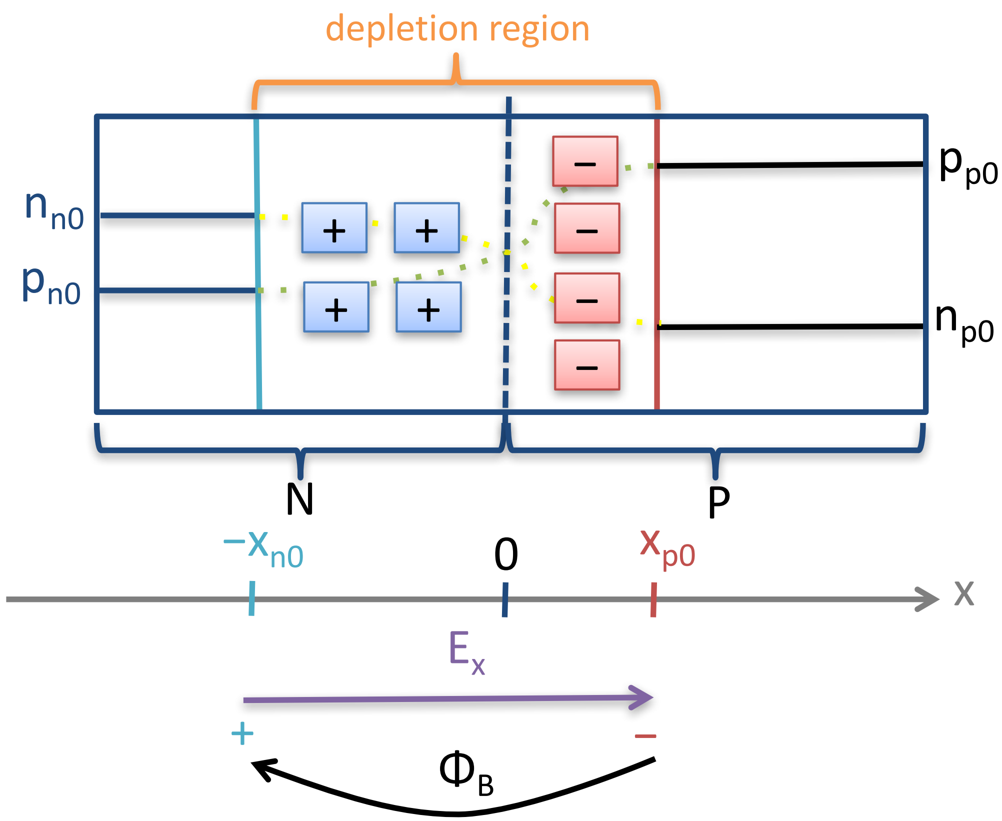
- Let's take one of the two equations and try to solve it:

$$p\cancel{q}\mu_p E_x = \cancel{q}D_p \frac{dp}{dx}$$

$$E_x = -\frac{dV}{dx}$$

$$-p\mu_p \frac{dV}{dx} = D_p \frac{dp}{dx}$$

Built-in voltage (equilibrium)



$$-p\mu_p \frac{dV}{\cancel{dx}} = D_p \frac{dp}{\cancel{dx}}$$

$$-\int_{V(-xn0)}^{V(xp0)} dV = \frac{D_p}{\mu_p}\int_{p(-xn0)}^{p(xp0)} \frac{dp}{p}$$

$$V(-x_{n0}) - V(x_{p0}) = V_T \ln \frac{p_{p0}}{p_{n0}}$$

$$\Phi_B = V_T \ln \frac{N_A}{n_i^2 / N_D}$$

$$\Updownarrow$$

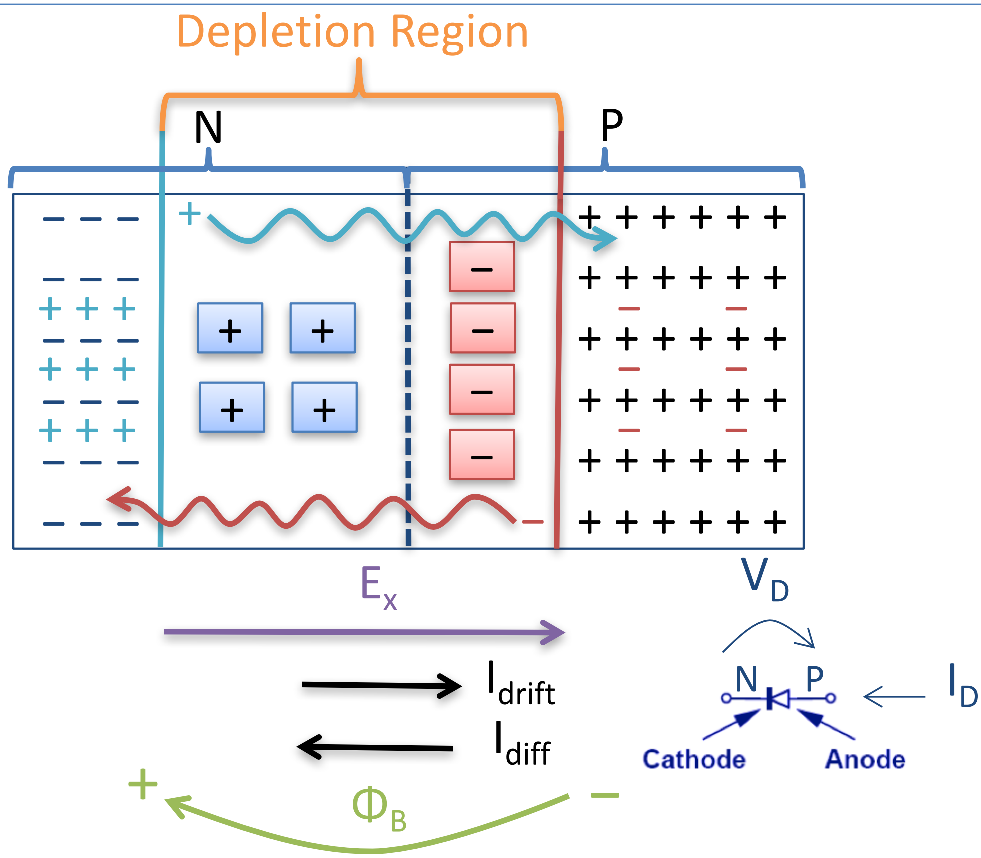
$$\boxed{\Phi_B = V_T \ln \frac{N_A N_D}{n_i^2}}$$

Example

Find built in voltage of a silicon PN junction at room temperature and with $N_A \approx 10^{18}$ and $N_D \approx 10^{17}$

$$\Phi_B \cong 26mV \times \ln \frac{10^{18}10^{17}}{10^{20}} \cong 26mV \times \ln(10) \times Log_{10} \frac{10^{18}10^{17}}{10^{20}} \cong$$
$$\cong 60mV \times 15 \cong 900mV$$

PN Junction in equilibrium (no bias applied)

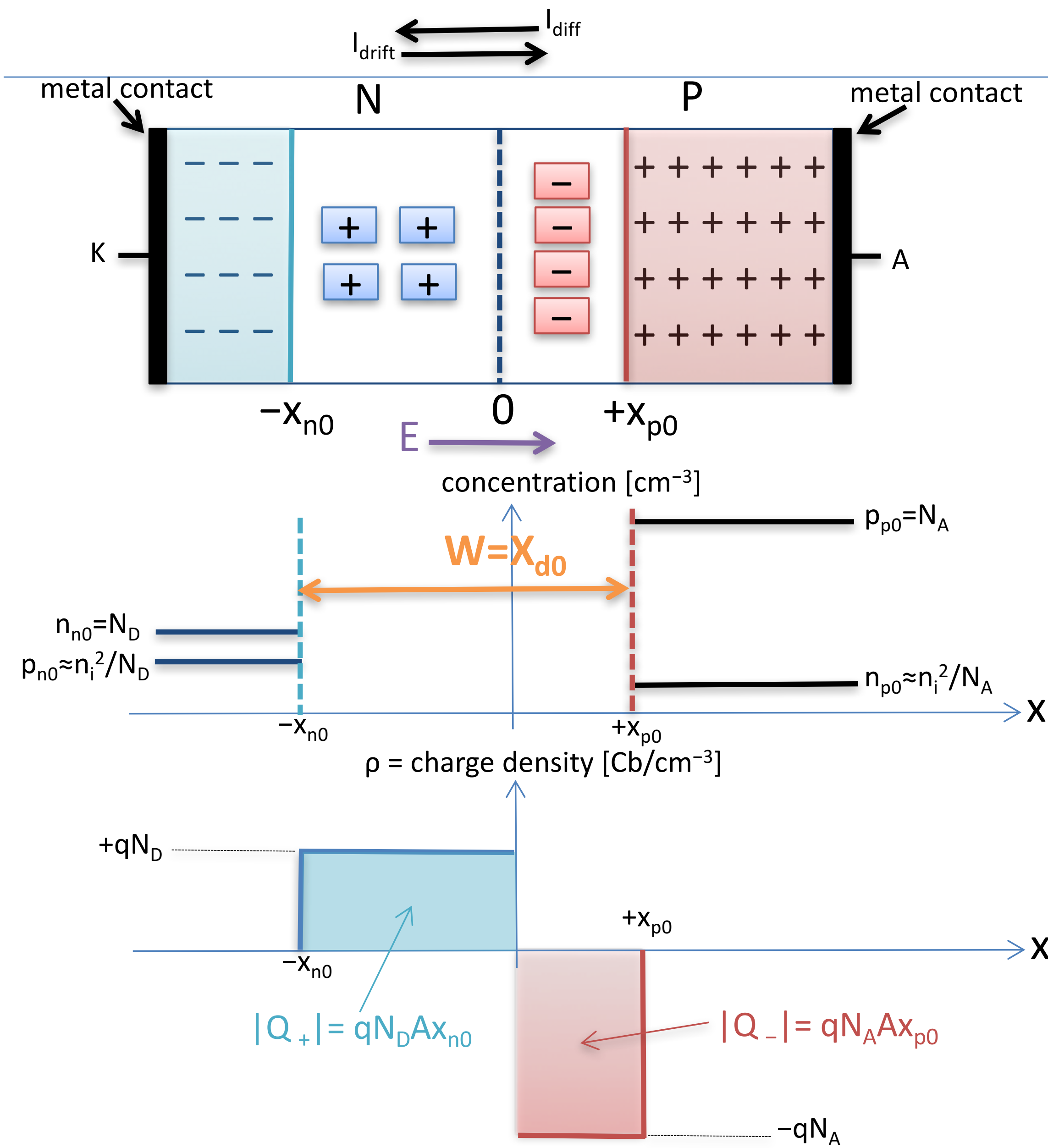


- Some of the thermally generated holes in the N region (minority carrier) moves toward the junction and reach the edge of the depletion region. There, they experience the strong electric force present in depletion region, and get swept all the way into the P region (where they become majority carriers).
- The same happen for the thermally generated minority carriers (free electrons) in the P region.

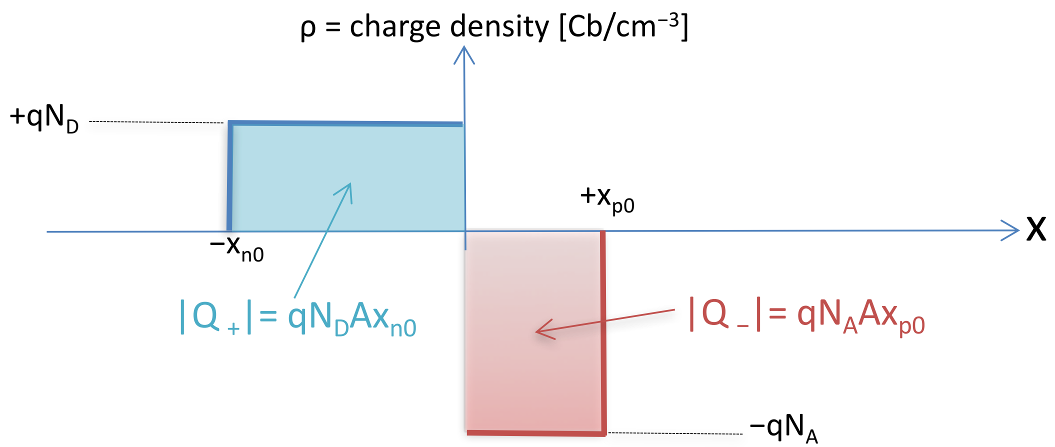
- At equilibrium the flow of carriers swept by the electric field (drift) and the flow of carriers due the gradient in carriers concentration (diffusion) cancel out

$$I_D = I_{diff} - I_{drift} = 0$$

Width of the Depletion region in equilibrium



Width of the Depletion region in equilibrium



Charge conservation:

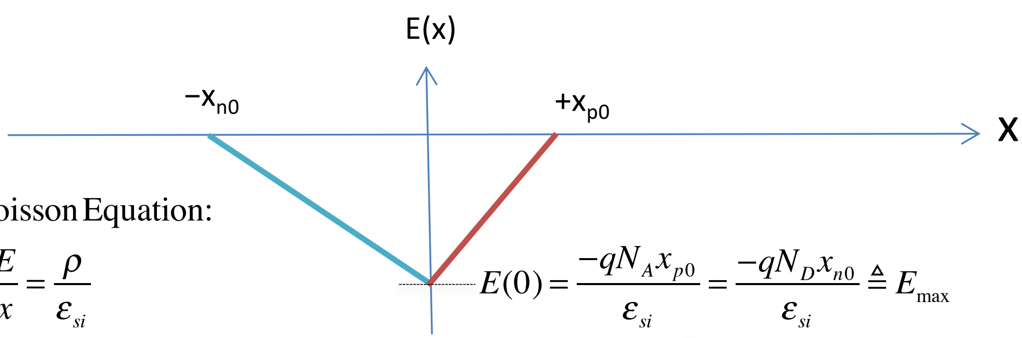
$$|Q_+| = |Q_-| \triangleq Q_{J0}$$

$$\Updownarrow$$

$$AqN_D x_{n0} = AqN_A x_{p0}$$

$$\Updownarrow$$

$$N_D x_{n0} = N_A x_{p0}$$



$$E(0) = \frac{-qN_A x_{p0}}{\varepsilon_{si}} = \frac{-qN_D x_{n0}}{\varepsilon_{si}} \triangleq E_{\max}$$

Poisson Equation:

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon_{si}}$$

$$\Updownarrow$$

$$dE = \frac{\rho}{\varepsilon_{si}} dx$$

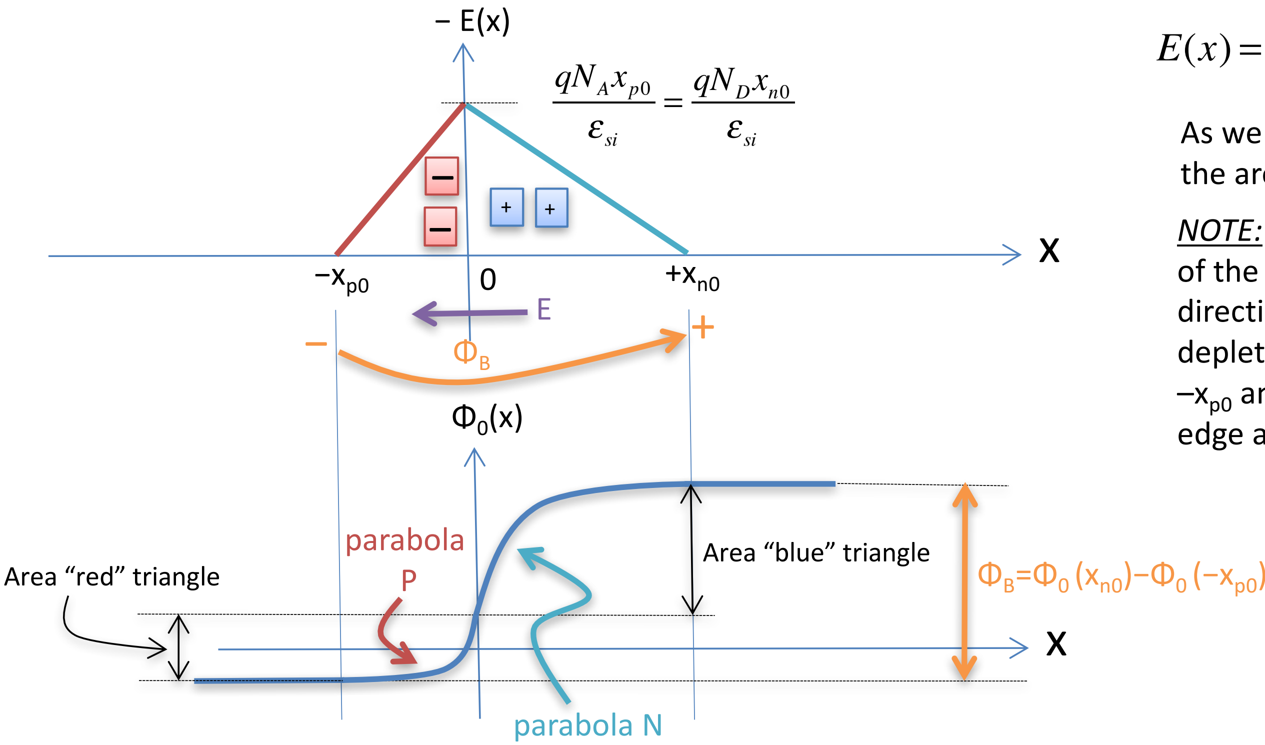
Permittivity of crystalline silicon:

$$\varepsilon_{si} \approx \underbrace{11.7}_{\varepsilon_r} \times \underbrace{8.85 \times 10^{-12}\ \text{F/m}}_{\varepsilon_0}$$

As long as we know how to compute the area of a rectangle we are fine with the math !!!

Width of the Depletion region in equilibrium

- Let’s go from the electric field to the voltage

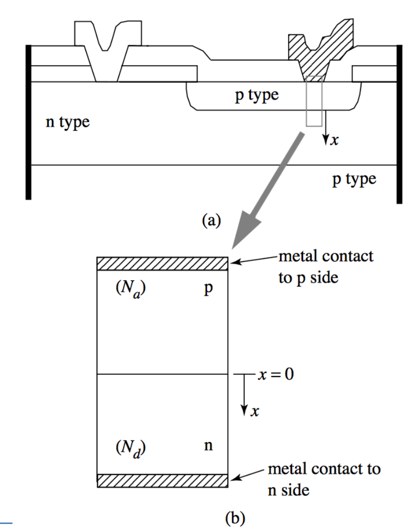


$$E(x) = -\frac{d\Phi_0}{dx} \Leftrightarrow -E(x)\cdot dx = d\Phi_0$$

As we move in the x direction (from $-x_{p0}$ to x_n) the area under –E(x) keeps growing

<u>*NOTE:*</u> for convenience I flipped the orientation of the p-side and the n-side with respect to the x direction so the name of the depletion region’s edge at the p-side becomes $-x_{p0}$ and the name of the depletion region’s edge at the n-side becomes $+x_{n0}$

source: Howe & Sodini



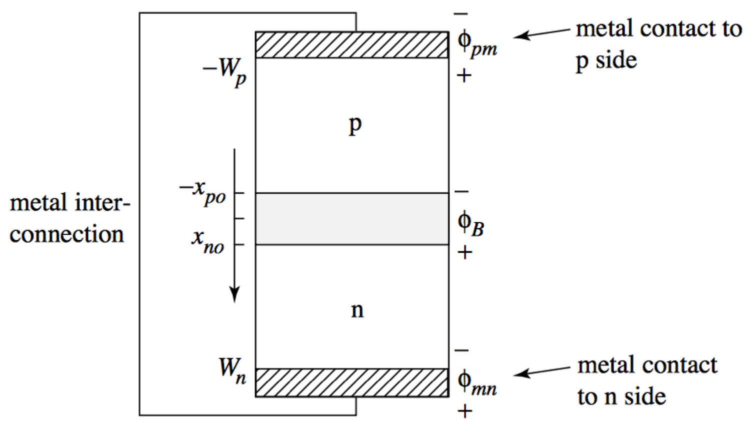
Area “red” triangle = $\frac{1}{2}\frac{qN_A x_{p0}^{\ 2}}{\varepsilon_{si}}$

Area “blue” triangle = $\frac{1}{2}\frac{qN_D x_{n0}^{\ 2}}{\varepsilon_{si}}$

$$\boxed{\Phi_B = \frac{1}{2}\frac{qN_A x_{p0}^{\ 2}}{\varepsilon_{si}} + \frac{1}{2}\frac{qN_D x_{n0}^{\ 2}}{\varepsilon_{si}}}$$

Metal-Semiconductor Contact Potential

source: Howe & Sodini



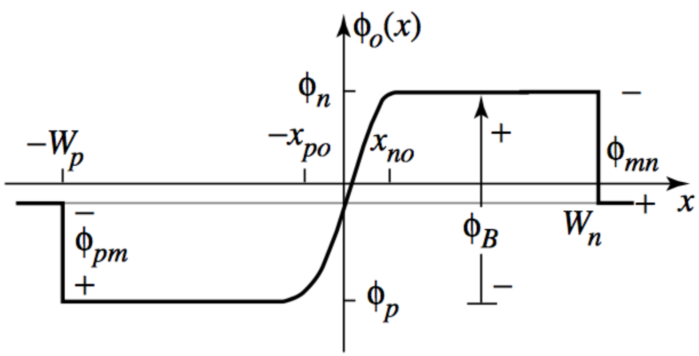
- Kirchhoff's Voltage Law:

$$0 = \phi_{pm} + \phi_B + \phi_{mn}$$

therefore, the built-in voltage is given by:

$$\phi_B = -\phi_{pm} - \phi_{mn}$$

- At first glance would seem strange that the potential drops at the contacts would have to add up to the built in potential of the junction. Were this is not the case, KVL would imply that current would flow in thermal equilibrium when a resistor is connected across the metal contacts to the p and n region. This conclusion violate our basic understanding of the meaning of equilibrium: namely the net current must be zero in equilibrium



Width of the Depletion region in equilibrium

$$W = x_{p0} + x_{n0} \;\Longrightarrow\; \left\{ \begin{array}{l} W = x_{n0}\left(1+\dfrac{N_D}{N_A}\right) = x_{n0}\left(\dfrac{N_A+N_D}{N_A}\right) \\[2ex] W = x_{p0}\left(1+\dfrac{N_A}{N_D}\right) = x_{p0}\left(\dfrac{N_A+N_D}{N_D}\right) \end{array} \right. \;\Longrightarrow\; \left\{ \begin{array}{l} x_{n0} = W\left(\dfrac{N_A}{N_A+N_D}\right) \\[2ex] x_{p0} = W\left(\dfrac{N_D}{N_A+N_D}\right) \end{array} \right.$$

Charge neutrality:

$$N_A x_{p0} = N_D x_{n0}$$

$$\Phi_B = \frac{qN_A}{2\varepsilon_{si}} x_{p0}{}^2 + \frac{qN_D}{2\varepsilon_{si}} x_{n0}{}^2 \;\Longrightarrow\; \Phi_B = \frac{qN_A}{2\varepsilon_{si}} W^2\left(\frac{N_D}{N_A+N_D}\right)^2 + \frac{qN_D}{2\varepsilon_{si}} W^2\left(\frac{N_A}{N_A+N_D}\right)^2 \;\Longrightarrow$$

$$\Phi_B = \frac{q}{2\varepsilon_{si}} W^2 \frac{N_A N_D{}^2 + N_D N_A{}^2}{(N_A+N_D)^2} \;\Longrightarrow\; W^2 = \frac{2\varepsilon_{si}}{q}\Phi_B \frac{(N_A+N_D)^2}{N_A N_D{}^2 + N_D N_A{}^2} = \frac{2\varepsilon_{si}}{q}\Phi_B \frac{(N_A+N_D)^{\cancel{2}}}{N_A N_D \cancel{(N_D+N_A)}}$$

$$\boxed{W = \sqrt{\frac{2\varepsilon_{si}}{q}\Phi_B \frac{N_A+N_D}{N_A N_D}} \triangleq X_{do}}$$

$$\boxed{x_{n0}} = \sqrt{\frac{2\varepsilon_{si}}{q}\Phi_B \frac{(N_A+N_D)}{N_A N_D} \frac{N_A{}^2}{(N_A+N_D)^2}} = \boxed{\sqrt{\frac{2\varepsilon_{si}}{q}\Phi_B \frac{N_A}{N_D(N_A+N_D)}}}$$

$$\boxed{x_{p0} = \sqrt{\frac{2\varepsilon_{si}}{q}\Phi_B \frac{N_D}{N_A(N_A+N_D)}}}$$

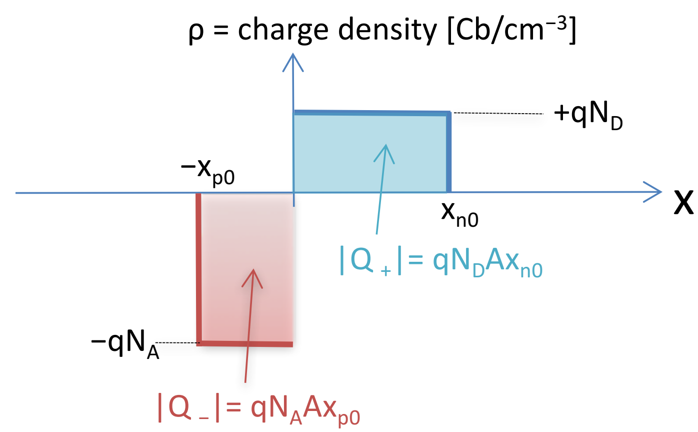
Charge on either side of the depletion region (in equilibrium)

$$Q_{J0} \triangleq |Q_+| = |Q_-| = AqN_D x_{n0}$$ ← Junction Charge

$$Q_{J0} = AqN_D \overbrace{W\frac{N_A}{N_A+N_D}}^{=x_{n0}} = Aq\frac{N_AN_D}{N_A+N_D}W$$

$$\begin{cases} x_{n0} = W\left(\dfrac{N_A}{N_A+N_D}\right) \\ x_{p0} = W\left(\dfrac{N_D}{N_A+N_D}\right) \end{cases}$$

$$\boxed{Q_{J0}} = Aq\frac{N_AN_D}{N_A+N_D}\underbrace{\sqrt{\frac{2\varepsilon_{si}}{q}\Phi_B\frac{N_A+N_D}{N_AN_D}}}_{=W=X_{d0}} = \boxed{A\sqrt{2\varepsilon_{si}q\frac{N_AN_D}{N_A+N_D}\Phi_B}}$$

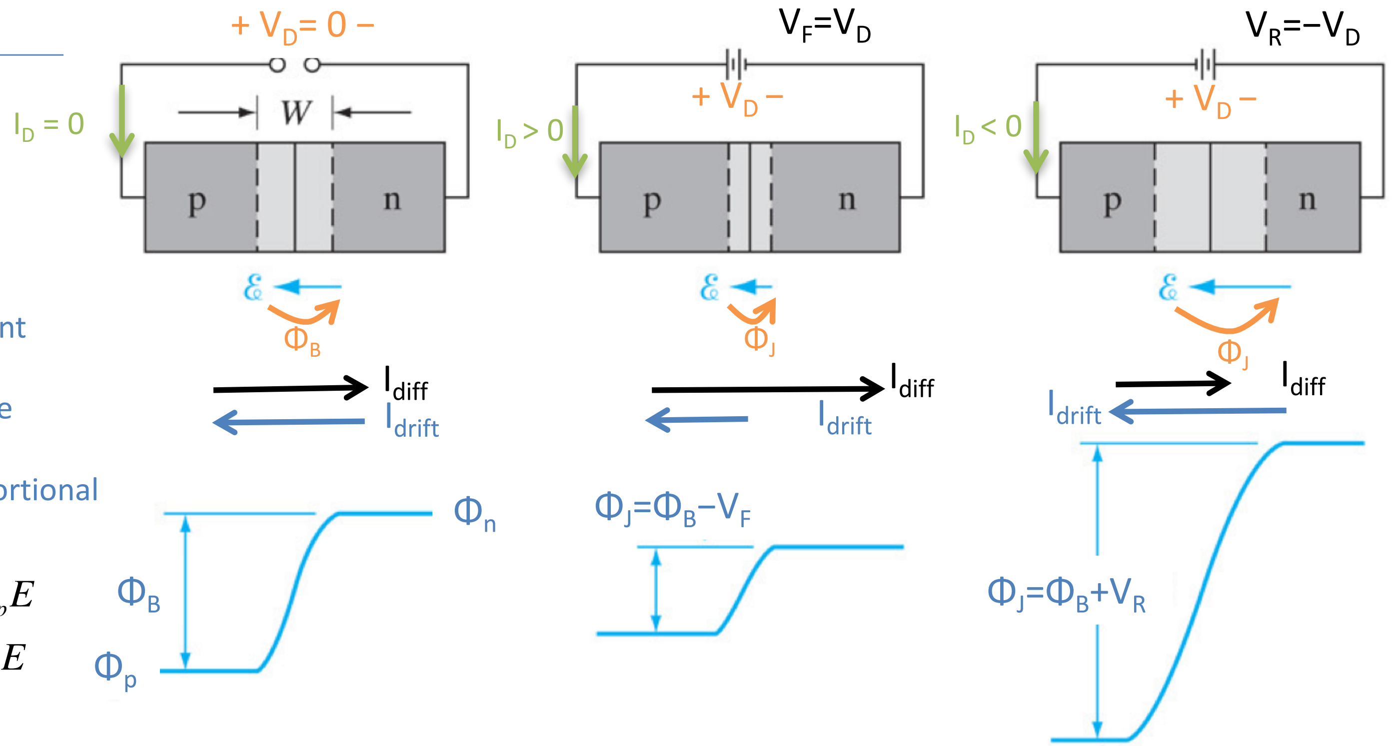


Effect of bias voltage on the PN Junction

$I_D = I_{diff} - I_{drift}$

source: Streetman

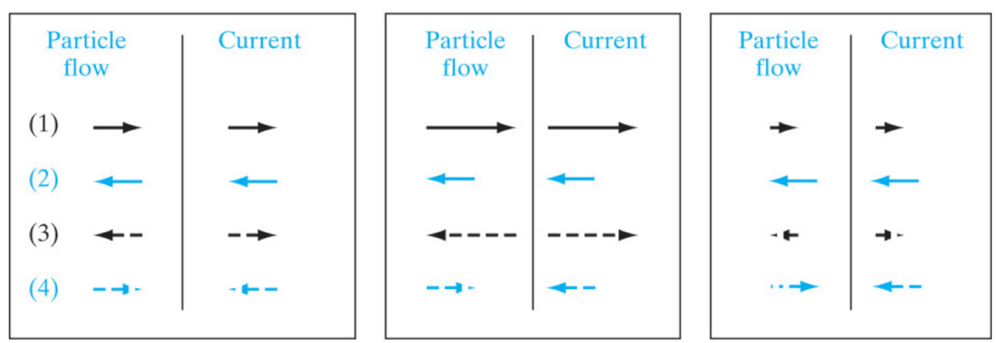
(a) Open circuit (Equilibrium)　(b) Forward Bias　(c) Reverse Bias



The drift current has the same direction of the electric field (and it is proportional to it)

$$J_{p,drift} = pq\mu_p E$$

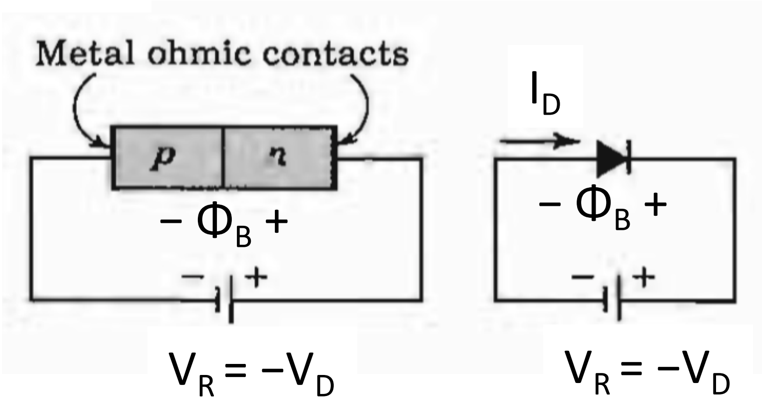
$$J_{n,drift} = nq\mu_n E$$



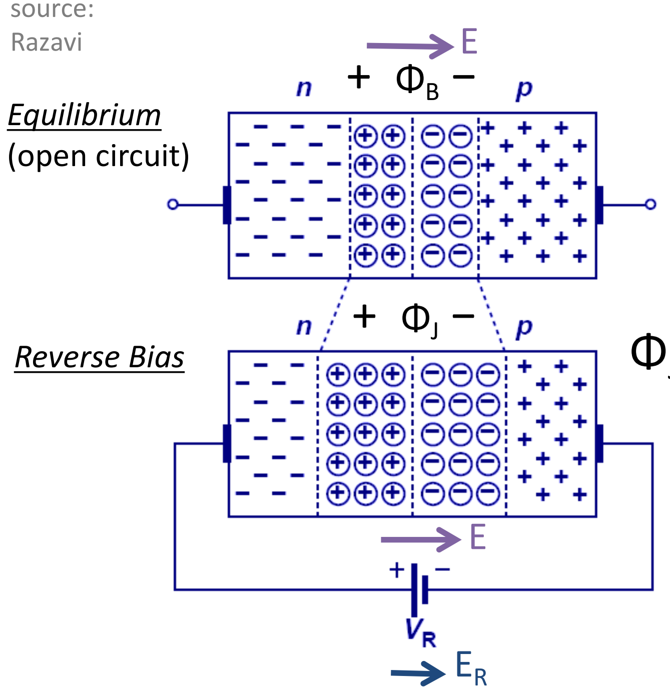
(1) Hole diffusion
(2) Hole drift
(3) Electron diffusion
(4) Electron drift

PN Junction under Reverse Bias

source:
Millman & Halkias



source:
Razavi



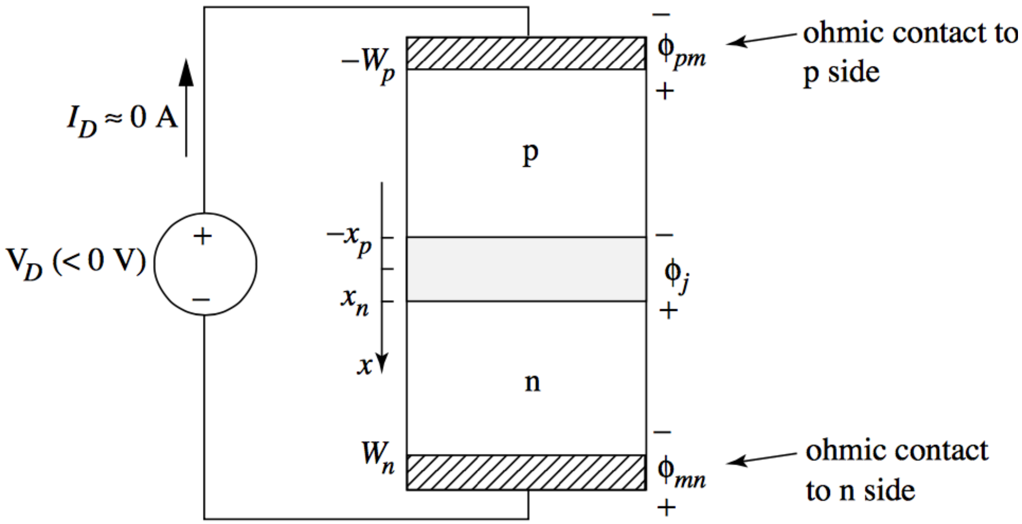
$$\Phi_J = \Phi_B + V_R$$

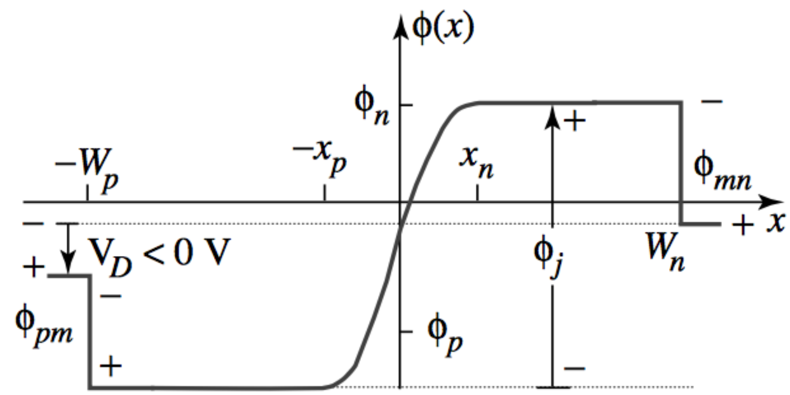
- The external voltage V_R is directed from the N side to the P side (same as Φ_B), therefore the electric field (E_R) due to the bias V_R enhances the built in field E. The depletion region W is widened

- Since the “barrier” potential rises higher than in equilibrium it gets harder for the majority carriers to cross the junction and easier for the minority carrier to be swept (drifted) across the junction.

- The current carried under reverse bias is negligible ⇨ the minority carriers are too few !!

PN junction under reverse bias





KVL: $V_D + \Phi_{mn} + \Phi_J + \Phi_{mn} = 0$

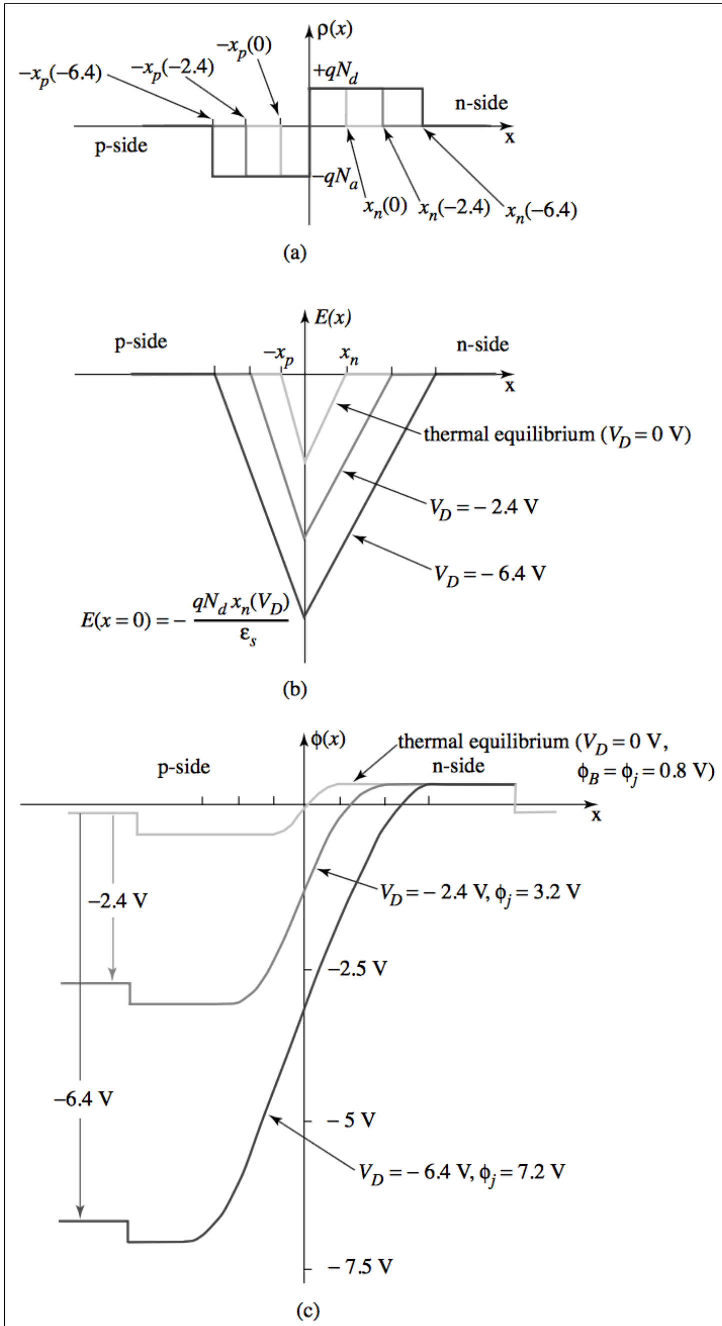
$$\Phi_J = -V_D \underbrace{- \Phi_{mn} - \Phi_{mp}}_{= \Phi_B} = \Phi_B - V_D$$

potential barrier in non-equilibrium

potential barrier in equilibrium

source: Howe & Sodini

PN junction under reverse bias



(a) charge density (b) electric field (c) potential

Example:

-
PN junction under reverse biases V_D=−2.4 V and V_D=−6.4V

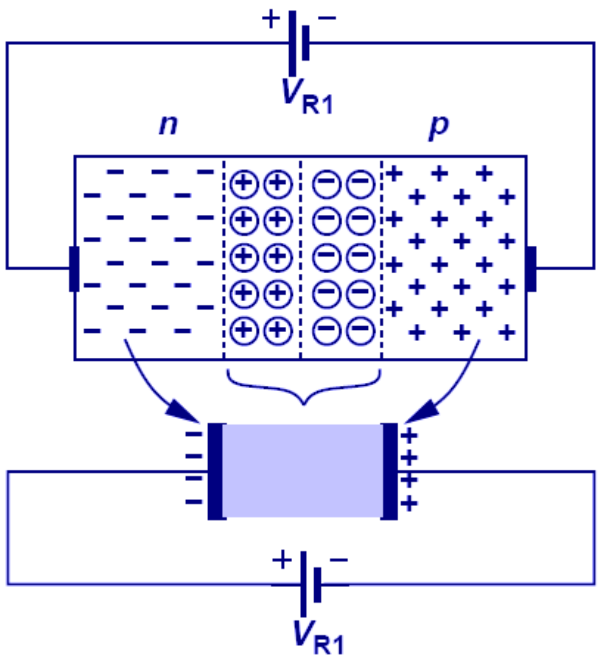
Assuming the potential barrier in equilibrium is Φ_B = 0.8V

As the PN junction gets **more reverse biased**

- the **depletion region** $X_d = x_p + x_n$ becomes **wider**
- the **potential barrier** becomes **stronger**

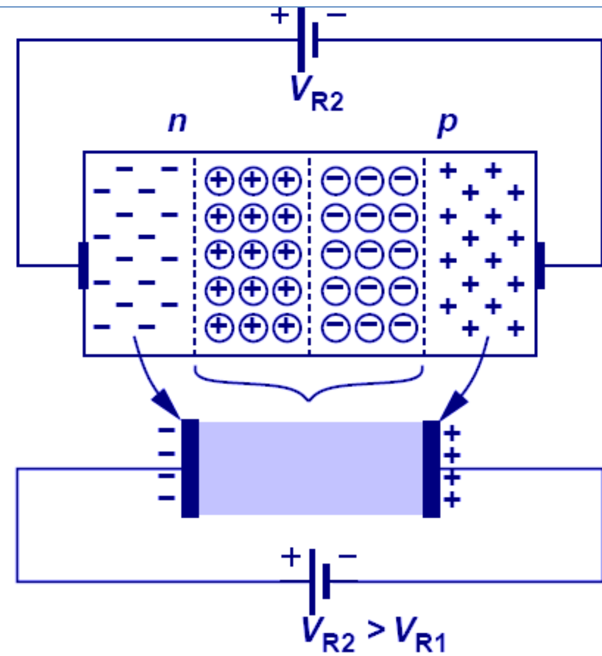
source: Howe & Sodini

Application of Reverse Biased PN Junction: Voltage Dependent capacitor



source: Razavi

(a)



(b)

Varactor

Qualitatively we see that if V_R goes up then C_J goes down

- A reverse biased PN junction can be used as a capacitor. By varying V_R, the depletion width changes, and so does the capacitance associated. Therefore a reverse biased PN junction can be seen as a voltage-dependent capacitor.

Capacitance of Reverse Biased Diode

$$X_d\big|_{@V_D} = \sqrt{\frac{2\varepsilon_{si}}{q}\left(\Phi_B - V_D\right)\frac{N_A+N_D}{N_A N_D}} = \sqrt{\frac{2\varepsilon_{si}}{q}\Phi_B\left(1-\frac{V_D}{\Phi_B}\right)\frac{N_A+N_D}{N_A N_D}} = \underbrace{X_{d0}}_{\text{width of depletion region in equilibrium}}\sqrt{1-\frac{V_D}{\Phi_B}}$$

careful: cap per unit area

$$C_J\big|_{@V_D} = \frac{\varepsilon_{si}}{X_d\big|_{@V_D}} = \sqrt{\frac{q\varepsilon_{si}}{2}\frac{N_A N_D}{\left(N_A+N_D\right)}\frac{1}{\left(\Phi_B - V_D\right)}} = \sqrt{\frac{q\varepsilon_{si}}{2}\frac{N_A N_D}{\left(N_A+N_D\right)}\frac{1}{\Phi_B}\frac{1}{\left(1-V_D/\Phi_B\right)}} =$$

$$= \frac{CJ0}{\sqrt{1-V_D/\Phi_B}}$$

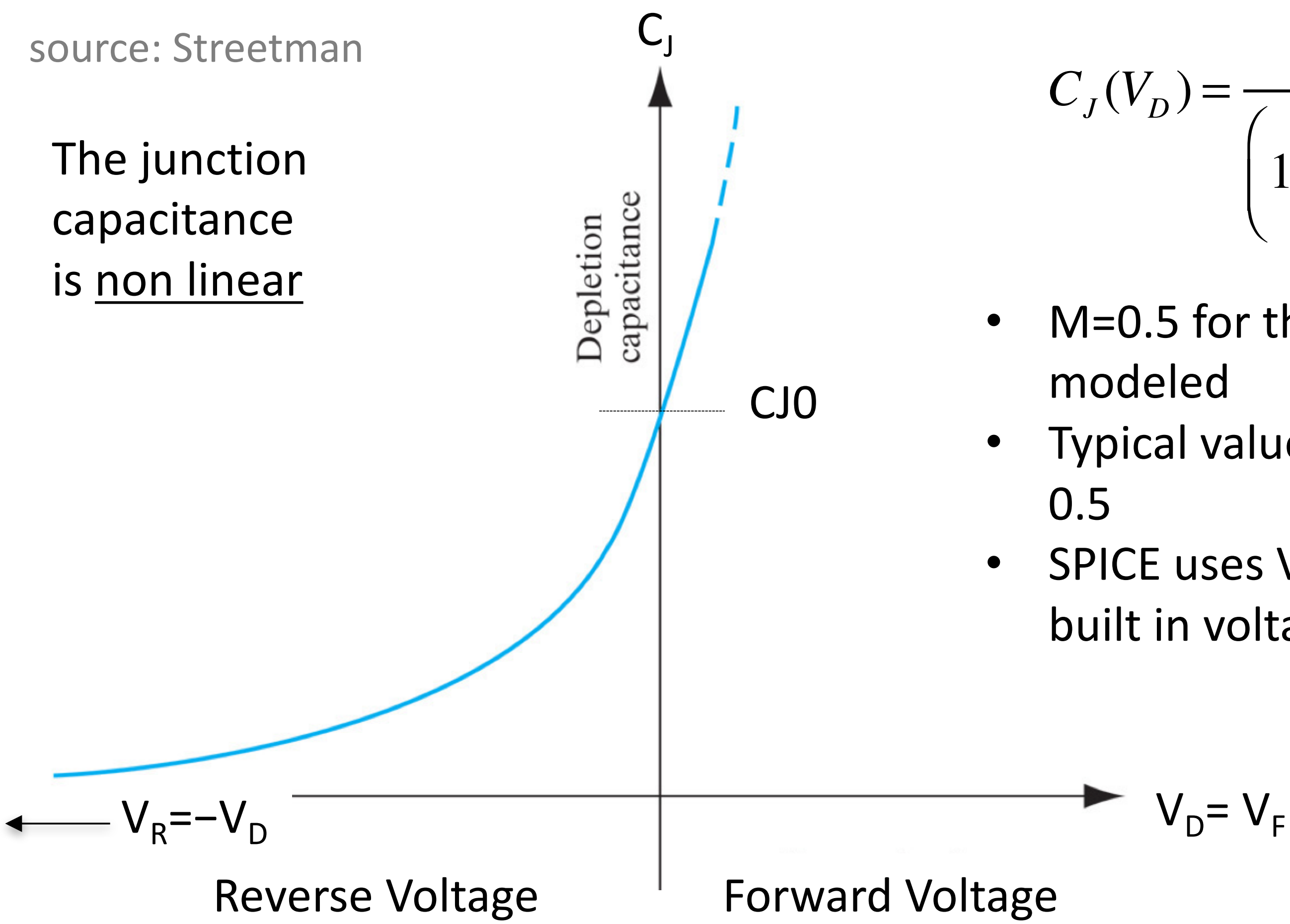
$$= CJ0 = \frac{\varepsilon_{si}}{X_{do}}$$

CJ0 = junction capacitance at zero bias (in equilibrium)

Junction capacitance of reverse biased diode

source: Streetman

The junction capacitance is <u>non linear</u>



$$C_J(V_D) = \frac{CJ0}{\left(1 - \frac{V_D}{\Phi_B}\right)^M} = \frac{CJ0}{\left(1 + \frac{V_R}{\Phi_B}\right)^M}$$

- M=0.5 for the abrupt PN junction we modeled
- Typical values for M range from 0.3 to 0.5
- SPICE uses VJ (or PB) to denote the built in voltage in equilibrium (Φ_B)

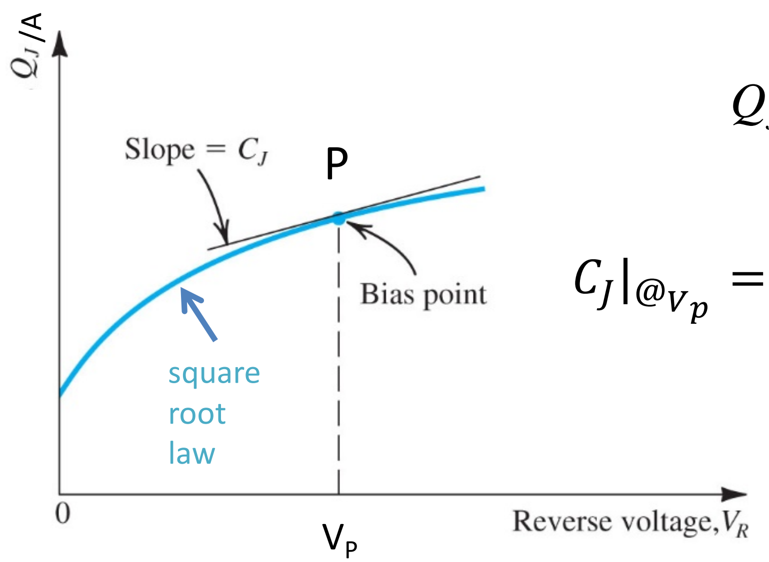
Junction capacitance of reverse biased diode

$$Q_J\big|_{@V_D} = \underbrace{A\sqrt{2\varepsilon_{si} q \frac{N_A N_D}{N_A+N_D}}}_{=\varsigma}\sqrt{(\Phi_B - V_D)} = A\sqrt{2\varepsilon_{si} q \frac{N_A N_D}{N_A+N_D}\Phi_B\left(1-\frac{V_D}{\Phi_B}\right)} = Q_{J0}\sqrt{1-\frac{V_D}{\Phi_B}} =$$

$$= Q_{J0}\sqrt{1+\frac{V_R}{\Phi_B}}$$

charge stored in either side of the depletion region in equilibrium

- The relation between charge Q_J and voltage ($V_R = -V_D$) across the diode is not linear
- It is difficult to define a capacitance that changes whenever V_R changes



source: Sedra & Smith

$$Q_J = A\sqrt{2\varepsilon_{si} q \frac{N_A N_D}{N_A+N_D}(\Phi_B + V_R)} = \varsigma\sqrt{\Phi_B + V_R}$$

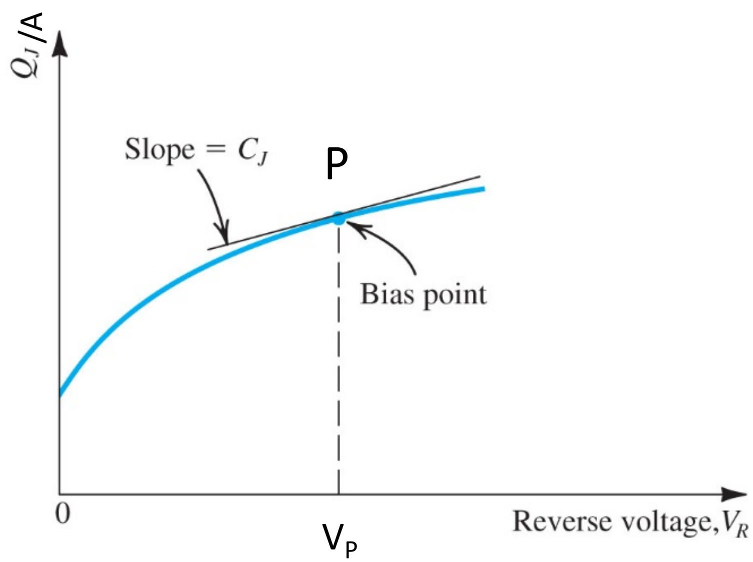
$$C_J\big|_{@V_P} = \frac{1}{A}\frac{dQ_J}{dV_R}\bigg|_{V_R=V_P} = \frac{1}{A}\frac{d}{dV_R}\left(\varsigma\sqrt{\Phi_B+V_R}\right)\bigg|_{@V_P} = \frac{\varsigma/A}{2\sqrt{\Phi_B+V_R}}\bigg|_{@V_P} =$$

$$= \frac{\varsigma/A}{2\sqrt{\Phi_B\left(1+\frac{V_R}{\Phi_B}\right)}}\Bigg|_{@V_P} = \frac{CJO}{\sqrt{1+\frac{V_R}{\Phi_B}}}\Bigg|_{@V_P}$$

Junction capacitance of reverse biased diode

Important result to remember
when using non linear devices

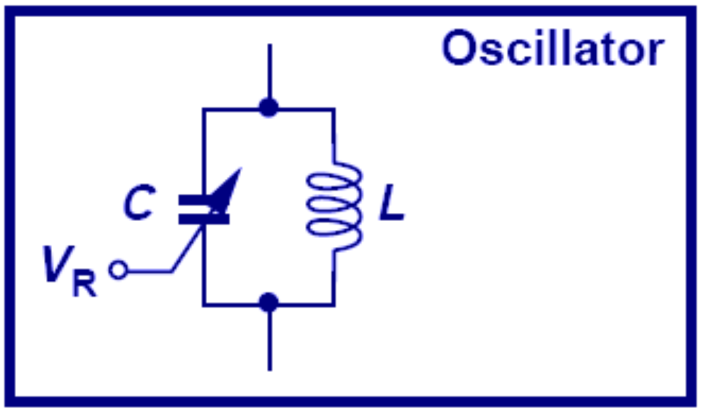
$$\left.\frac{dQ_J}{dV_R}\right|_{V_R=V_P} \boxed{\neq} \left.\frac{Q_J(V_R)}{V_R}\right|_{V_R=V_P}$$



source: Sedra & Smith

Application of a reverse biased diode

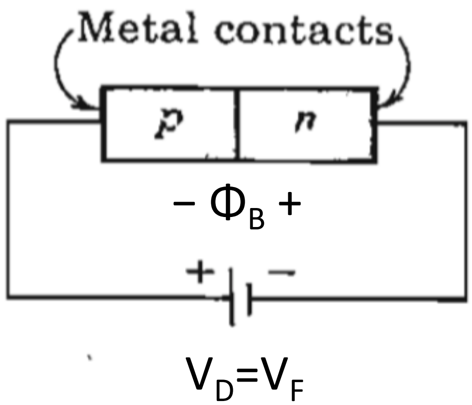
source: Razavi

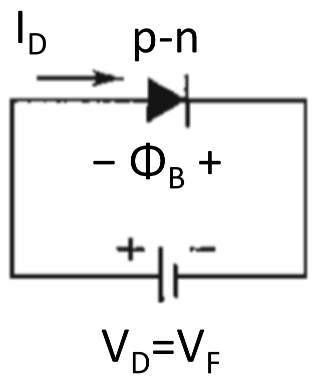


$$f_{res} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

PN Junction in Forward Bias

source:
Millman & Halkias

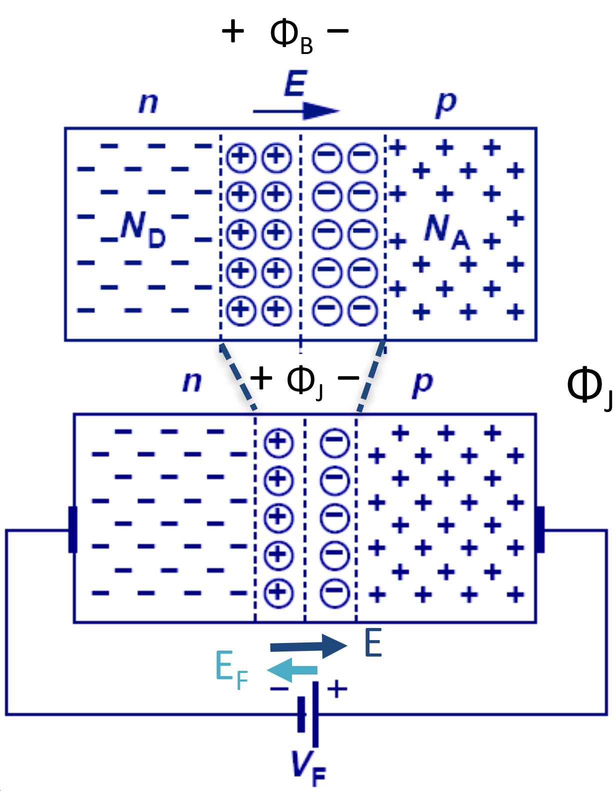




source:
Razavi

Equilibrium
(open circuit)

Forward Bias

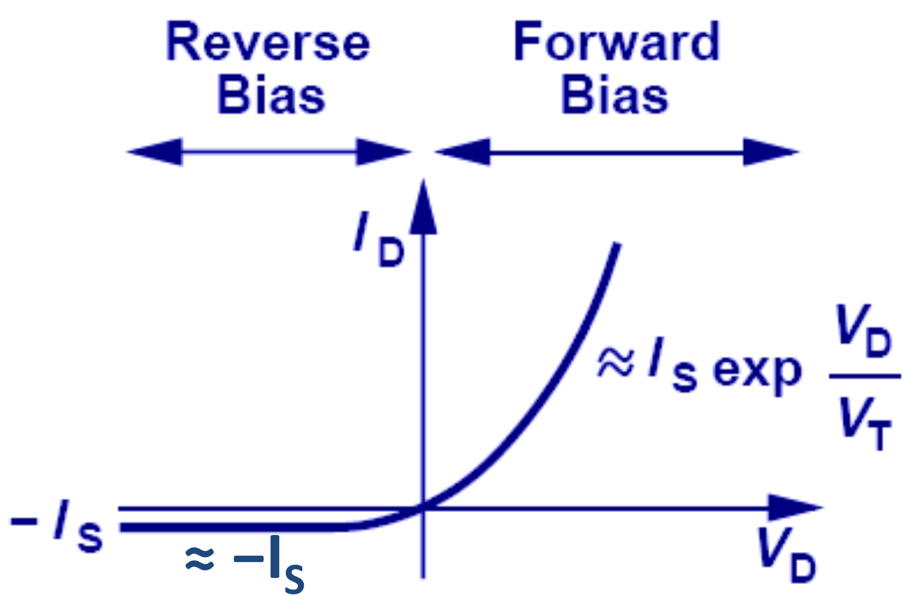


$\Phi_J = \Phi_B - V_F$

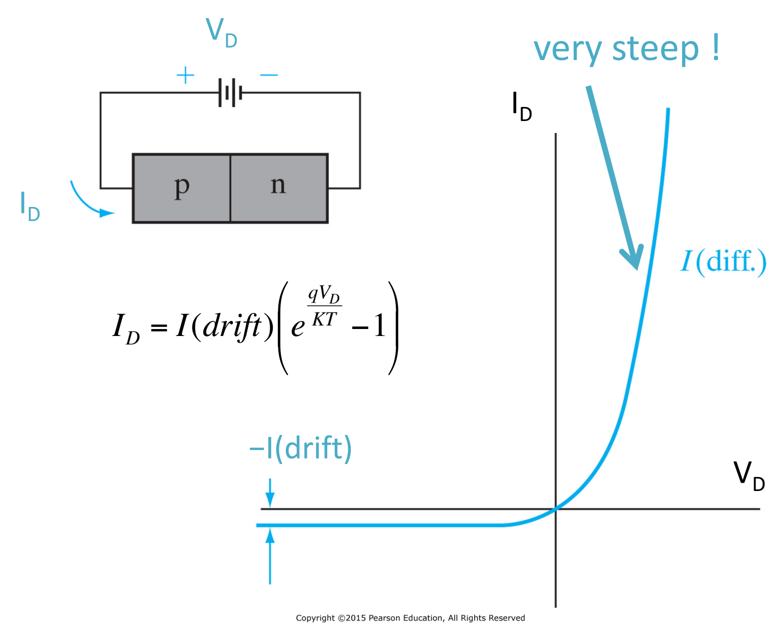
- The electric field (E_F) due to the bias (V_F) is opposite to the built in field E. The depletion region W is shortened
- Since the “barrier” potential becomes lower than in equilibrium it gets easier for the majority carriers to diffuse through the junction and harder for the minority carrier to be swept (drifted) across the junction.
- The majority carriers are a lot, so we expect a considerable current flow:
  $I_D = I_{diff} - I_{drift}$

Diode's Current/Voltage Characteristic

source: Razavi



$$I_D = I_S\left(e^{\frac{V_D}{V_T}} - 1\right)$$



source: Streetman

NOTE:

Consider $|V_D/V_T| \approx 2.3$ → $\begin{cases} \exp(2.3) \approx 9.97 \approx 10 \\ \exp(-2.3) \approx 0.1 \end{cases}$

At room temperature $V_D = 2.3 \times V_T \approx 59.8$ mV ≈ 60 mV

Diode's Current/Voltage Characteristic

- How can we quantify the I/V behavior of the diode ?
- We need to look at how electrons and holes move under the effect of the bias voltage
- In equilibrium:

holes at x_{n0}

holes at $-x_{p0}$

$$\Phi_B = V_T \ln \frac{p_{p0}}{p_{n0}} \Leftrightarrow p_{n0} = p_{p0} \cdot e^{-\Phi_B/V_T} \Leftrightarrow p_{n0} = \frac{p_{p0}}{e^{\Phi_B/V_T}}$$

electrons at x_{n0}

electrons at $-x_{p0}$

$$n_{p0} = \frac{n_{n0}}{e^{\Phi_B/V_T}}$$

$-x_{p0}$ and x_{n0} are the edges of the depletion region in equilibrium

- With an external bias voltage applied the situation is similar but now the potential barrier is $\Phi_B - V_D$ instead of only Φ_B

Law of the Junction

holes at x_n

holes at $-x_p$

$$p_n = p_p \cdot \frac{e^{V_D/V_T}}{e^{\Phi_B/V_T}}$$

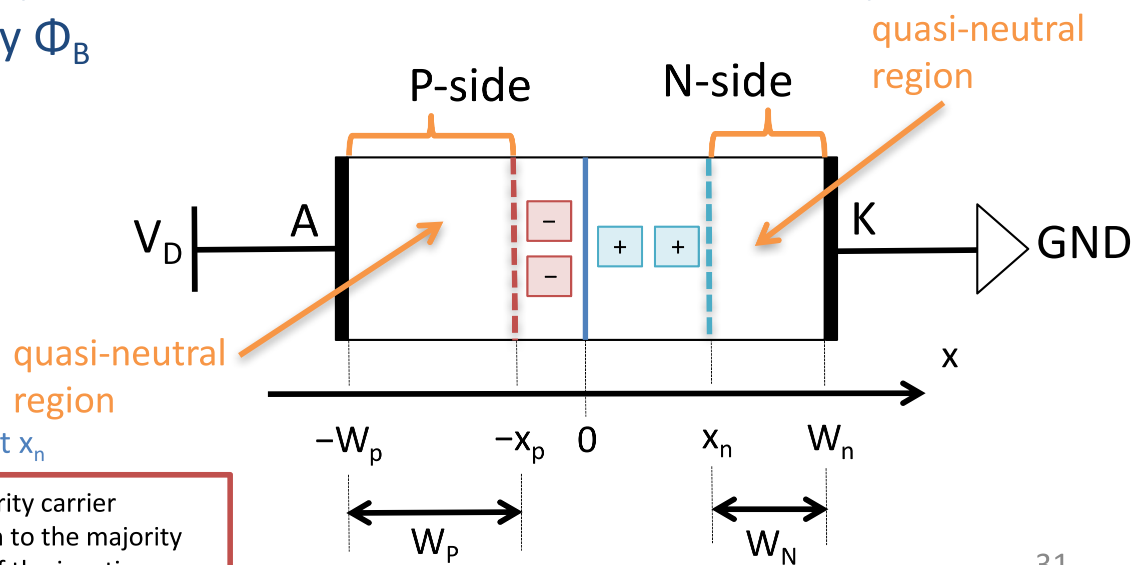
electrons at $-x_p$

$$n_p = n_n \cdot \frac{e^{V_D/V_T}}{e^{\Phi_B/V_T}}$$

electrons at x_n

$-x_p$ and x_n are the edges of the depletion region with non zero bias

The Law of the Junction relates the minority carrier concentration on one side of the junction to the majority carrier concentration on the other side of the junction



Diode's Current/Voltage Characteristic

- When we apply a forward bias V_D the potential barrier is reduced from Φ_B to $\Phi_B - V_D$. This reduces the drift field and upsets the balance between diffusion and drift that exists at zero bias.

- Electrons can now diffuse from the N side (where they are majority carriers) to the P side (where they become minority carriers). Similarly, holes diffuse from the P side into the N side. This migration of carriers from one side to the other is called **injection.**

- As a result of injection, more electrons are now present at $-x_p$ and more holes appear at x_n than when the barrier was higher
  - There is an excess of minority carriers at the edges of the depletion region

- Let's make some simplifying assumptions to help quantifying $p_n(x_n)$, $p_p(-x_p)$, $n_n(x_n)$ and $n_p(x_p)$
  - There is electric field only in the space charge region (that is between $-x_p$ and x_n). The regions from $-W_p$ to $-x_p$ (a.k.a. P bulk region) and from x_n to W_n (a.k.a. N bulk region) are quasi-neutral (it is like they were perfect conductors and in perfect conductors there is no electric field inside)

Diode's Current/Voltage Characteristic

- The requirement that the bulk regions on either side remain charge-neutral even under applied bias (remember: E(x) = ρ(x)/ε, so no field means no charge density) implies that any increases in the minority carriers due to injection must be counter balanced by in an increase of majority carriers
  - On the P side $\rho(x=-x_p) = 0$ implies that:

$$p_p(-x_p) = N_A + n_p(-x_p) \approx p_{p0} + n_p(-x_p)$$

  - On the N side $\rho(x=x_n) = 0$ implies that:

$$n_n(x_n) = N_D + p_n(x_n) \approx n_{n0} + p_n(x_n)$$

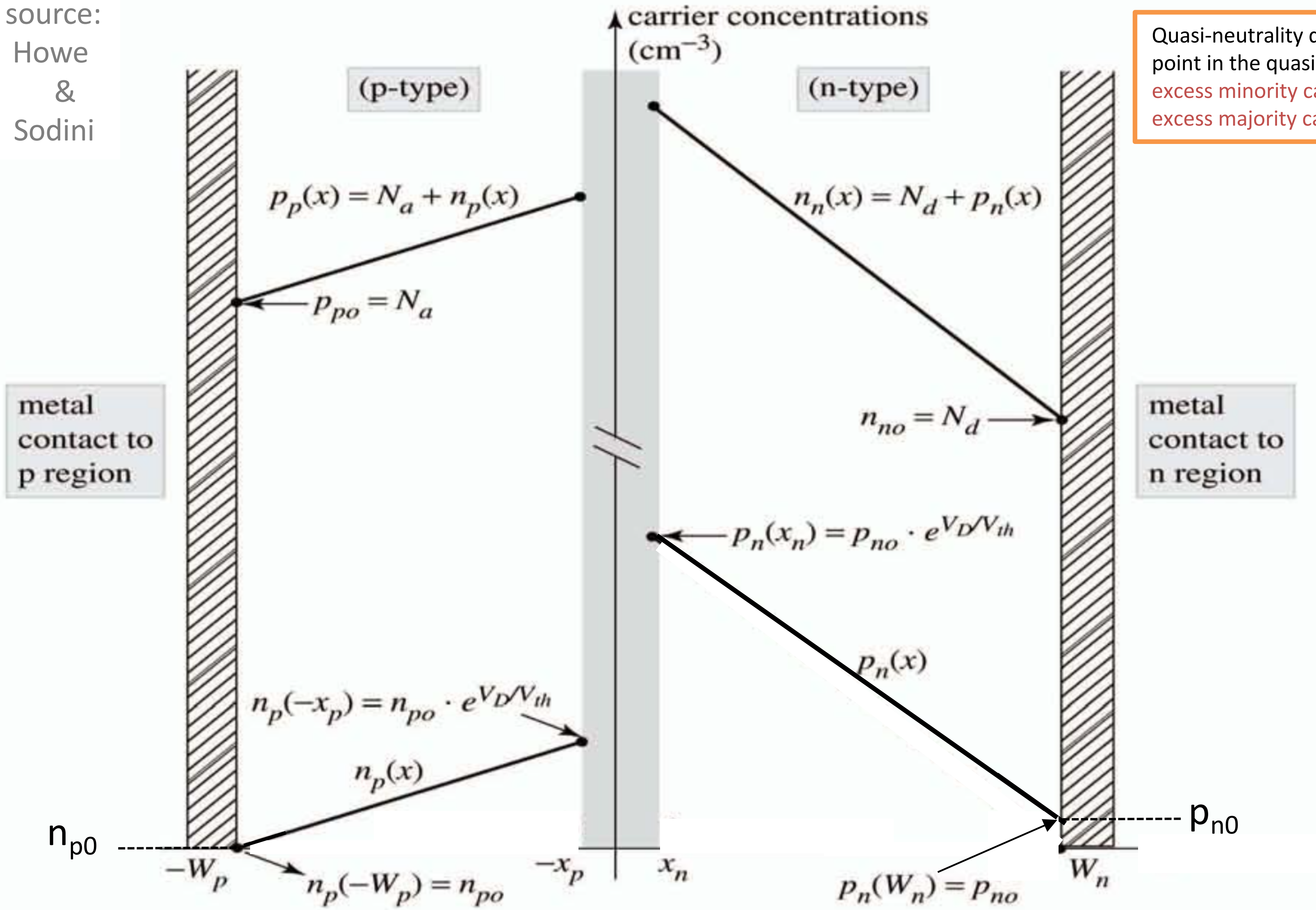
- Assuming the level of **injection is low** we can neglect the slight increase in majority carrier concentration due to carrier injection and write:

$$p_p(-x_p) \approx p_{p0} \approx N_A \qquad n_p(-x_p) << p_{p0}$$

$$n_n(x_n) \approx n_{n0} \approx N_D \qquad p_n(x_n) << n_{n0}$$

Minority and Majority carriers concentrations in a forward biased diode

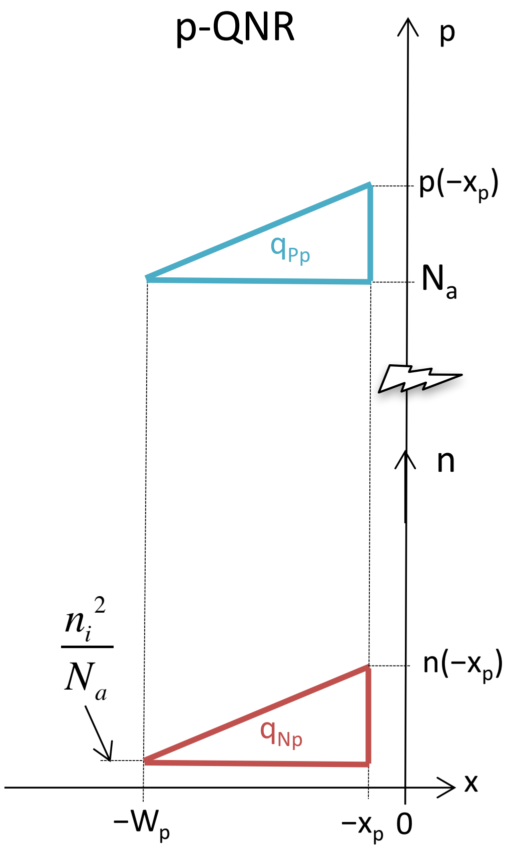
source: Howe & Sodini

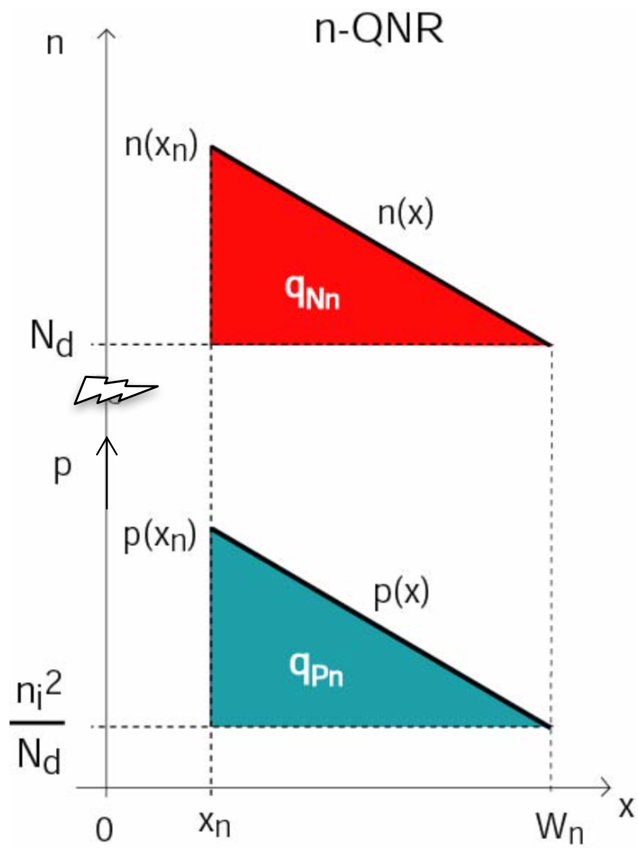


Quasi-neutrality demands that at every point in the quasi-neutral regions: excess minority carrier concentration = excess majority carriers concentration

Minority and Majority carriers concentrations in a forward biased diode

Quasi-neutrality demands that at every point in the quasi-neutral regions:
excess minority carrier concentration = excess majority carriers concentration





__NOTE:__
at this point we did not put much thinking in the profile of the minority carriers diffusing in the QNRs, however if we assume the diffusion occurs over a short distance we can approximate the curves with straight lines !

source: Sodini

Diode's Current/Voltage Characteristic

- Under the low injection condition we can approximate the **Law of the Junction** as follows:

$$p_n(x_n) = \frac{p_p(-x_p)}{e^{\Phi_B/V_T}} \cdot e^{V_D/V_T} \approx \frac{p_{p0}}{e^{\Phi_B/V_T}} \cdot e^{V_D/V_T} = p_{n0} \cdot e^{V_D/V_T}$$

$$n_p(-x_p) = \frac{n_n(x_n)}{e^{\Phi_B/V_T}} \cdot e^{V_D/V_T} \approx \frac{n_{n0}}{e^{\Phi_B/V_T}} \cdot e^{V_D/V_T} = n_{p0} \cdot e^{V_D/V_T}$$

The concentration of minority carriers at the edges of the space charge region are an exponential function of the applied bias

$$p_n(x_n) \approx p_{n0} \cdot e^{V_D/V_T} \approx \frac{n_i^2}{N_D} \cdot e^{V_D/V_T}$$

$$n_p(-x_p) \approx n_{p0} \cdot e^{V_D/V_T} \approx \frac{n_i^2}{N_A} \cdot e^{V_D/V_T}$$

source: G.W. Neudeck

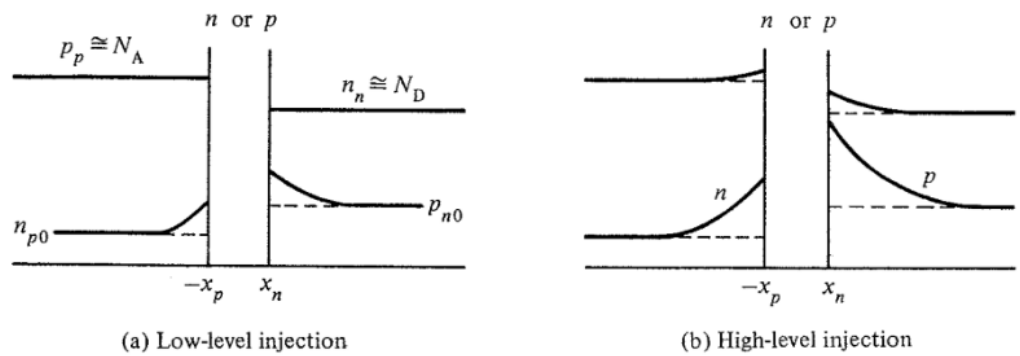


Fig. 4.9 Carrier concentrations: (a) low-level injection; (b) high-level injection.

Minority carriers in a forward biased PN junction

- The excess concentration of minority carriers at the edges is:

$$\Delta n_p \triangleq n_{p0} \cdot e^{V_D/V_T} - n_{p0} = n_{p0}\left(1 - e^{V_D/V_T}\right) \qquad \Delta p_n \triangleq p_{n0} \cdot e^{V_D/V_T} - p_{n0} = p_{n0}\left(1 - e^{V_D/V_T}\right)$$

- As the injected carriers diffuse through the quasi-neutral regions they recombine with the majority carriers and eventually disappear. The excess minority carriers decay exponentially with distance, and the decay constant is called diffusion length L.

source:
Sedra & Smith

It can be shown through the **current continuity equation** that under the low injection condition the concentration's profile of the minority carriers diffusing in the quasi neutral regions is **exponential**

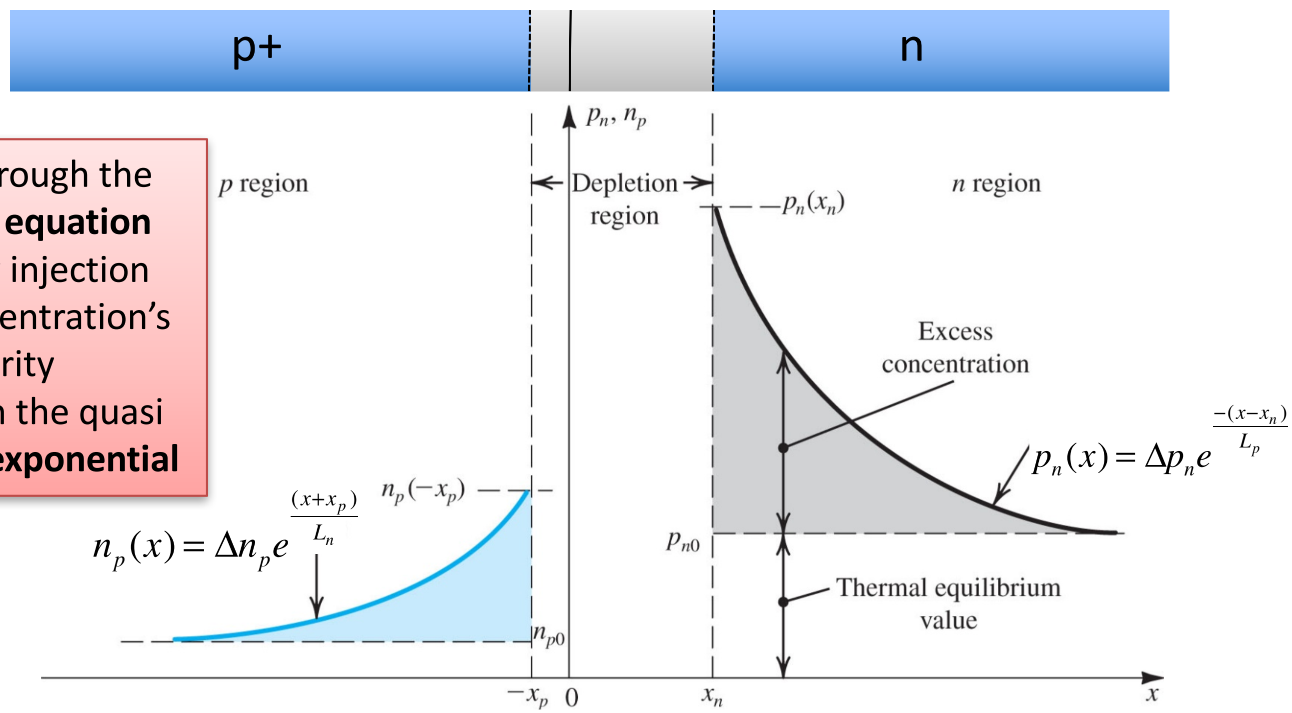


Figure 3.12 Minority-carrier distribution in a forward-biased *pn* junction. It is assumed that the *p* region is more heavily doped than the *n* region; $N_A \gg N_D$.

Minority carrier concentrations Equilibrium vs. Forward Bias

source: G.W. Neudeck

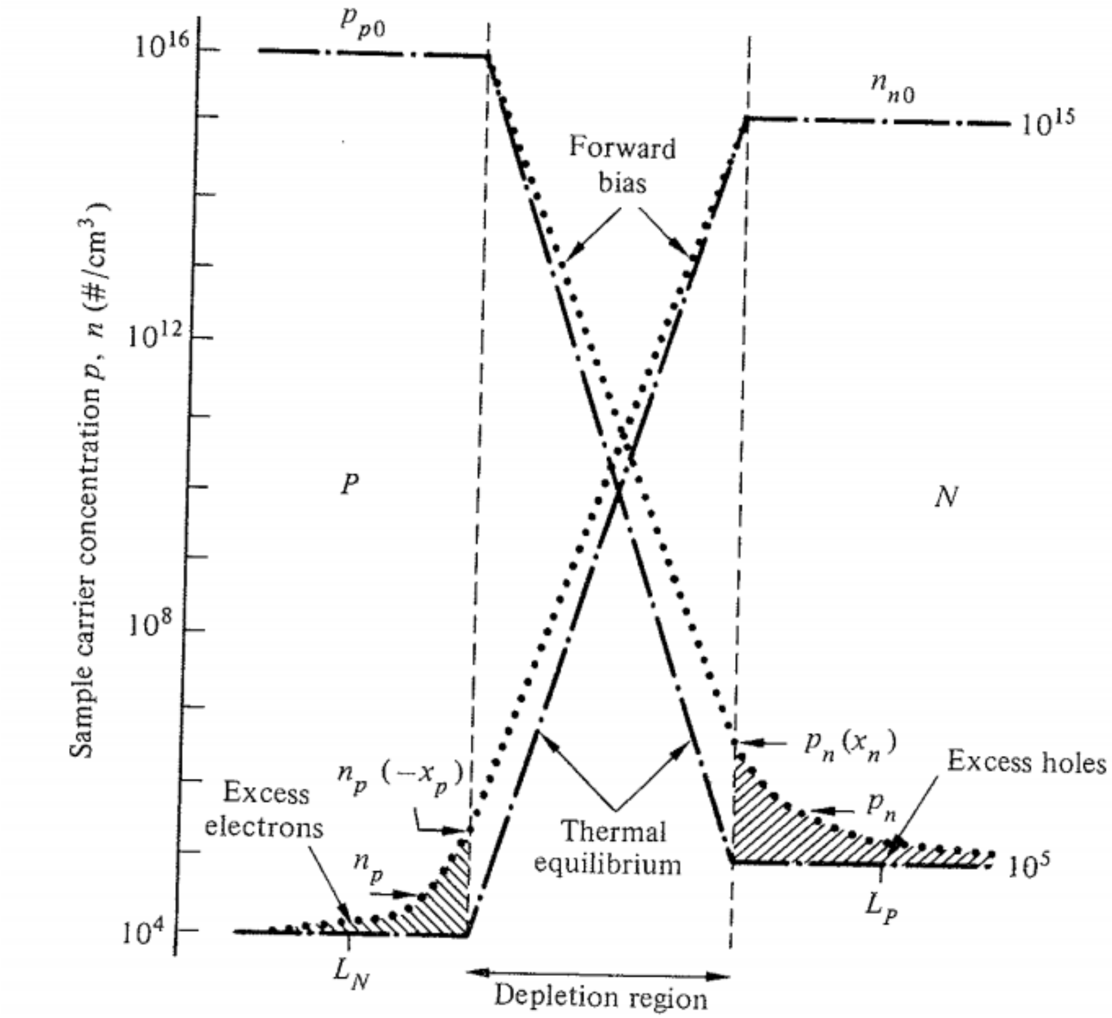


Fig. 3.11 Carrier concentrations at thermal equilibrium and at forward bias.

Aside (a simple trick to simplify the math)

- Change the axis selection for the bulk regions as follows:

$$\begin{cases} x'' \triangleq 0'' \equiv -x_p \\ x' \triangleq 0' \equiv x_n \end{cases}$$

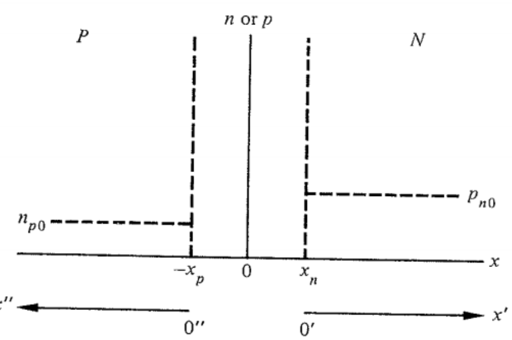


Fig. 3.7 Axis selection for bulk regions.

source: G.W. Neudeck

- This way we can write the minority carriers concentrations as follows:

$$\begin{cases} n_p(x'') = \Delta n_p e^{-\frac{x''}{L_n}} \\ p_n(x') = \Delta p_n e^{-\frac{x'}{L_p}} \end{cases}$$

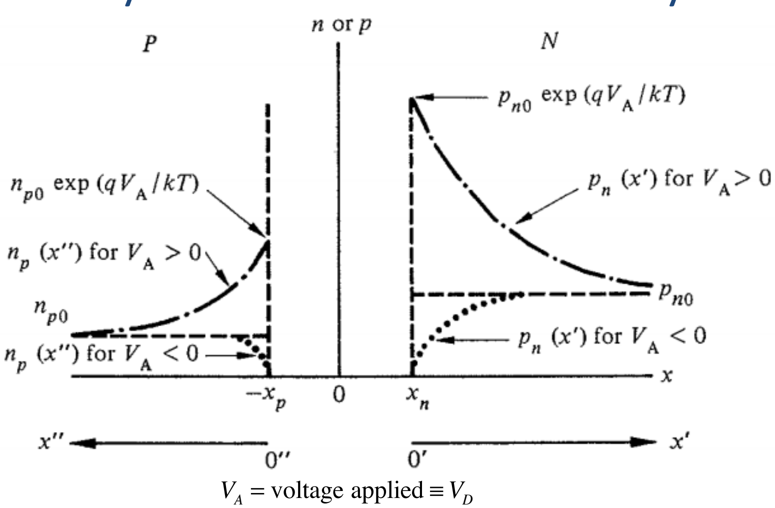


Fig. 3.8 Minority carrier concentrations in the bulk regions.

source: G.W. Neudeck

$$L_n = \sqrt{D_n \tau_n}$$

$$L_p = \sqrt{D_p \tau_p}$$

L_n = Diffusion length for electrons [m]
L_p = Diffusion length for holes [m]
D_n = Diffusion coefficient for electron [m^2/s]
D_p = Diffusion coefficient for holes [m^2/s]
τ_n = mean lifetime for electrons [s]
τ_p = mean lifetime for holes [s]

Physical interpretation of L, D, and τ

- Physically L_p and L_n represents the average distance minority carriers can diffuse into a "sea" of majority carriers before being annihilated

$$L_p \triangleq \frac{\int_0^\infty x\Delta p_n(x)dx}{\int_0^\infty \Delta p_n(x)dx} \qquad L_n \triangleq \frac{\int_0^\infty x\Delta n_p(x)dx}{\int_0^\infty \Delta n_p(x)dx}$$

- Physically τ_p and τ_n represents the average time minority carriers can diffuse into a "sea" of majority carriers before being annihilated

- Physically the D_p and D_n represents the ease with which holes and electrons diffuse

$$D_p \triangleq \mu_p \frac{KT}{q} \qquad D_n \triangleq \mu_n \frac{KT}{q}$$

PN Junction current

- The current that flows through any "transverse" section x must be the same and is formed by both holes and electrons

$$J(x) = J_n(x) + J_p(x)$$
$$J_n(x) = J_{n,drift}(x) + J_{n,diff}(x)$$
$$J_p(x) = J_{p,drift}(x) + J_{p,diff}(x)$$

- If we look at the current through a section x in the quasi-neutral regions, since in the quasi-neutral regions there is no electric field there will be no drift

injected electrons diffusing on P-side (minority carriers)

$$J_n(x) = J_{n,diff}(x) = qD_n \frac{dn_p(x)}{dx} = q\frac{D_n}{L_n} n_{p0}\left(e^{V_D/V_T} - 1\right)e^{+(x+x_p)/L_n} \quad for\ -W_p \le x \le -x_p$$

$$J_p(x) = J_{p,diff}(x) = -qD_p \frac{dp_n(x)}{dx} = q\frac{D_p}{L_p} p_{n0}\left(e^{V_D/V_T} - 1\right)e^{-(x-x_n)/L_p} \quad for\ x_n \le x \le W_n$$

injected holes diffusing on N-side (minority carriers)

PN Junction currents

- The easier sections for evaluating the diffusion currents are the edges of the space charge region ($x=-x_p$ and $x=x_n$)

$$J_n(-x_p) = J_{n,diff}(-x_p) = q\frac{D_n}{L_n} n_{p0}\left(e^{V_D/V_T} - 1\right)$$

$$J_p(x_n) = J_{p,diff}(x_n) = q\frac{D_p}{L_p} p_{n0}\left(e^{V_D/V_T} - 1\right)$$

- The total current through a section of the diode can be found as follows:

We do not know have this value (yet) !

$$J = J_n(-x_p) + J_p(-x_p) = J_n(x_n) + J_p(x_n)$$

We do not know have this value (yet) !

- If we make the simplifying assumption that there is no recombination-generation in the space-charge region, this means that the electrons/holes current at $x=-x_p$ and at $x=x_n$ do not change in the space-charge region and therefore we can write:

$$J_n(-x_p) = J_n(x_n)$$

$$J_p(x_n) = J_p(-x_p)$$

PN Junction current

- Therefore we can add together the diffusion currents at $x=-x_p$ and $x=x_n$

$$J = J_n(-x_p) + J_p(x_n) = q\frac{D_n}{L_n}n_{p0}\left(e^{V_D/V_T}-1\right) + q\frac{D_p}{L_p}p_{n0}\left(e^{V_D/V_T}-1\right)$$

$$J = \left(q\frac{D_n}{L_n}n_{p0} + q\frac{D_p}{L_p}p_{n0}\right)\left(e^{V_D/V_T}-1\right)$$

$$I_D = J\times A = I_{n,diff} + I_{p,diff} = \left(Aq\frac{D_n}{L_n}n_{p0} + Aq\frac{D_p}{L_p}p_{n0}\right)\left(e^{V_D/V_T}-1\right)$$

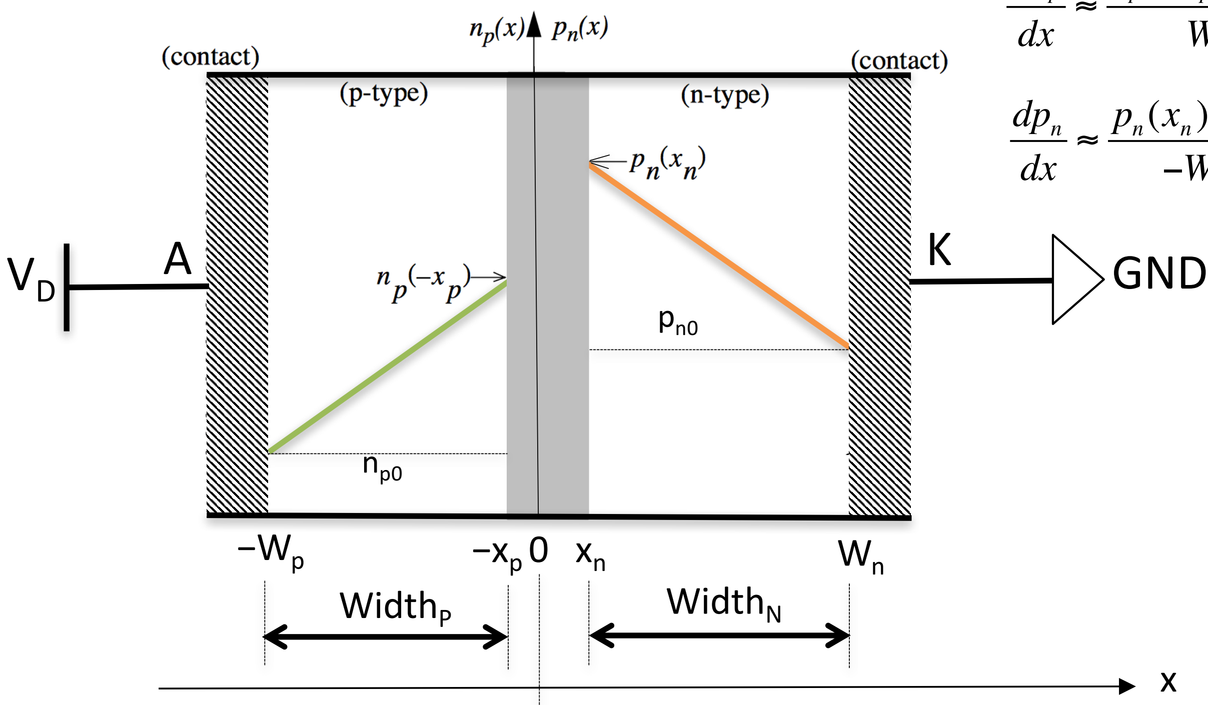
$$\boxed{I_D = Aq\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)\left(e^{V_D/V_T}-1\right) = \overbrace{I_S}^{\text{scale current (a.k.a. saturation current)}}\left(e^{V_D/V_T}-1\right)}$$

Minority concentrations for a forward biased short base diode

- If the diode is short the exponential decays can be approximated with straight lines

$$\frac{dn_p}{dx} \approx \frac{n_p(-x_p) - n(-W_p)}{Width_P} = \frac{n_{p0}e^{V_D/V_T} - n_{p0}}{Width_P}$$

$$\frac{dp_n}{dx} \approx \frac{p_n(x_n) - p_n(W_n)}{-Width_N} = -\frac{p_{n0}e^{V_D/V_T} - p_{n0}}{Width_N}$$



$$\boxed{I_D = Aq\left(\frac{D_n}{Width_P}\underline{n_{p0}} + \frac{D_p}{Width_N}\underline{p_{n0}}\right)\left(e^{V_D/V_T} - 1\right) = I_S\left(e^{V_D/V_T} - 1\right)}$$

Current densities in the forward bias case

No matter which section x we consider J_p and J_n must add to the same total value J

source: G.W. Neudeck

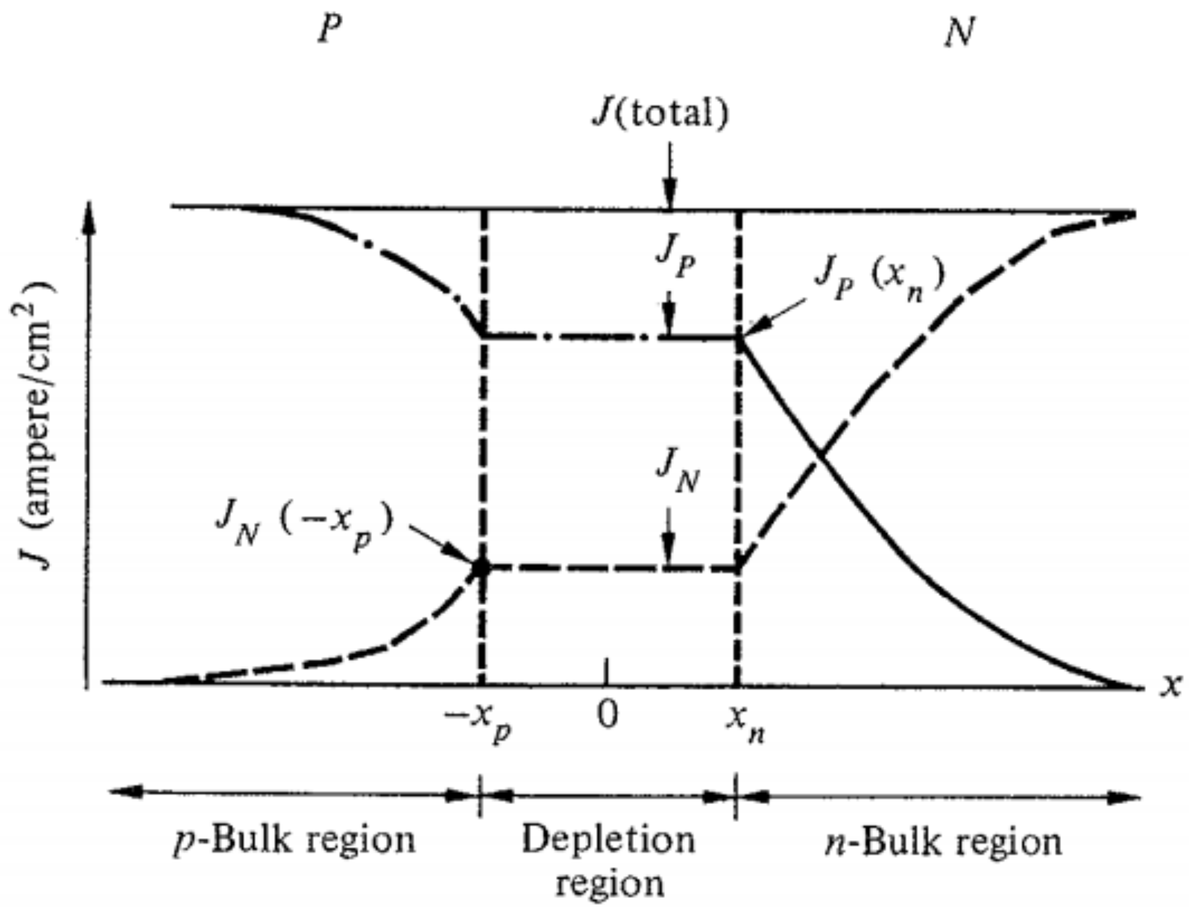


Fig. 3.9 Current density in the forward biased case.

Assumptions used to derive the Ideal diode equation

- Depletion approximation (in depletion region: $N_A >> p_{p0}(x)$, $N_D >> n_{n0}(x)$)
- Negligible fields in the quasi neutral regions (QNR). This means that the applied voltage V_D is dropped entirely across the space charge region (SCR)
- Quasi-neutral regions that are so long that the injected carriers all recombine before reaching the end contacts
- Low levels of injection
- No recombination-generation in the space-charge region. This means that the electron current and the hole current are constant in the space-charge region (that is $J_n(-x_p)=J_n(x_n)$ and $J_p(x_n)=J_p(-x_p)$)

- Real diodes may deviate considerably from the ideal equation.

$$I_D = I_S^*(e^{\frac{V_D}{nV_T}} - 1)$$

n = emission coefficient (a.k.a. ideality factor)
Typically the ideality factor n varies from 1 to 2

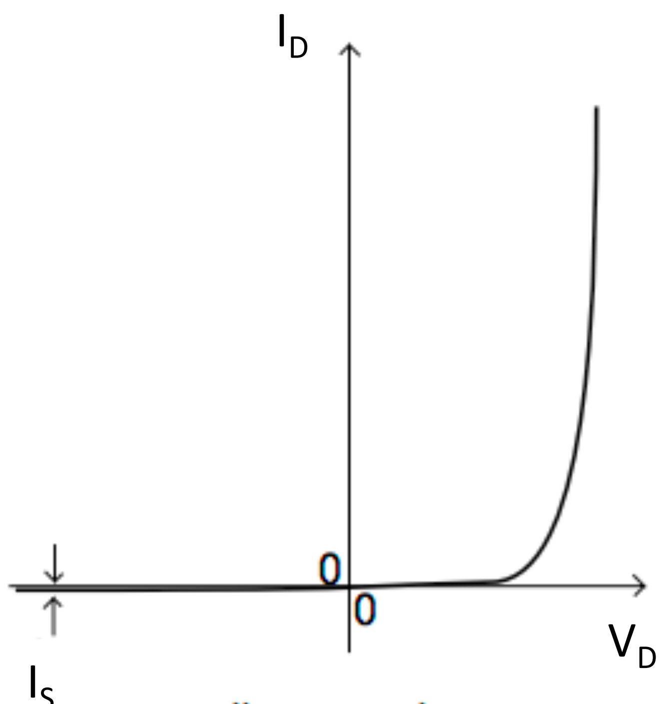
Ideal diode

source: Sodini

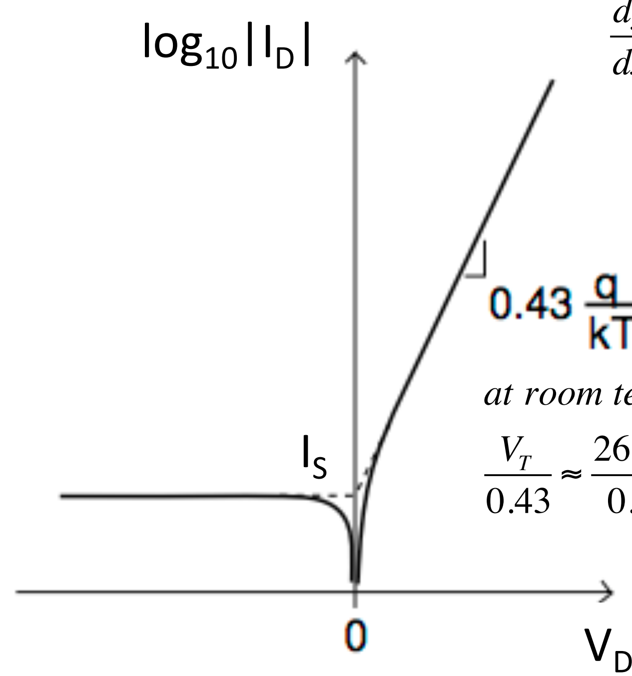
$$\begin{cases} y = \log_{10}\left(I_S \cdot e^{\frac{V_D}{V_T}}\right) = \log_{10}(I_S) + \log_{10}(e) \cdot \dfrac{V_D}{V_T} \approx \log_{10}(I_S) + 0.43\dfrac{V_D}{V_T} \\ x = V_D \end{cases}$$

$$\Downarrow$$

$$\frac{dy}{dx} \approx \frac{0.43}{V_T}$$



linear scale



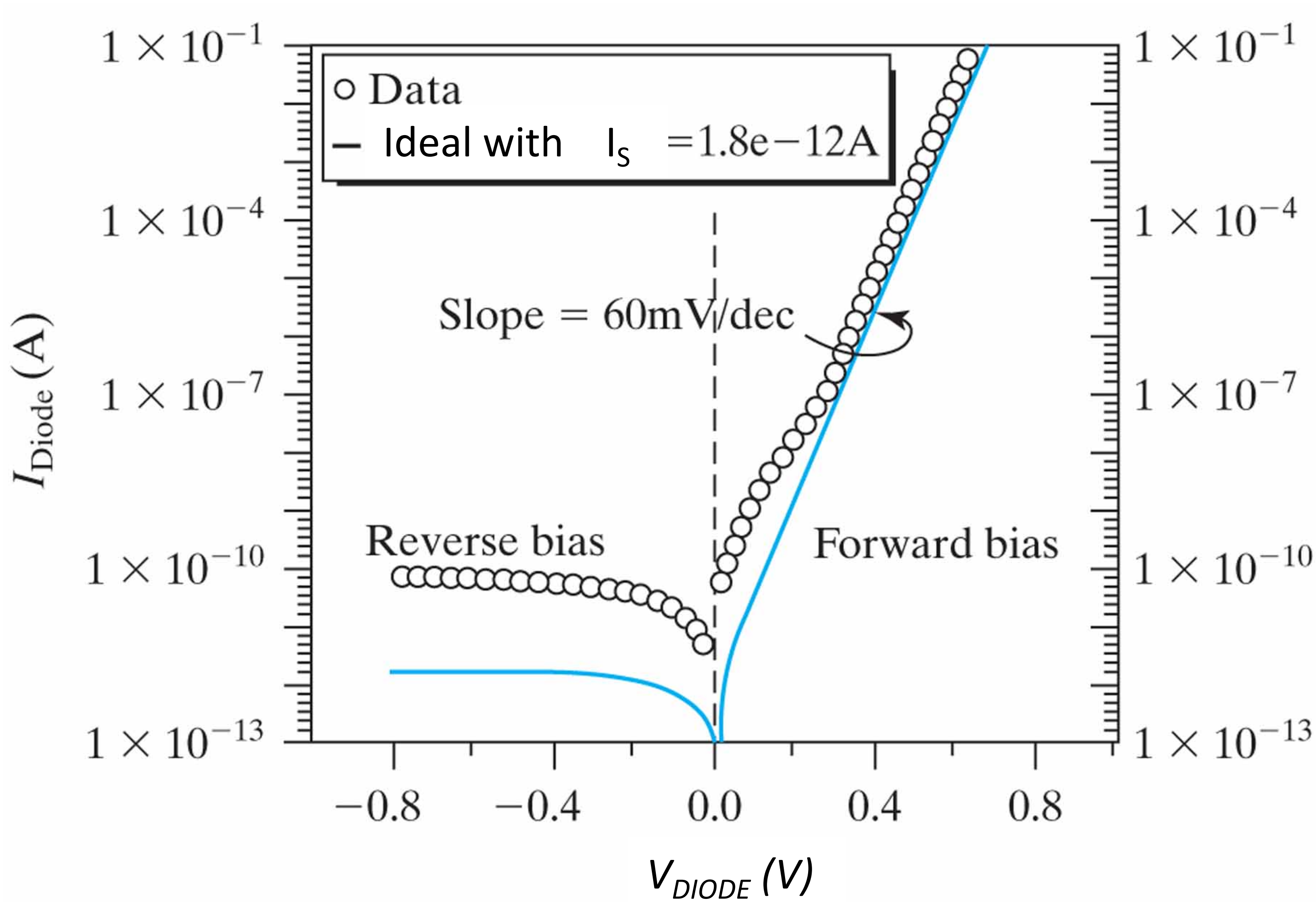
semilogarithmic scale

at room temperature $(300K)$:

$$\frac{V_T}{0.43} \approx \frac{26mV}{0.43} \approx \frac{60mV}{10}$$

every 60 mV the current changes by a decade

Deviations from the ideal diode



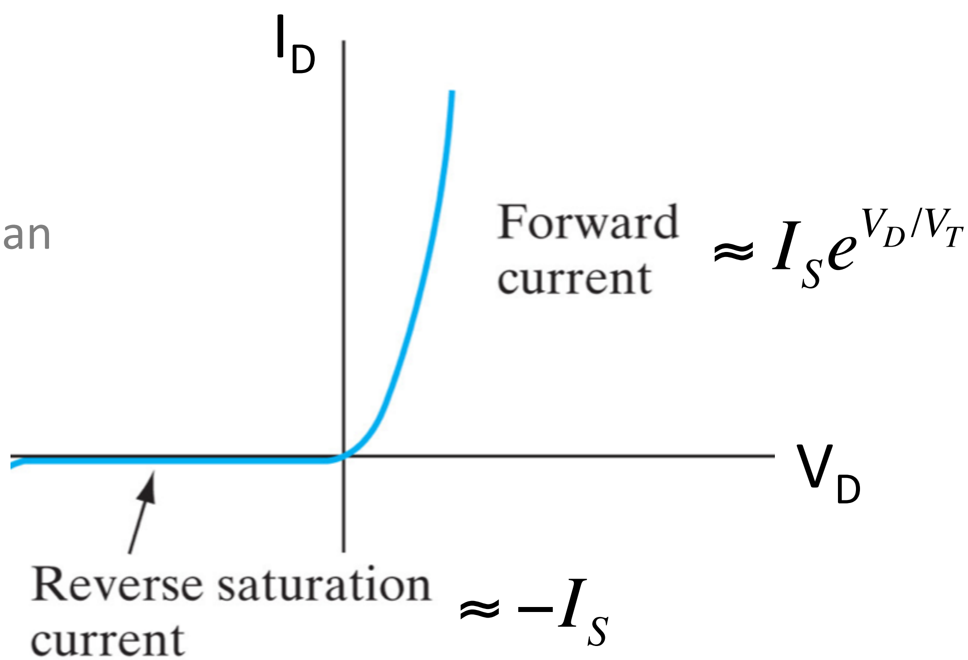
source: Hu

Ideal diode equation

- Although we derived the ideal diode equation by subjecting the diode to a forward bias, the steps followed can also be applied for the case of reverse bias (the only difference is that now $V_D < 0$).
- The equation works for both forward and reverse region.

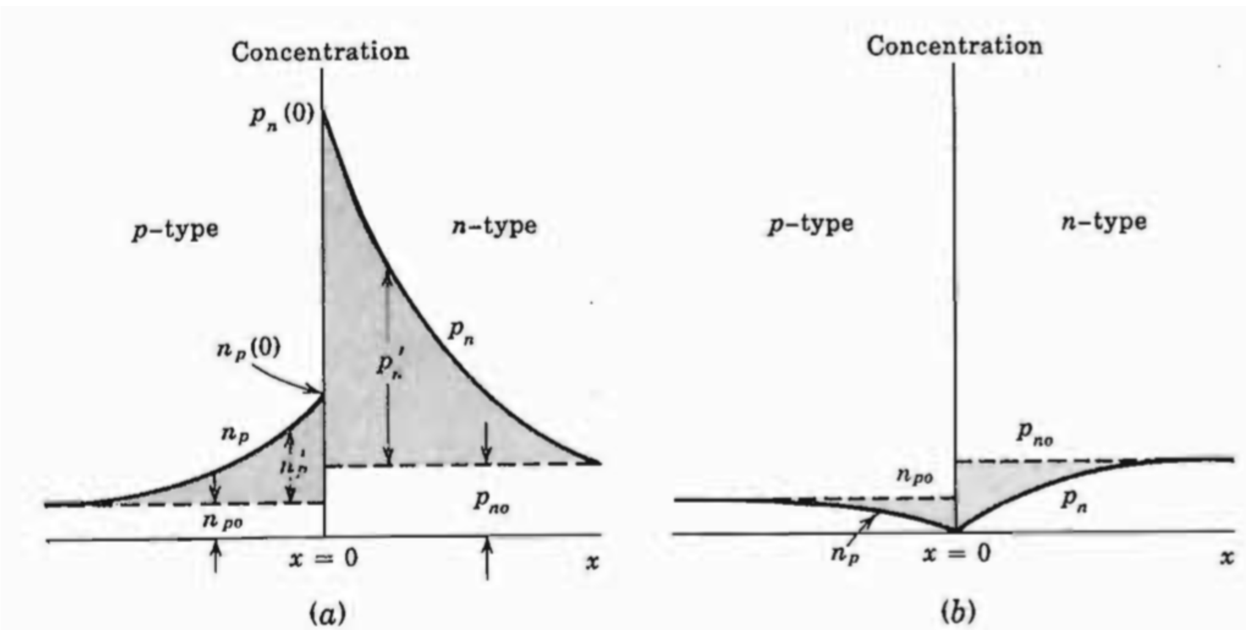
$$I_D = I_S\left(e^{V_D/V_T} - 1\right)$$

source: Streetman



Minority concentrations for reverse biased diode

- In forward bias, minority carriers are injected into quasi-neutral regions
- In reverse bias, minority carriers are extracted (swept away) from the quasi-neutral regions
- In reverse bias there is a very small amount of carriers available for extraction
  - I_D saturates to small value



Minority-carrier density distribution as a function of the distance from the junction.
(a) Forward biased junction
(b) Reverse biased junction
(The depletion region is assumed so small relative to the diffusion length that is not indicated in the figure)

source: Millman & Halkias

Minority carrier concentrations
Equilibrium vs. Reverse Bias

source: G.W. Neudeck

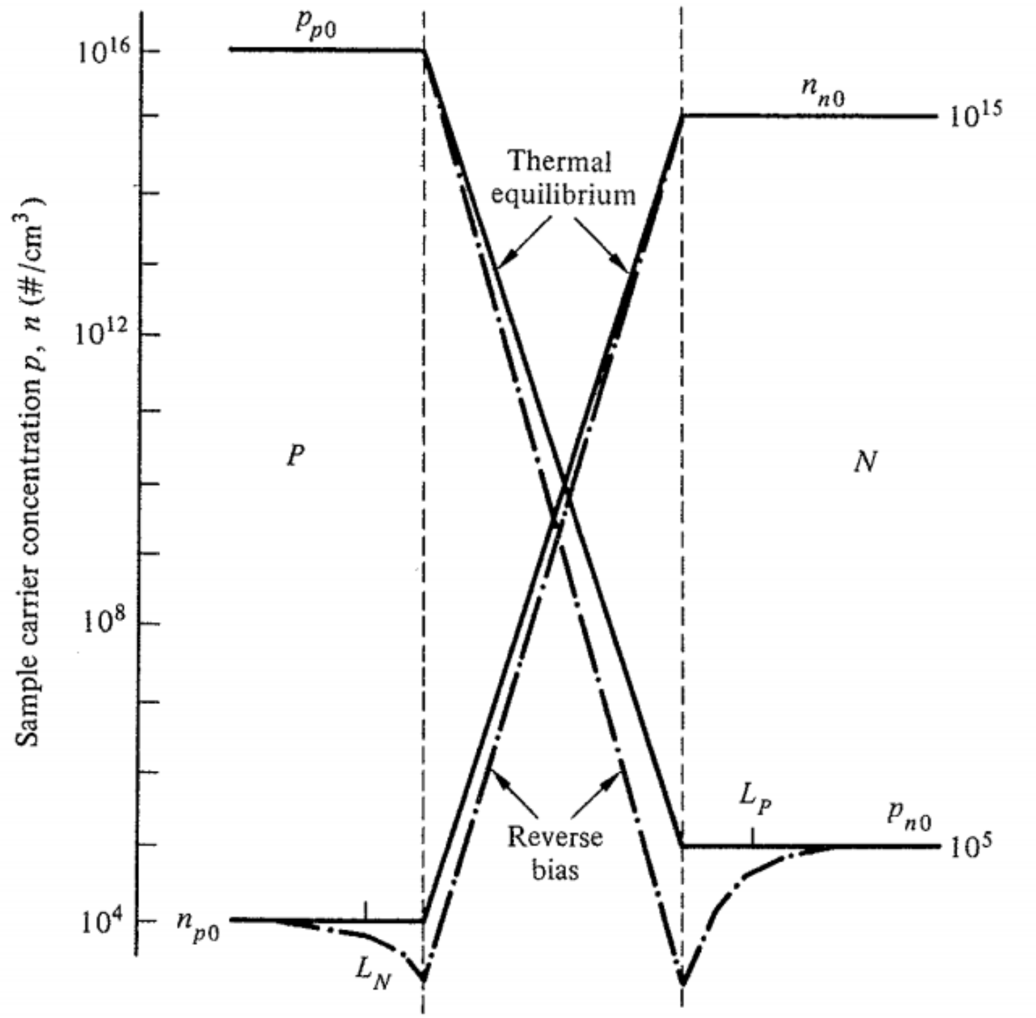
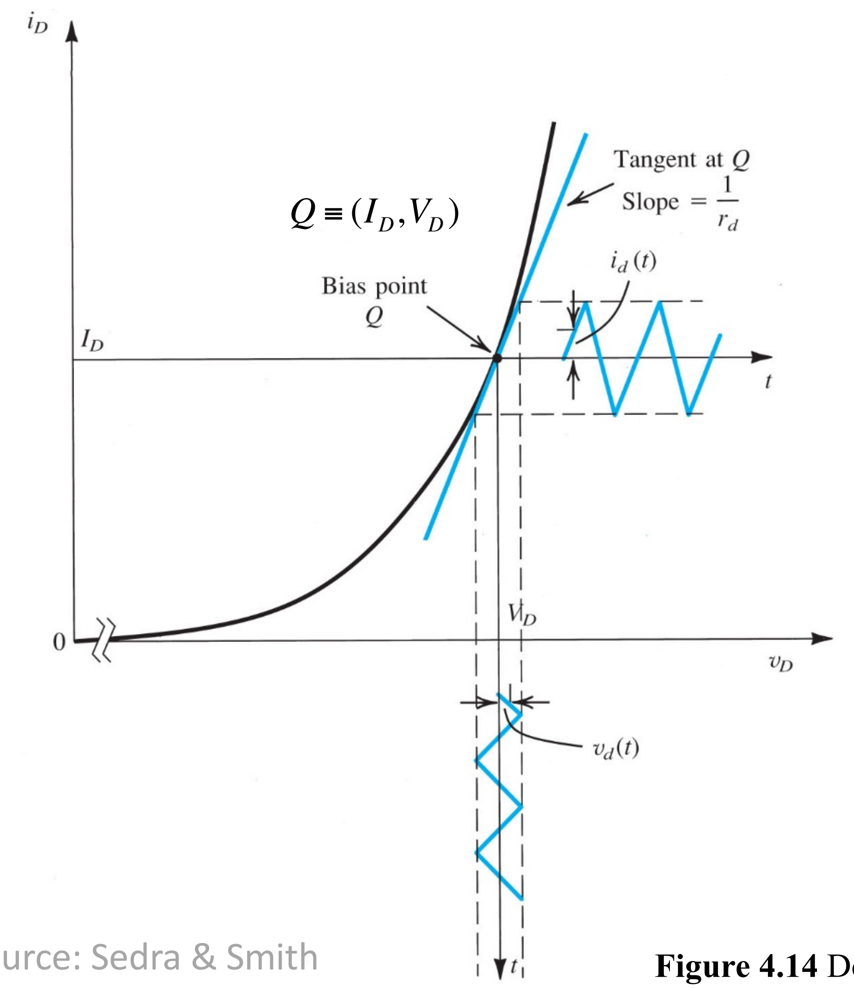


Fig. 3.12 Carrier concentrations for thermal equilibrium and reverse bias.

Small-signal equivalent model for diode

- Examine the effect of a small signal overlapping the bias:



$$v_D(t) \equiv V_D + v_d(t)$$

i_D is a function of v_D:

$$i_D = I_S\left(e^{\frac{v_D}{V_T}} - 1\right) = I_S\left(e^{\frac{V_D+v_d}{V_T}} - 1\right)$$

$$i_D \overset{*}{\cong} i_D\big|_{V_D} + \left.\frac{di_D}{dv_D}\right|_{V_D}(v_D - V_D) = I_D + \left.\frac{di_D}{dv_D}\right|_{V_D} v_d$$

$\equiv g_d$

$$i_D - I_D \cong g_d v_d \iff i_d \cong g_d v_d$$

$$\left.\frac{di_D}{dv_D}\right|_{V_D} = \frac{d}{dv_D}\left[I_S\left(e^{\frac{v_D}{V_T}} - 1\right)\right]_{V_D} = \frac{I_S e^{\frac{V_D}{V_T}}}{V_T} = \frac{I_S e^{\frac{V_D}{V_T}} + I_S - I_S}{V_T} =$$

$$= \frac{I_D + I_S}{V_T}$$

* Applied Taylor

source: Sedra & Smith

Figure 4.14 Development of the diode small-signal model.

Small-signal equivalent model for diode

- From a small signal point of view the diode behaves as a (non linear) resistance of value:

$$r_{diode} = \frac{1}{g_d} = \frac{V_T}{I_D + I_S}$$

$r_{diode} = 1/g_d$

- r_{diode} depends on bias: $g_d = \frac{I_D + I_S}{V_T}$

- In forward bias: $g_d \approx \frac{I_D}{V_T}$ In reverse bias: $g_d \approx 0$

- Under the assumption that the signal overlapping the bias is small the i-v relationship of the diode can be linearized:

$$v_D = V_D + v_d \quad \Leftrightarrow \quad i_D \cong I_D + g_d v_d = I_D + i_d$$

source: Sedra & Smith

Figure 4.14 Development of the diode small-signal model.

Small-signal equivalent model for diode

- Besides having current flowing through it (resistive behavior) the diode also exhibit charge displacement mechanisms that can be modeled as a capacitive behavior

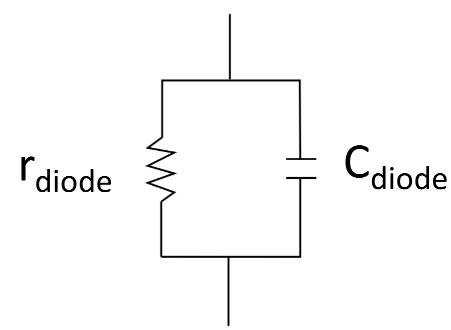
- The charge response to the voltage applied across the diode is non-linear

- As long as the voltage signal superimposed to the bias voltage is small, the relationship between charge and voltage can be linearized.

$$v_D = V_D + v_d$$

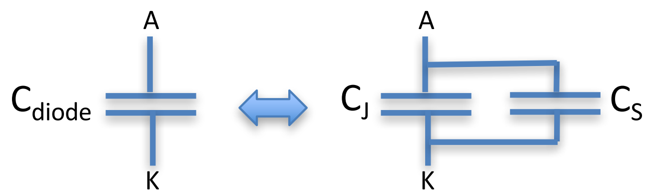
$$q_D \approx Q_D\big|_{V_D} + \overbrace{\left.\frac{dq_D}{dv_D}\right|_{V_D}}^{\equiv C_{diode}} (v_D - V_D) = Q_D\big|_{V_D} + C_{diode} \cdot v_d$$

- Complete small signal model for diode



Capacitive effects in the PN junction

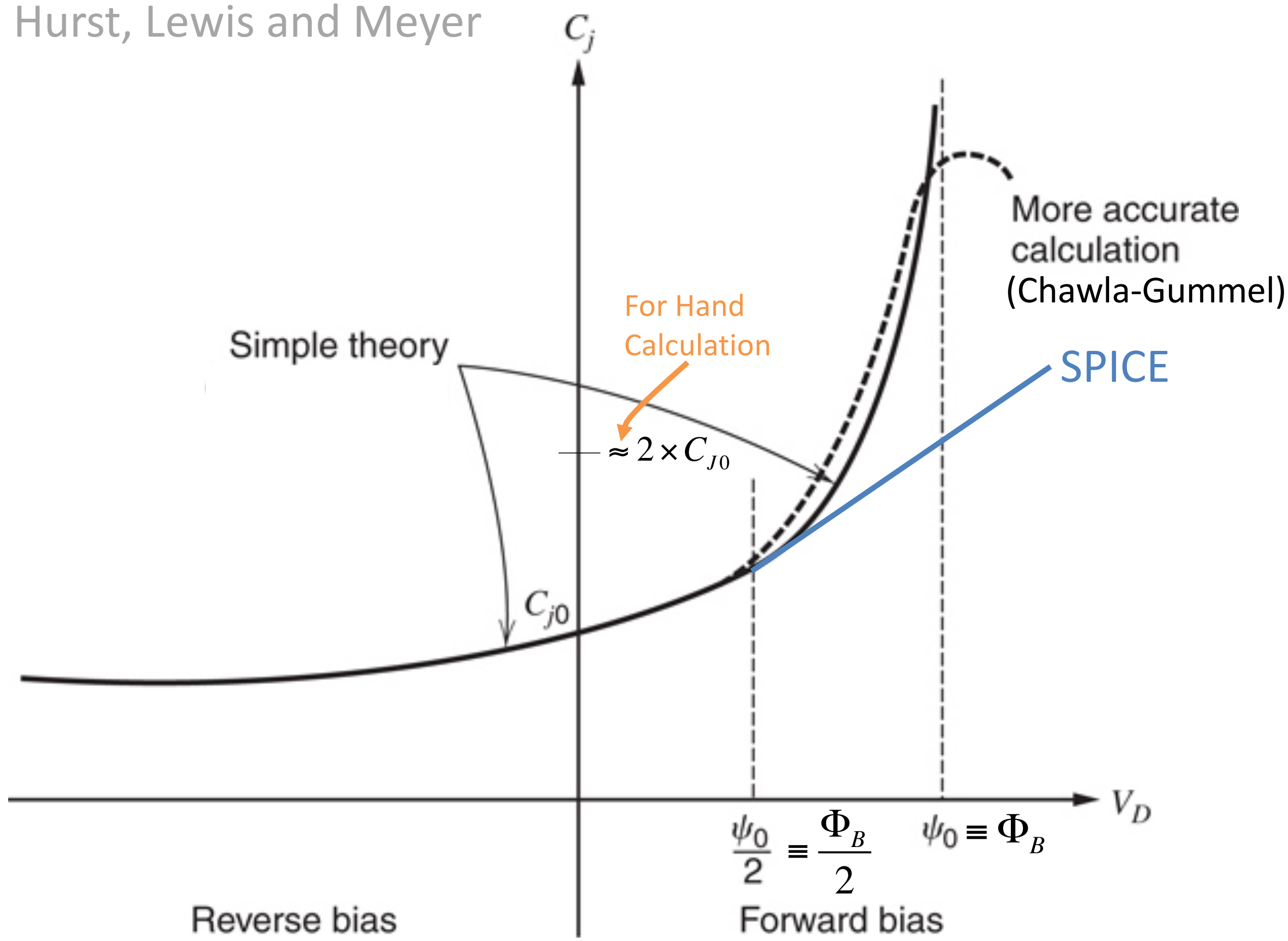
- Diodes have two capacitive effects
- But … please careful:
  there are way too many letters "D":
  - depletion capacitance = junction capacitance = C_J
  - diffusion capacitance = storage capacitance = C_S
  - $C_{diode} = C_J + C_S$



- We already know quite a bit about the junction cap. that develops when the diode is in reverse bias (… but what about forward bias?)
- So far we did not even think about the existence of storage (diffusion) capacitance
- We need to dig more into the capacitive effects …

Junction Capacitance

source: Gray, Hurst, Lewis and Meyer



Behavior of junction capacitance as a function of bias voltage V_D

Junction Capacitance

- Earlier, we derived the capacitance associated with depletion region (junction capacitance) for a **reverse biased** diode as:

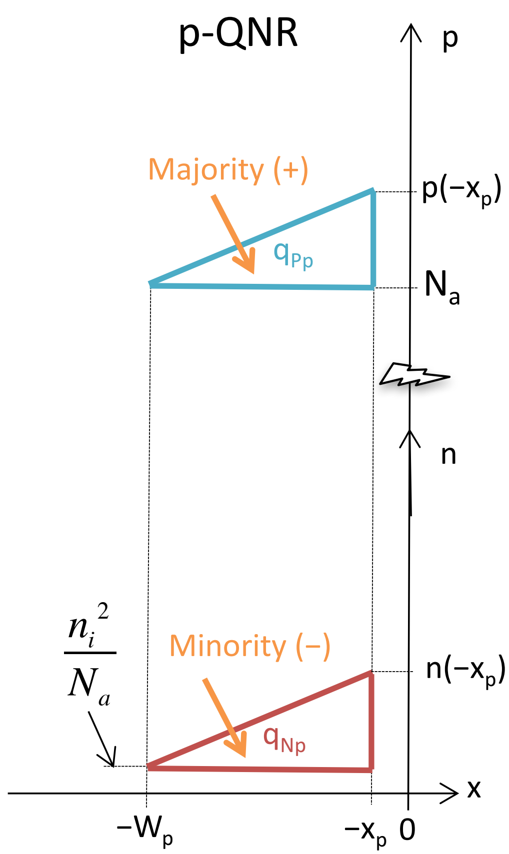
$$C_j\Big|_{V_D} = \frac{CJO}{\sqrt{1-\dfrac{V_D}{\Phi_B}}} \qquad CJO = \text{zero-bias junction capacitance} = \sqrt{\frac{\varepsilon_{si} q}{2}\cdot\frac{N_A N_D}{N_A+N_D}\cdot\frac{1}{\Phi_B}}$$

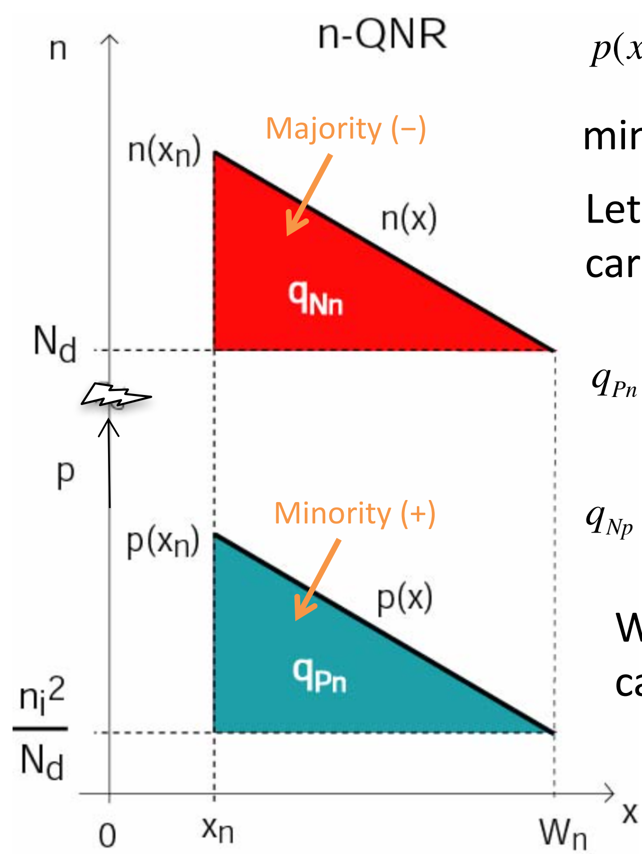
- Under significant forward bias $V_D > \Phi_B/2$ the approximation that there are no mobile carriers in the depletion region becomes invalid (this was the basic assumption behind the derivation !!)
- Therefore, under **forward bias** a common approximation is to take the value of C_J at $V_D = \Phi_B/2$ (or being slightly more pessimistic)

$$C_j\Big|_{V_D=\frac{\Phi_B}{2}} = \frac{CJO}{\sqrt{1-1/2}} = \sqrt{2}\times CJO \approx 2\times CJO$$

Diffusion capacitance

- The diffusion capacitance is due to charge carrier storage occurring in QNRs (to maintain quasi-neutrality)





$$p(x_n) = \frac{n_i^2}{N_d} e^{\frac{V_D}{V_T}} \qquad n(-x_p) = \frac{n_i^2}{N_a} e^{\frac{V_D}{V_T}}$$

minority carriers charge in QNRs?

Let’s compute charge in minority carriers “triangles”:

$$q_{Pn} = \frac{qA}{2}(W_n - x_n)\left(\frac{n_i^2}{N_d} e^{\frac{V_D}{V_T}} - \frac{n_i^2}{N_d}\right) = -q_{Nn}$$

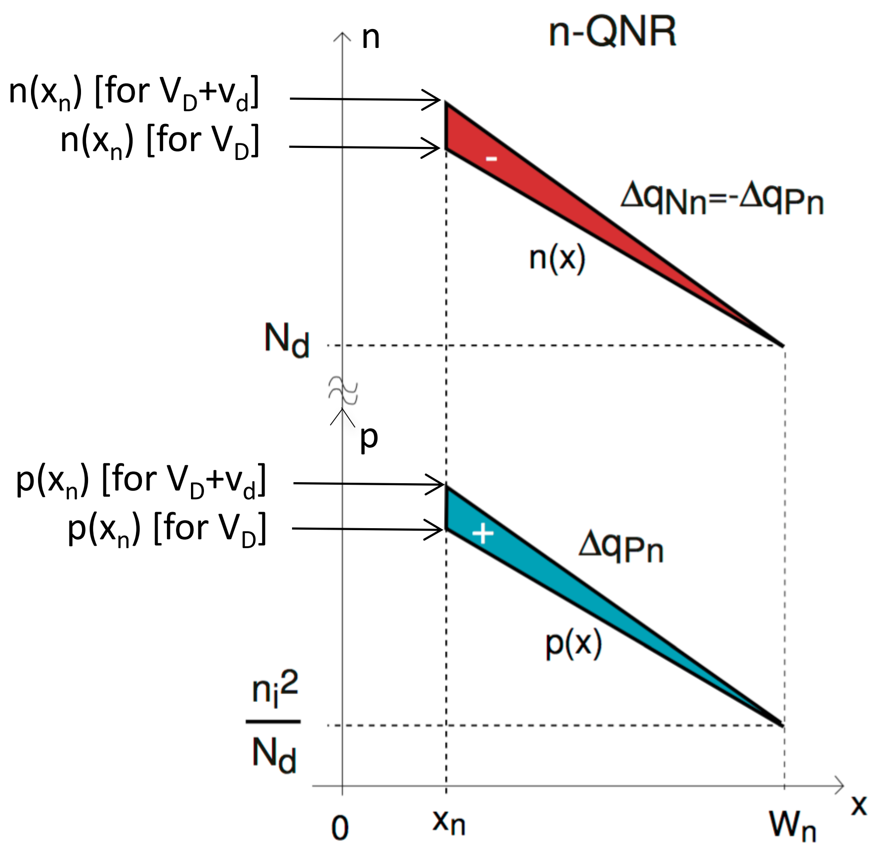
$$q_{Np} = -\frac{qA}{2}(W_p - x_p)\left(\frac{n_i^2}{N_a} e^{\frac{V_D}{V_T}} - \frac{n_i^2}{N_a}\right) = -q_{Pp}$$

What happen to the majority carriers in the QNRs ?

Quasi-neutrality demands that at every point in the quasi-neutral regions:
excess minority carrier concentration = excess majority carriers concentration

Diffusion capacitance

- Let's examine what happen when we apply a small increase in voltage (v_D=V_D+v_d=V_D+Δv_D)



Small increase in V_D

Small increase in q_{Pn}

Small increase in q_{Nn}

Behaves as a capacitor of capacitance:

$$C_{dn} = \left.\frac{dq_{Pn}}{dv_D}\right|_{v_D=V_D} = \frac{qA}{2}\left(W_n - x_n\right)\frac{n_i^2}{N_d}\frac{1}{V_T}e^{\frac{V_D}{V_T}}$$

Diffusion capacitance

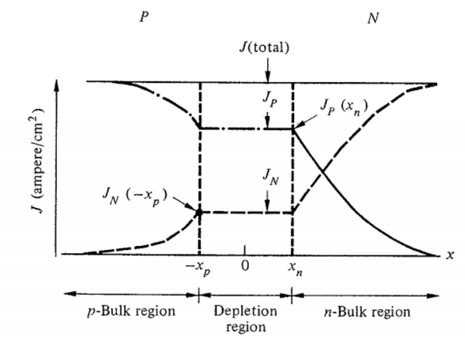


Fig. 3.9 Current density in the forward biased case.

- If we recall the equation of the short-base diode:

$$I_D = Aq\left(\frac{D_n}{W_P - x_p}\frac{n_i^2}{N_a} + \frac{D_p}{W_N - x_n}\frac{n_i^2}{N_d}\right)\left(e^{V_D/V_T} - 1\right) = I_{D,n}(-x_p) + I_{D,p}(x_n)$$

- using some math manipulation we can rewrite the diffusion capacitance of the n-QSR region as a function of the portion of diode current due to the holes in n-QNR ($I_{D,p}$) :

$$C_{dn} = \frac{qA}{2}\left(W_n - x_n\right)\frac{n_i^2}{N_d}\frac{1}{V_T}e^{\frac{V_D}{V_T}}\frac{\left(W_n - x_n\right)}{\left(W_n - x_n\right)}\frac{D_p}{D_p} = \frac{1}{V_T}\frac{\left(W_n - x_n\right)^2}{2D_p}qA\frac{\left(W_n - x_n\right)}{D_p}\frac{n_i^2}{N_d}e^{\frac{V_D}{V_T}} \cong \frac{1}{V_T}\frac{\left(W_n - x_n\right)^2}{2D_p}I_{D,p}$$

- We define transit time of the holes through the n-QNR the average time for a hole to diffuse through N-QNR:

$$\tau_{Tp} \equiv \frac{\left(W_n - x_n\right)^2}{2D_p}$$

Diffusion capacitance

- Then in summary: $C_{dn} \cong \tau_{Tp} \dfrac{I_{D,p}}{V_T}$
- Aside:
  - we already had a name for the average time it takes a hole to diffuse through an n-region. It is the hole's lifetime τ_p

$$\tau_p = \frac{L_p^2}{D_p}$$

  - In fact, if we compute the amount of excess charge given by the holes in n-QSR accurately (see slide 39):

$$q_{Pn} = qA\int_0^\infty p_n(x')\cdot dx' = qA\int_0^\infty \Delta p_n e^{-\frac{x'}{L_p}}\cdot dx' = qA\int_0^\infty p_{n0}\left(e^{V_D/V_T}-1\right)e^{-\frac{x'}{L_p}}dx' =$$

$$= qAp_{n0}\left(e^{V_D/V_T}-1\right)\int_0^\infty e^{-\frac{x'}{L_p}}dx' = qAp_{n0}\left(e^{V_D/V_T}-1\right)L_p$$

Diffusion capacitance

$$\tau_p \equiv \frac{L_p^2}{D_p}$$

– Therefore:

$$C_{dn} = \left.\frac{dq_{Pn}}{dv_D}\right|_{v_D=V_D} = qA\frac{n_i^2}{N_d}\frac{L_p}{V_T}e^{V_D/V_T} = \frac{L_p^2}{D_p}\frac{1}{V_T}qA\frac{n_i^2}{N_d}\frac{D_p}{L_p}e^{V_D/V_T} \approx \tau_p\frac{I_{D,p}}{V_T}$$

Note: The increment in the stored hole charge comes from injected holes from the P-SIDE (anode) to the N-side (cathode)

– Similarly for the excess charge and capacitance in the p-QNR

$$q_{Np} = -qA\int_0^\infty n_p(x'')\cdot dx'' = -qA\int_0^\infty \Delta n_p e^{-\frac{x''}{Ln}}\cdot dx'' = -qAn_{p0}\left(e^{V_D/V_T}-1\right)L_n$$

$$C_{dp} = \left.\frac{-dq_{Np}}{dv_D}\right|_{v_D=V_D} = \left.\frac{\left|dq_{Np}\right|}{dv_D}\right|_{v_D=V_D} = \frac{L_n^2}{D_n}\frac{1}{V_T}qA\frac{n_i^2}{N_a}\frac{D_n}{L_n}e^{V_D/V_T} \approx \tau_n\frac{I_{D,n}}{V_T}$$

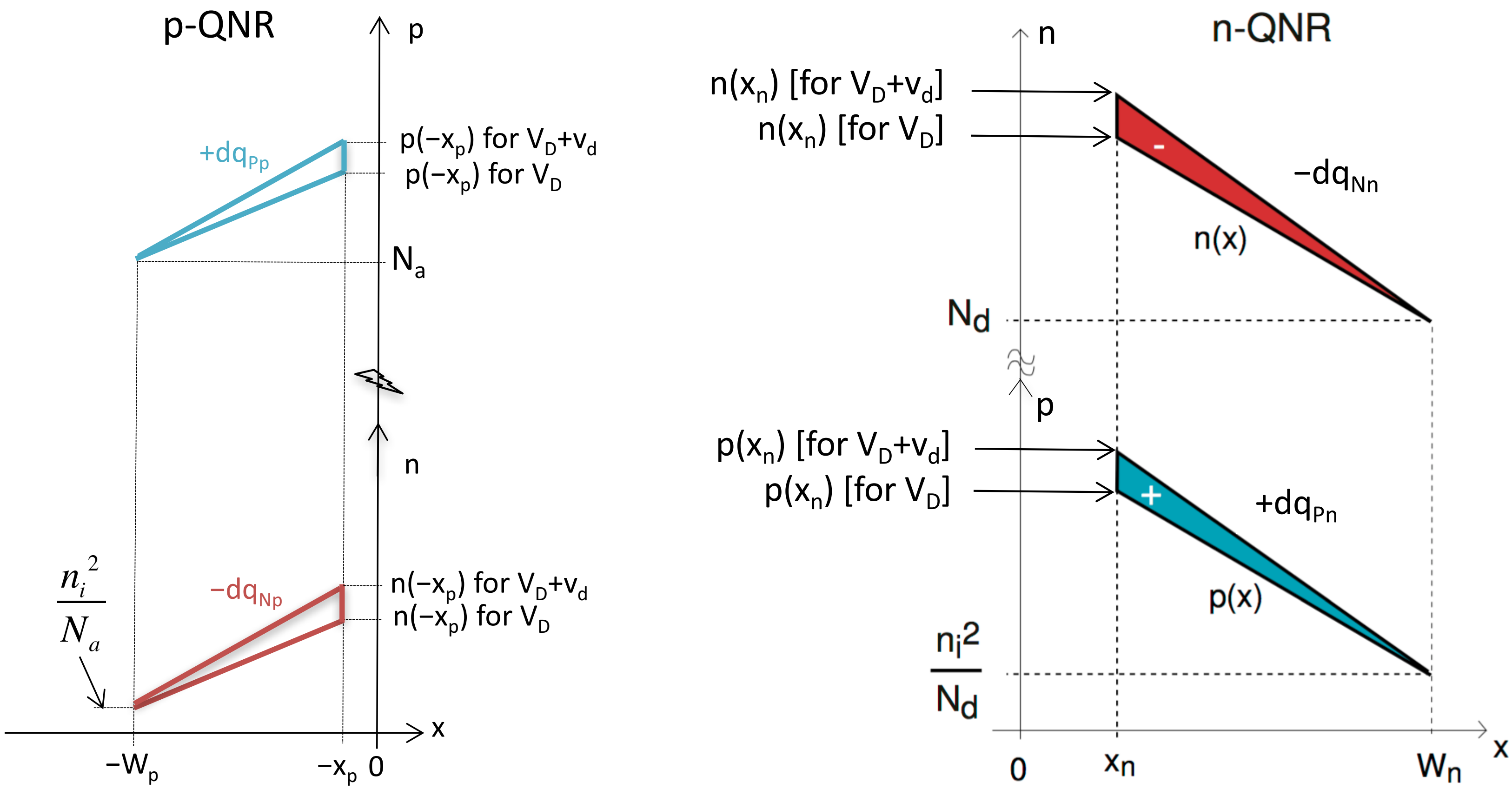
Note: The increment in stored electron charge is negative, but results from electrons that are injected from the cathode (N-side) to the anode (P-SIDE)

– The total excess charge stored and the associated total capacitance is:

$$q_s = q_{Pn} + \left|q_{Np}\right| = \tau_p I_{D,p} + \tau_n I_{D,n}$$

$$C_S = \left.\frac{dq_s}{dv_D}\right|_{v_D=V_D} = C_{dn} + C_{dp} \approx \tau_n\frac{I_{D.n}}{V_T} + \tau_p\frac{I_{D.p}}{V_T}$$

Diffusion capacitance



Diffusion capacitance: physical intuition

- If we had looked more carefully at the expressions of the charge stored in the QNRs rather than immediately starting to crunch equations we would have noticed that:

$$q_{Pn} = -q_{Nn} \propto \left(e^{\frac{V_D}{V_T}} - 1 \right)$$

$$-q_{Np} = q_{Pp} \propto \left(e^{\frac{V_D}{V_T}} - 1 \right)$$

- Therefore, the stored charge q_S, is proportional to the diode's current I_D, which is also proportional to $\exp(V_D/V_T)-1$

$$q_s = q_{Pn} + |q_{Np}| \propto \left(e^{\frac{V_D}{V_T}} - 1 \right) \propto I_D \Leftrightarrow q_S = \tau_T I_D$$

- The constant of proportionality is called charge-storage time or transit time

$$\boxed{C_S \cong \tau_T \frac{I_D}{V_T} \cong \tau_T g_d = \frac{\tau_T}{r_{diode}}}$$

Diffusion capacitance: physical intuition

- There is a very simple explanation to this proportionality, I_D is the rate of minority charge injection into the diode.
- In steady state. This rate must be equal to the rate of charge recombination, which is q_S/τ_T
- In a one-sided junction the transit time τ_T is the recombination lifetime on the lighter-doping side, where most of the charge injection and recombination take place
- In general, τ_T is an average of the recombination lifetimes on the N-side and P-side
- *In any event, I_D and q_S are simply linked through a charge storage time τ_T (a.k.a. τ_S)*

Diffusion capacitance: physical intuition

Example

- $N_A >> N_D$
  - N-side is more lightly doped than the P-side
  - Diffusion of minority carriers on N-side dominates

$$I_D \approx I_{D,p}(x_n) \approx qA\frac{D_p}{L_p}p_{n0}e^{\frac{V_D}{V_T}} = qA\frac{D_p}{L_p}\frac{n_i^2}{N_d}e^{\frac{V_D}{V_T}}$$

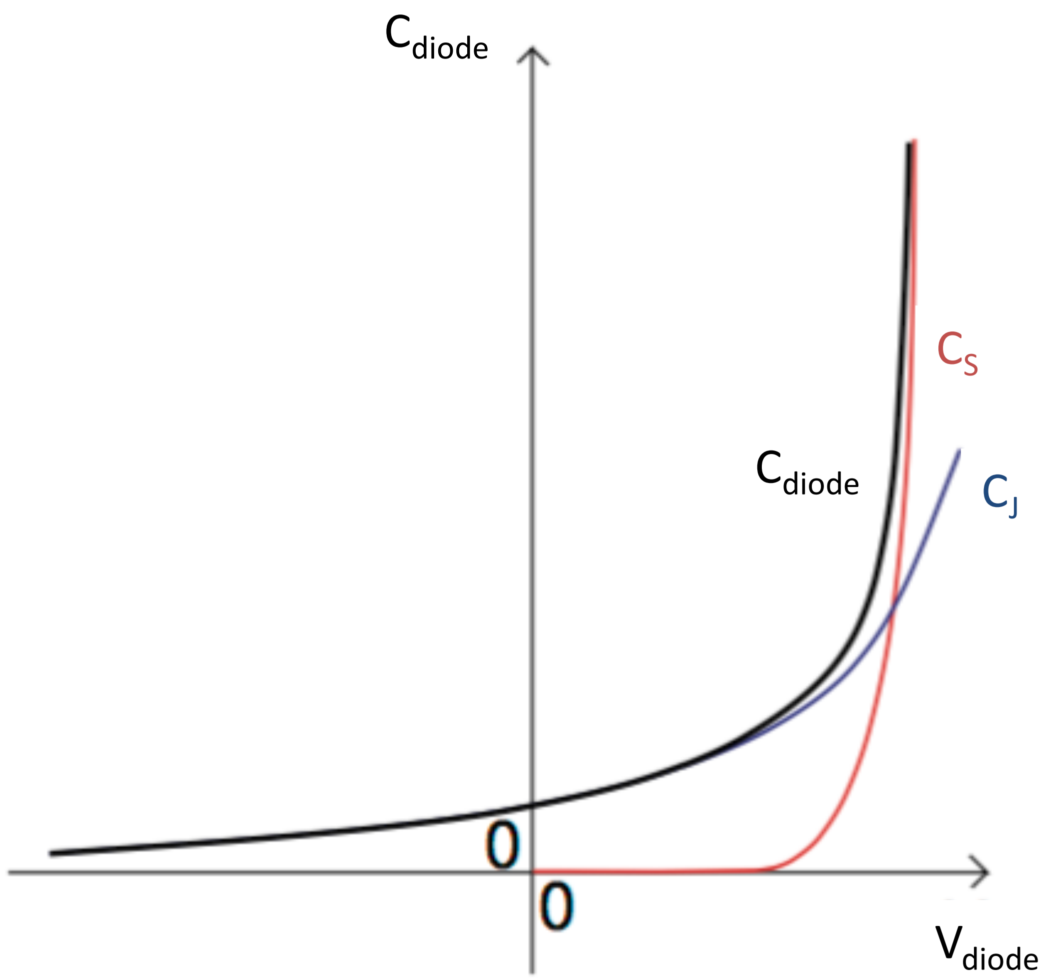
$$\tau_p \equiv \text{hole lifetime} \equiv \frac{L_p^2}{D_p}$$

  - The excess charge stored is: $q_S \approx q_{Pn} \approx \tau_p I_{D,p}(x_n) \approx \tau_p I_D$

$$\tau_S \approx \tau_p$$

  - And finally: $C_S \approx \tau_p \frac{I_D}{V_T} \approx \tau_p \times g_D$

Bias Dependence of C_J and C_S



- C_J (associated with charge modulation in depletion region) dominates in reverse bias and small forward bias

$$C_J \propto \frac{1}{\sqrt{\Phi_B - V_D}}$$

- C_S (associated with charge storage in QNRs to maintain quasi-neutrality) dominates in strong forward bias

$$C_S \propto e^{\frac{V_D}{V_T}}$$

source: Sodini

Summary:

source:
Sedra & Smith

Table 3.1 Summary of Important Equations

Quantity	Relationship	Values of Constants and Parameters (for Intrinsic Si at $T = 300$ K)
Carrier concentration in intrinsic silicon (cm^{-3})	$n_i = BT^{3/2}e^{-E_g/2kT}$	B = 5×10^{15} $\text{cm}^{-3}\text{K}^{-3/2}$ E_g = 1.12 eV $k = 8.62 \times 10^{-5}$ eV/K $n_i = 1\times10^{10}$ cm^{-3}
Diffusion current density (A/cm^2)	$J_p = -qD_p \frac{dp}{dx}$ $J_n = qD_n \frac{dn}{dx}$	$q = 1.60 \times 10^{-19}$ coulomb $D_p = 12\ \text{cm}^2/\text{s}$ $D_n = 34\ \text{cm}^2/\text{s}$
Drift current density (A/cm^2)	$J_{\text{drift}} = q(p\mu_p + n\mu_n)E$	$\mu_p = 480\ \text{cm}^2/\text{V}\cdot\text{s}$ $\mu_n = 1350\ \text{cm}^2/\text{V}\cdot\text{s}$
Resistivity ($\Omega\cdot\text{cm}$)	$\rho = 1/[q(p\mu_p + n\mu_n)]$	μ_p and μ_n decrease with the increase in doping concentration
Relationship between mobility and diffusivity	$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$	$V_T = kT/q \simeq 25.9$ mV
Carrier concentration in n-type silicon (cm^{-3})	$n_{n0} \simeq N_D$ $p_{n0} = n_i^2/N_D$	
Carrier concentration in p-type silicon (cm^{-3})	$p_{p0} \simeq N_A$ $n_{p0} = n_i^2/N_A$	
Junction built-in voltage (V)	$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$	
Width of depletion region (cm)	$\frac{x_n}{x_p} = \frac{N_A}{N_D}$ $W = x_n + x_p$ $= \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$	$\epsilon_s = 11.7\epsilon_0$ $\epsilon_0 = 8.854 \times 10^{-14}$ F/cm

Summary:

source:
Sedra & Smith

Table 3.1 *continued*

Quantity	Relationship	Values of Constants and Parameters (for Intrinsic Si at $T = 300$ K)
Charge stored in depletion layer (coulomb)	$Q_J = q\frac{N_A N_D}{N_A + N_D}AW$	
Forward current (A)	$I = I_p + I_n$ $I_p = Aqn_i^2 \frac{D_p}{L_p N_D}\left(e^{V/V_T} - 1\right)$ $I_n = Aqn_i^2 \frac{D_n}{L_n N_A}\left(e^{V/V_T} - 1\right)$	
Saturation current (A)	$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$	
I–V relationship	$I = I_S\left(e^{V/V_T} - 1\right)$	
Minority-carrier lifetime (s)	$\tau_p = L_p^2/D_p \qquad \tau_n = L_n^2/D_n$	$L_p, L_n = 1\ \mu\text{m to } 100\ \mu\text{m}$ $\tau_p, \tau_n = 1\text{ ns to } 10^4\text{ ns}$
Minority-carrier charge storage (coulomb)	$Q_p = \tau_p I_p \qquad Q_n = \tau_n I_n$ $Q = Q_p + Q_n = \tau_T I$	
Depletion capacitance (F)	$C_{j0} = A\sqrt{\left(\frac{\epsilon_s q}{2}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)\frac{1}{V_0}}$ $C_j = C_{j0} \Big/ \left(1 + \frac{V_R}{V_0}\right)^m$	$m = \frac{1}{3} \text{ to } \frac{1}{2}$
Diffusion capacitance (F)	$C_d = \left(\frac{\tau_T}{V_T}\right)I$	

The Five Semiconductor Equations

- Poisson's Equation: $\frac{d^2\Phi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{q}{\varepsilon}\left(p - n + N_D - N_A\right)$

$$E(x) = -\frac{d\Phi}{dx} \quad Gauss' \;\; Law$$

$$\Updownarrow$$

$$\frac{d^2\Phi}{dx^2} = -\frac{dE}{dx} = -\frac{\rho}{\varepsilon}$$

- Electron current density: $J_n = q\mu_n E + qD_n \frac{dn}{dx}$
- Hole current density: $J_p = q\mu_p E - qD_p \frac{dp}{dx}$
- Continuity of electrons: $\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + \left(G_n - R_n\right)$
- Continuity of holes: $\frac{\partial p}{\partial t} = \frac{1}{q}\frac{\partial J_p}{\partial x} + \left(G_p - R_p\right)$

SPICE Diode model

source: Antognetti & Massobrio

TABLE 1-1 SPICE Diode Model

Symbol	SPICE 2G keyword	Parameter name	Default value	Unit
I_S	IS	Saturation current	10^{-14}	A
r_S	RS	Ohmic resistance	0	Ω
n	N	Emission coefficient	1	
τ_T	TT	Transit time	0	s
CJ(0)	CJO	Zero-bias junction capacitance	0	F
Φ_B	VJ†	Junction potential	1	V
m	M	Grading coefficient	0.5	
E_g	EG	Energy gap: 1.11 for Si 0.69 for SBD 0.67 for Ge	1.11	eV
p_t	XTI†	Saturation current temperature exponent: 3.0 for *pn*-junction diode 2.0 for SBD	3.0	
FC	FC	Coefficient for forward-bias depletion capacitance formula	0.5	
BV	BV	Reverse breakdown voltage (positive number)	∞	V
IBV	IBV	Reverse breakdown current (positive number)	10^{-3}	A
k_f	KF	Flicker-noise coefficient	0	
a_f	AF	Flicker-noise exponent	1	
T	‡	Nominal temperature for simulation and at which all input data is assumed to have been measured	27	°C

† In some SPICE versions, VJ is replaced by PB and XTI by PT.

‡ Remember, the temperature T is regarded as an operating condition and not as a model parameter.

SPICE characteristic of diode

General Sxntax:

```
D[name] <anode_node> <cathode_node> <modelname>
.model <modelname> D (<paramX>=<valueX> <paramY>=<valueY> ...)
```

Example:

```
D1 1 2 diode_1N4004
.model diode_1N4004 D (IS=18.8n RS=0 BV=400
+ IBV=5.00u CJO=30 M=0.333 N=2)

D2 3 4 idealmod_d
.model idealmod_d D
+ IS=10f
+ n = 0.01
+ IBV=0.1n
* By default BV = inf
```

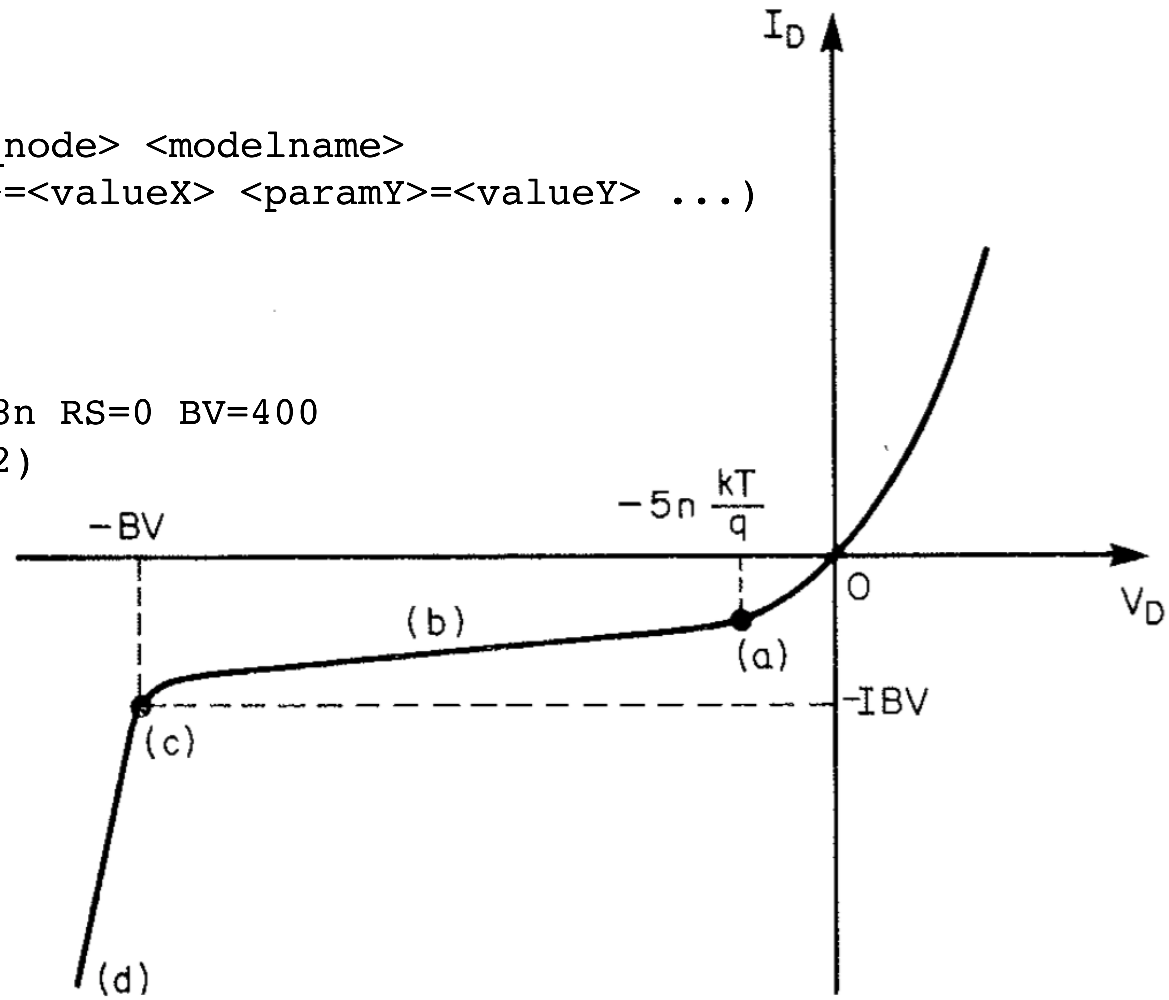


Figure 1-15 SPICE characteristic of the real diode.

source: Antognetti & Massobrio