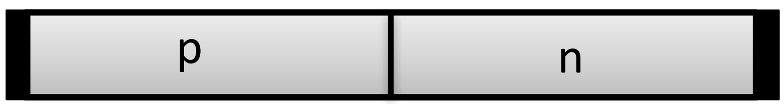
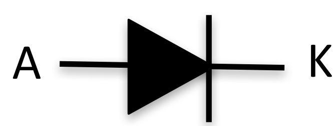
Chapter 4

Electronics I - Diode models





Fall 2017

Effect of Temperature on I/V curves

Source: Hu

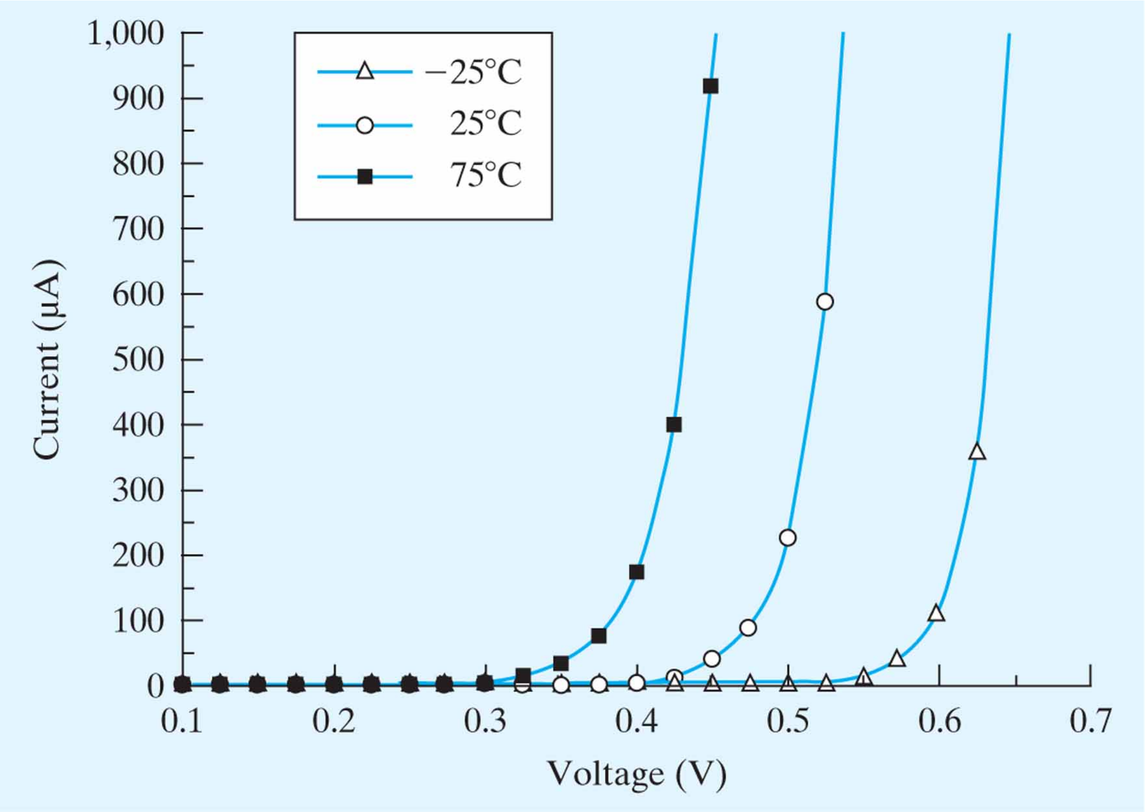


Figure 4.21 The *IV* curves of the silicon PN diode shift to lower voltages with increasing temperature

Effect of Temperature on I/V curves

- For a given I_D we want to understand how the diode voltage V_D varies with temperature
- The diode's I/V equation contains temperature in two places: V_T = KT/q and I_S

$$I_D = I_S\left(e^{\frac{V_D}{V_T}} - 1\right) \qquad \text{where: } I_S = qAn_i^2\left(\frac{D_p}{L_pN_d} + \frac{D_n}{L_nN_a}\right) = qAn_i^2\left(\frac{\mu_pV_T}{L_pN_d} + \frac{\mu_nV_T}{L_nN_a}\right)$$

- Inside the expression of I_S the terms n_i and μ also depends on temperature:

$$\mu \propto T^{-3/2} \qquad n_i^2 = N_CN_Ve^{\frac{-E_G}{KT}} = A_CA_VT^3e^{\frac{-E_G}{KT}}$$

- N_v and N_c are respectively the effective density of energy states in valence band and conduction band

$$N_C = A_CT^{3/2} \quad and \quad N_V = A_VT^{3/2}$$

Effect of Temperature on I/V curves

- If for simplicity we ignore the weak temperature dependence of the effective mass density of the conduction band electrons (m_n^*) and the valence band holes (m_p^*) the values of A_C and A_V are about constant (sources: Pierret p. 51):

$$\left.\begin{aligned} n_i^2 &= A_C A_V T^3 e^{\frac{-E_G}{KT}} \quad with: \\ A_C &= 2\left[\frac{m_n^* k}{2\pi\hbar^2}\right]^{3/2} \approx 6.2\times10^{15}\, cm^{-3}K^{-3/2} \\ A_V &= 2\left[\frac{m_p^* k}{2\pi\hbar^2}\right]^{3/2} \approx 3.52\times10^{15}\, cm^{-3}K^{-3/2} \end{aligned}\right\} \Rightarrow \begin{aligned} & n_i^2 = A_C A_V \times T^3 e^{\frac{-E_G}{KT}} \\ & \qquad \Updownarrow \\ & n_i = \sqrt{A_C A_V} \times T^{3/2} e^{\frac{-E_G}{2KT}} = B \times T^{3/2} e^{\frac{-E_G}{2KT}} \end{aligned}$$

- k = Boltzmann's constant = 8.62×10^{-5} eV/K = 1.38×10^{-23} Joule/K
- ħ= Reduced Planck's constant = 1.055×10^{-34} Joule-sec
- $m_n^* = 1.18 \times m_0$
- $m_p^* = 0.81 \times m_0$
- m_0 = electron rest mass = 9.11×10^{-31} Kg

Effect of Temperature on I/V curves

- Therefore≈: $$n_i = B \times T^{3/2} e^{\frac{-E_G}{2KT}} \quad with\, B \cong 5 \times 10^{15}\, cm^{-3} K^{-3/2}$$

- If for simplicity we ignore the weak temperature dependence of the band gap energy:

source: Plummer

$$E_G = E_{G0} - \frac{4.73 \times 10^{-4} \times T^2}{T + 636} \approx \quad 1.16 - 3 \times 10^{-3} \times T$$

where E_{G0} is the band gap energy at 0K (E_{G0}≈1.17 eV) and the temperature T is expressed in K

- We can finally write I_S as follows:

$$I_S = qAB^2T^3 e^{\frac{-E_G}{kT}} \left(\frac{\kappa_{\mu,p} T^{-1.5} kT/q}{L_p N_d} + \frac{\kappa_{\mu,n} T^{-1.5} kT/q}{L_n N_a} \right) \equiv \varsigma \cdot T^{2.5} \cdot e^{\frac{-E_G}{kT}}$$

Effect of Temperature on I/V curves

- First pass:

$$I_S = qAB^2T^3e^{\frac{-E_G}{kT}}\left(\frac{\kappa_{\mu,p}T^{-1.5}kT/q}{L_pN_d}+\frac{\kappa_{\mu,n}T^{-1.5}kT/q}{L_nN_a}\right)\equiv \varsigma\cdot T^{2.5}\cdot e^{\frac{-E_G}{kT}}$$

- the temperature dependence caused by the polynomial term is weak compared to the one caused by the exponential term, so we will ignore it

$$I_S = \underbrace{\varsigma\cdot T^{2.5}}_{=\zeta}\cdot e^{\frac{-E_G}{kT}} \equiv \zeta\cdot e^{\frac{-E_G}{kT}}$$

- Let's get back to our objective:
  given a fixed value of current I_D we want to find out the voltage variation occurring ΔV_D when there is a temperature variation ΔT occurring

$$TC \equiv \left.\frac{dV_D}{dT}\right|_{@I_D=const}$$

Effect of Temperature on I/V curves

- Let's assume ***forward bias*** and flip the I/V eq.:

$$I_D \approx I_S\left(e^{\frac{V_D}{V_T}}\right) \Rightarrow \frac{I_D}{I_S} = e^{\frac{V_D}{V_T}} \Rightarrow V_D \approx V_T \ln\frac{I_D}{I_S} = \frac{kT}{q}\ln\left(\frac{I_D}{I_S}\right)$$

$$\Downarrow$$

$$\frac{dV_D}{dT} = \frac{d}{dT}\left[\frac{kT}{q}\ln\left(\frac{I_D}{I_S}\right)\right] = \frac{k}{q}\ln\left(\frac{I_D}{I_S}\right) + \frac{kT}{q}\frac{d}{dT}\left(\ln I_D - \ln I_S\right) \cong$$

$$\cong \frac{k}{q}\cdot\frac{V_D}{\frac{kT}{q}} + \frac{kT}{q}\frac{d}{dT}\left(\ln I_D - \ln\zeta + \frac{E_G}{kT}\right) \cong \frac{V_D}{T} + \frac{kT}{q}\frac{d}{dT}\left(\frac{E_G}{kT}\right) \cong$$

$$\cong \frac{V_D}{T} - \frac{E_G}{qT}$$

constant

constant

E_G and k are constants take them out of the derivative

And let's keep in mind a couple of useful math rules:

$$\frac{d(uv)}{dx} = v\frac{du}{dx} + u\frac{du}{dx}$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

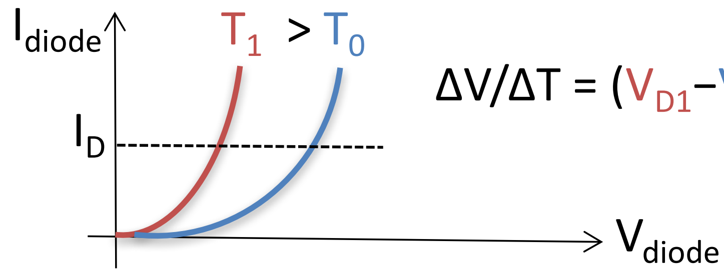
$$\frac{d}{dx}\left(x^N\right) = N\cdot x^{N-1}$$

Effect of Temperature on I/V curves

$$TC \equiv \frac{dV_D}{dT} \cong \frac{V_D}{T} - \frac{E_G / q}{T}$$

- Assuming V_D=0.5V and T=300K: $TC = \dfrac{0.5-1.12}{300} = -2.1\ mV/K = -2.1\ mV/degree$

- At room temperature, a diode's forward voltage-drop has a thermal coefficient of about −2mV per degree



$\Delta V/\Delta T = (V_{D1} - V_{D0})/(T_1 - T_0) \cong -2mV/degree$

- If we know the diode voltage at some reference temperature T_0 we can estimate the diode voltage at any other temperature T as follows:

$$V_D(T) \cong V_D(T_0) - 2m\text{V/degree} \times (T - T_0) = V_D(T_0) + TC \times (T - T_0)$$

Aside: second pass

- If we want to consider also the temperature dependence caused by the polynomial term, all we have to do is a little more work taking the derivative:

$$\frac{dV_D}{dT} = \frac{d}{dT}\left[\frac{kT}{q}\ln\left(\frac{I_D}{I_S}\right)\right] = \frac{k}{q}\ln\left(\frac{I_D}{I_S}\right) + \frac{kT}{q}\frac{d}{dT}\left(\ln I_D - \ln I_S\right) \cong$$

$$I_S = \varsigma \cdot T^{2.5} \cdot e^{\frac{-E_G}{kT}}$$

$$\cong \frac{V_D}{T} + \frac{kT}{q}\frac{d}{dT}\left(\ln I_D - \ln\zeta - \ln T^{2.5} + \frac{E_G}{kT}\right) \cong$$

$$\frac{d}{dT}\left(\ln T^m\right) = \frac{1}{T^m}\frac{d}{dT}\left(T^m\right) = \frac{m}{T^m}T^{m-1} = \frac{m}{T}$$

$$\cong \frac{V_D}{T} - \frac{2.5V_T}{T} - \frac{E_G/q}{T}$$

- The term V_D/T results from the temperature dependence on V_T. The negative terms results from the temperature dependence of I_S, and does not depend on the voltage across the diode

- Assuming V_D=0.5V and T=300K: $TC = \frac{0.5 - 2.5\times 26\times 10^{-3} \quad -1.12}{300} = -2.3 \text{ mV/degree}$

- So for conservative design it is common to assume: $TC = \frac{\mathrm{dV_D}}{dT} \cong -2.5 \text{ mV/degree}$

Effect of Temperature on I/V curves

- In *reverse bias* we find out that the variation of I_S with T is about 14.6% percent/degree:

Recalling:
$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dT}(\ln I_S) = \frac{1}{I_S}\frac{dI_S}{dT} \Rightarrow \frac{dI_S}{dT} = I_S \cdot \frac{d(\ln I_S)}{dT}$$

$$\frac{d}{dT}\left[\ln\underbrace{\left(\varsigma \cdot T^{2.5} \cdot e^{\frac{-E_G}{kT}}\right)}_{=I_S}\right] = \frac{d}{dT}\left(\ln\varsigma + \ln T^{2.5} - \frac{E_G}{kT}\right) = \frac{2.5}{T} + \frac{E_G}{kT^2} = \frac{2.5}{T} + \frac{E_G}{kT^2}\frac{q}{q} = \frac{2.5}{T} + \frac{E_G/q}{T \cdot V_T}$$

$$\boxed{\frac{dI_S}{dT} = I_S\left(\frac{2.5}{T} + \frac{E_G/q}{T \cdot V_T}\right)}$$

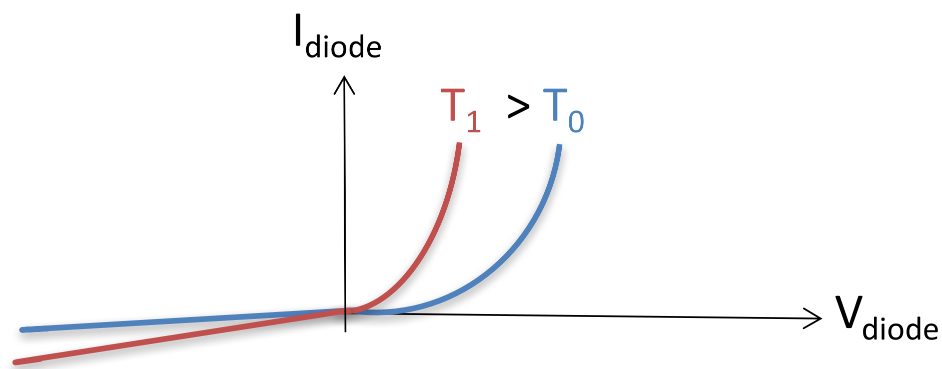
- At room temperature: $$\frac{dI_S}{dT} = I_S\left(\frac{2.5}{300} + \frac{1.12}{300 \cdot 26 \cdot 10^{-3}}\right) = I_S \times \frac{15.2}{100} \text{ [A/degree]}$$

- Since $(1.152)^5 \approx 2$ we conclude that the saturation current approximately doubles for every 5 degrees rise in temperature

Effect of Temperature on I/V curves

- In *reverse bias:*

$$\boxed{I_S(T) = I_S(T_0) \times 2^{(T-T_0)/5}}$$



- Although still quite small, real diodes exhibit reverse currents that are much larger than I_S. A large part of the reverse current is due to leakage effects. These leakage effects are proportional to the junction area A just as I_S is.

- As a rule of thumb I_R doubles for every 10 degree rise in temperature

$$I_R(T) = I_R(T_0) \times 2^{(T-T_0)/10}$$

Different Models

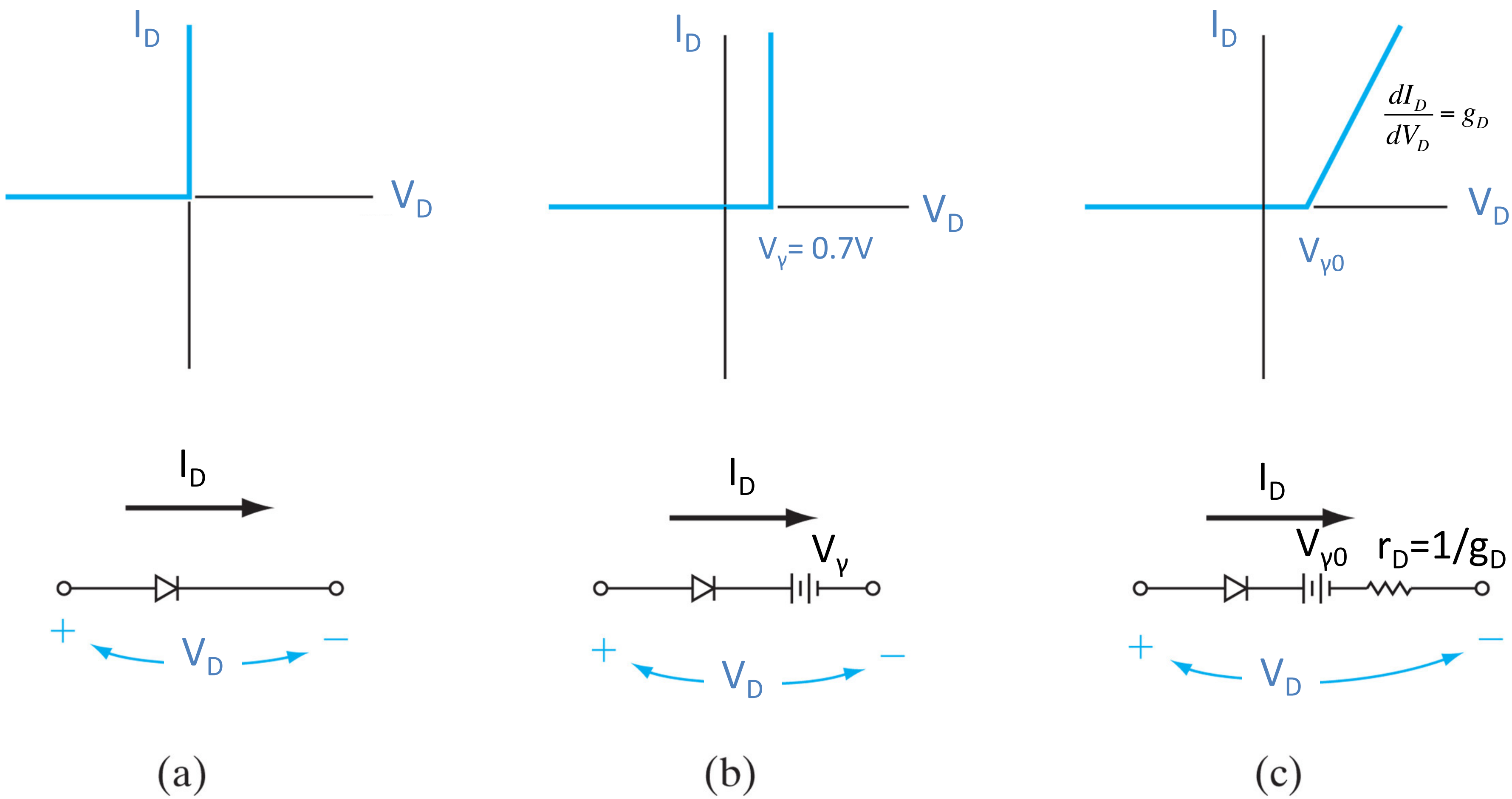
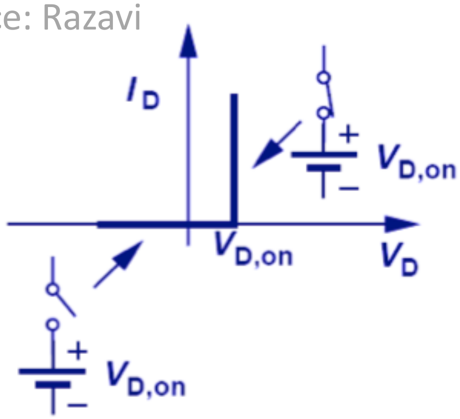


Figure 5–23 Streetman
approximations of junction diode characteristics: (a) the ideal diode; (b) ideal diode with an offset voltage; (c) ideal diode with an offset voltage and a resistance to account for slope in the forward characteristic.

Ideal diode with voltage offset

source: Razavi



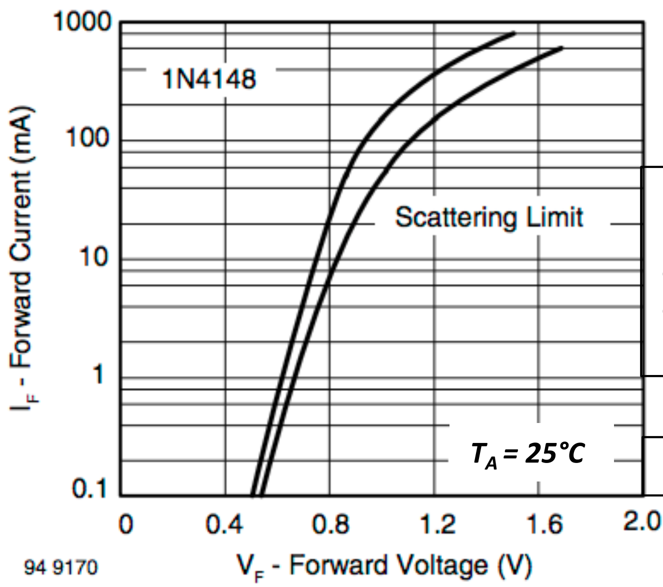


Fig. 2 - Forward Current vs. Forward Voltage

1N4148 Forward continuous current: IF = 300 mA

for VF=0.7V, IF≈3 mA

- Constant voltage model:
  - the constant voltage is called cut-in voltage, turn-on voltage, or threshold voltage and is usually denoted as $V_{D,on}$ or V_γ

- This model is based on the observation that a forward-conducting diode has a voltage drop that varies in a relatively narrow range (e.g. 0.6 to 0.8). We'll assume $V_\gamma \approx 0.7V$

- Below V_γ the current is very small (less than 1% of the maximum rated value).

$$\text{for } T > 25^\circ C \Rightarrow P_{D,\max} = 500 - 1.68 \times (T - 25)\ [mW]$$

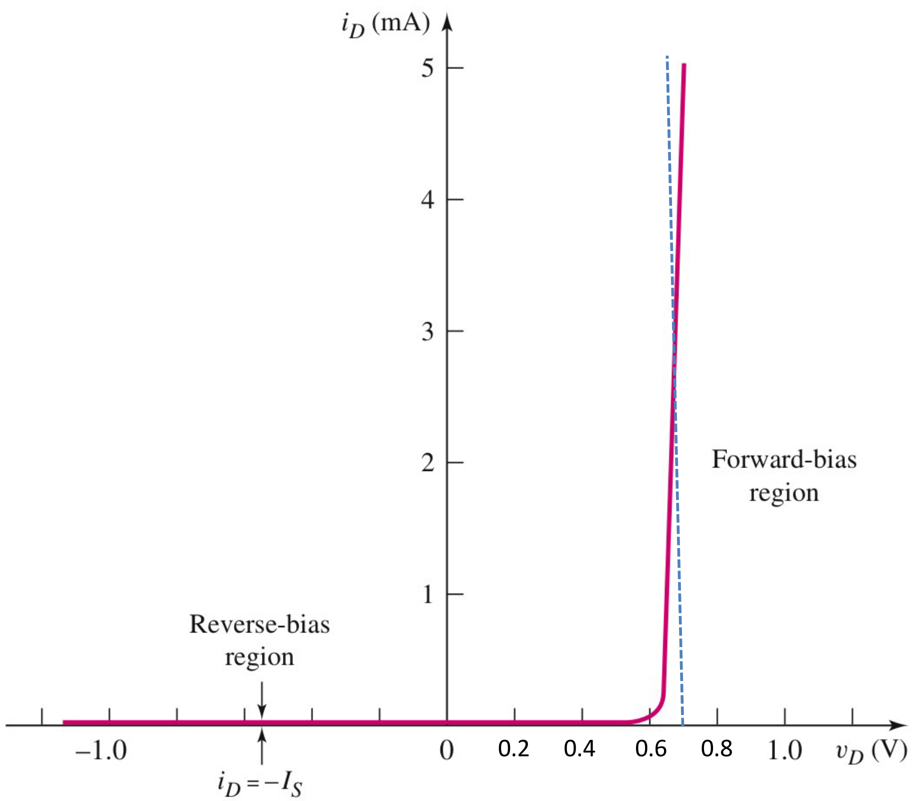
$$T_J = T_A + R_{\Theta JA} \times P_D$$

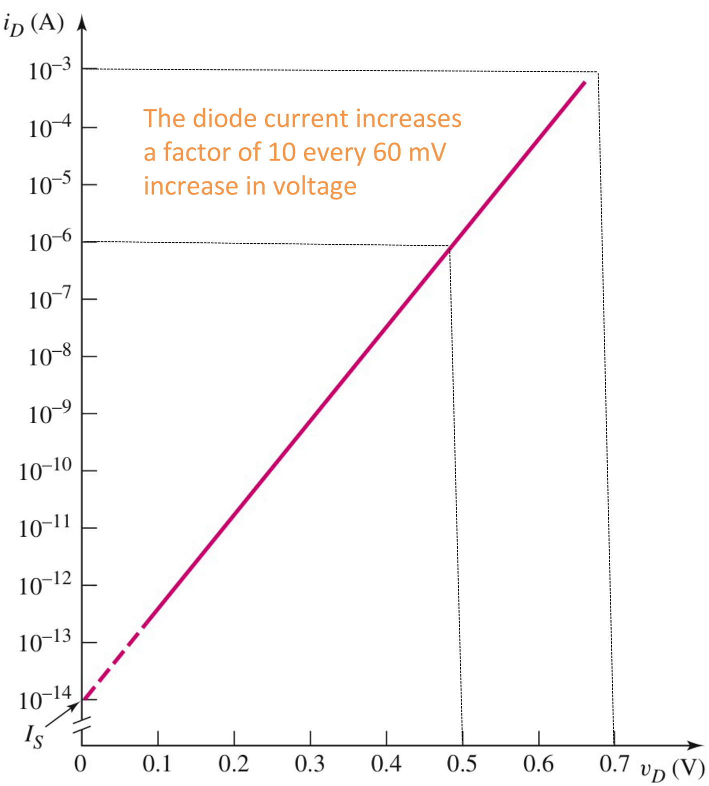
Thermal Characteristics

Characteristic	Symbol	Value	Unit
Power Dissipation (Note 1) Derate Above 25°C	P_D	500 1.68	mW mW/°C
Thermal Resistance, Junction to Ambient Air (Note 1)	$R_{\theta JA}$	300	°C/W
Operating and Storage Temperature Range	T_J, T_{STG}	-65 to +175	°C

Ideal diode with voltage offset

source: Neamen





- I/V characteristic of a theoretical diode with $I_S=10^{-14}$A = 10 fA

- Forward-bias part of the characteristics with current plotted on a log scale

NOTE: real diodes exhibit reverse currents that are considerably larger than I_S (this is mainly due to holes and electrons being generated within the space charge region). A typical value of reverse-bias current maybe 1nA (still small and negligible in most cases)

Is the model good enough ?

- Should we worry about the fact that the diode has resistance ?

$$\boxed{r_{diode}} = \left(\frac{dI_D}{dV_D}\right)^{-1} = \frac{d}{dV_D}\left[I_S\left(e^{\frac{V_D}{V_T}} - 1\right)\right]^{-1} = \left(\frac{I_S}{V_T}e^{\frac{V_D}{V_T}}\right)^{-1} = \left(\frac{I_S e^{\frac{V_D}{V_T}} - I_S + I_S}{V_T}\right)^{-1} = \boxed{\frac{V_T}{I_D + I_S}}$$

- Forward Region: $r_{diode} = \frac{V_T}{I_D + I_S} \underset{I_D >> I_S}{\cong} \frac{V_T}{I_D}$

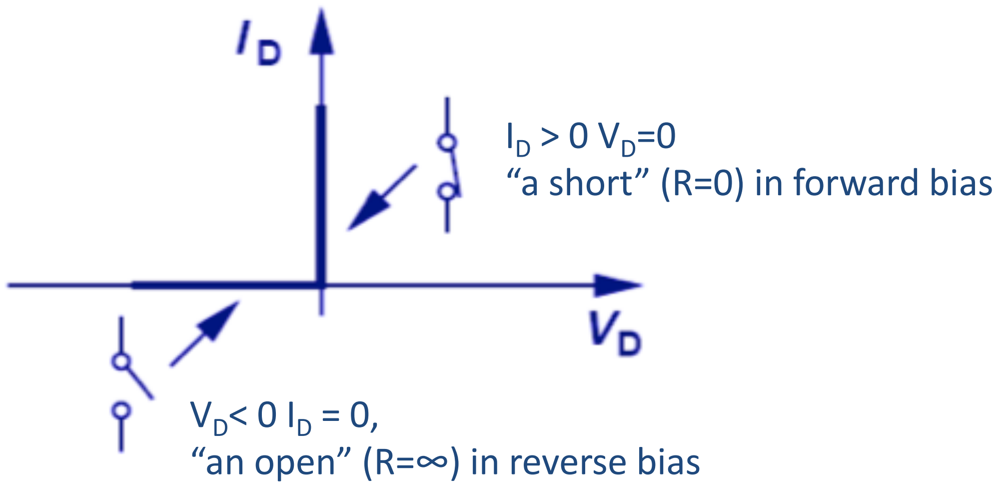
$\boxed{V_T/I_D \text{ is only a few } \Omega}$

Example:
$I_S \approx 10$ fA => $I_D \approx I_S \exp(V_D/V_T) = 10^{-14}\exp(0.7/26m) \approx 4.9$ mA => $V_T/I_D = 26/4.9 \approx 5.3\ \Omega$

- Reverse Region: $r_{diode} = \frac{V_T}{I_D + I_S} \underset{I_D \cong -I_S}{\cong} \infty\ [\Omega]$

Ideal Diode

source: Razavi

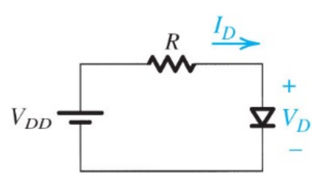


- In applications that involve voltages much greater than the diode voltage drop (0.6V-0.8V) we may neglect the diode voltage drop altogether.

Piecewise linear model

- This model is useful when there is a small varying signal superimposed to the biasing voltage

- Let's say we want to forward bias a diode so that it operates at a given value of I_D, we need to find the corresponding V_D (Q = operating point = (I_D,V_D))

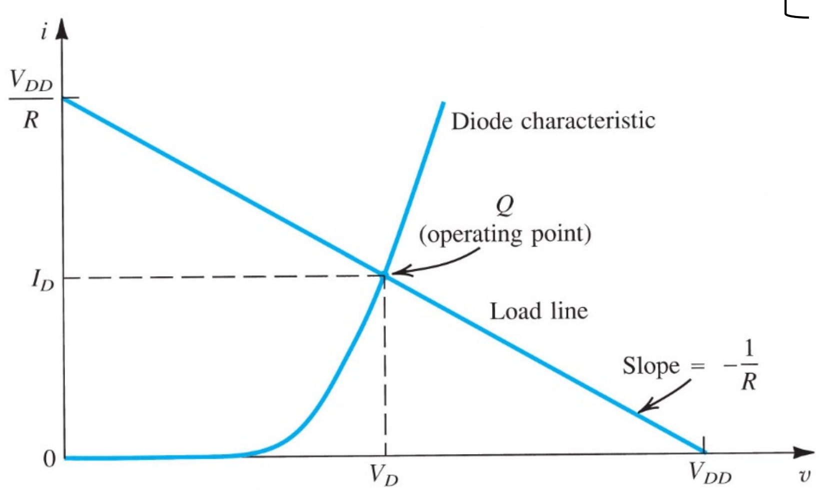


$$\begin{cases} I_D = \dfrac{V_{DD}-V_D}{R} \Leftrightarrow I_D = \dfrac{V_{DD}}{R} - \dfrac{V_D}{R} \\ I_D \approx I_S e^{\frac{V_D}{V_T}} \end{cases}$$

KVL = topological eq.

constitutive eq. of device

This is the eq, of a line (I_D vs. V_D) with slope −1/R



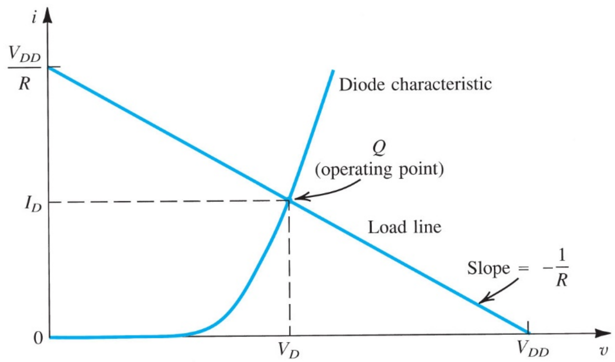
$NOTE: \ln(x) = \alpha \cdot Log(x) \rightarrow \ln(e) = 1 = \alpha \cdot Log(e) = \alpha \cdot 0.43 \rightarrow \alpha \approx 2.3$

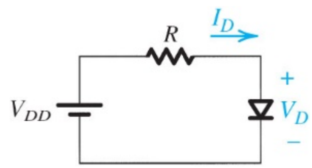
Piecewise linear model

Example

We have V_{DD}=5V and the diode at V_{D0}=0.7V has a current of I_{D0}=1mA. We want $I_D \cong$ 4.3mA.

Let's assume $V_D \cong$ 0.7V (and iterate until we find the right value corresponding to I_D)





$$R = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 0.7}{4.3m} = 1K\Omega$$

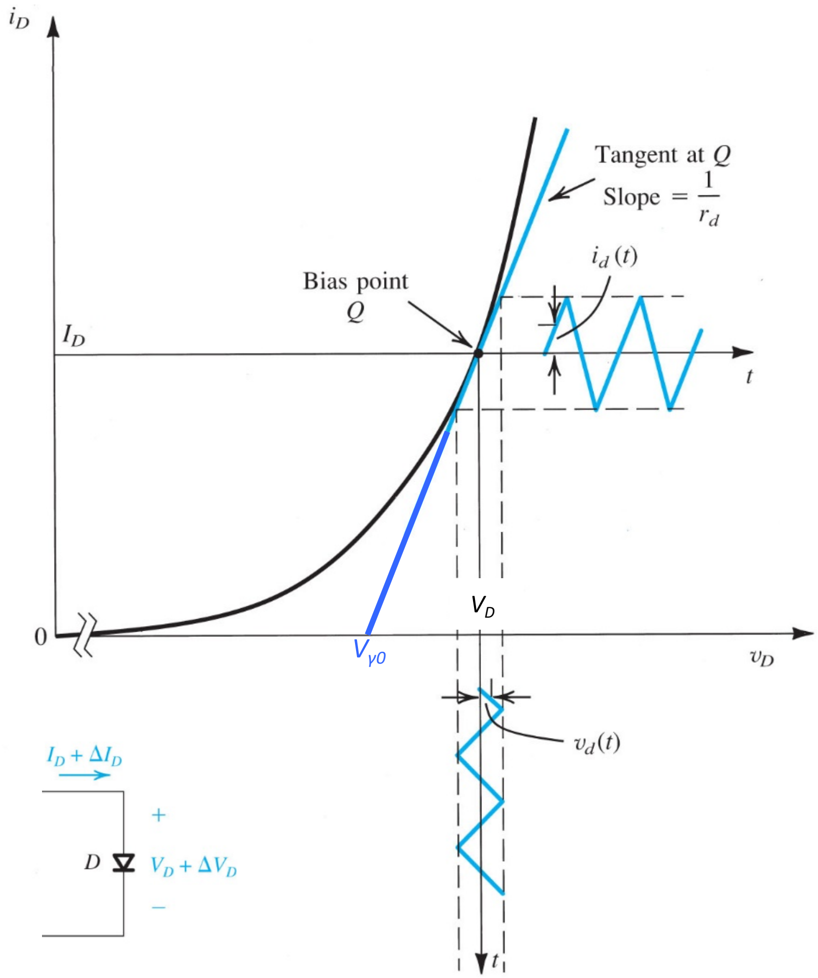
A voltage increase of 60 mV corresponds to a 10 x current increase

$$V_{D1} - V_{D0} = 2.3V_T \times Log\left(\frac{I_{D1}}{I_{D0}}\right) \Rightarrow V_{D1} = V_{D0} + 60mV \times Log\left(\frac{I_{D1}}{I_{D0}}\right) = 0.7 + 60mV \times Log\left(\frac{4.3m}{1m}\right) \approx 0.738V$$ (iteration 1)

$$I_{D2} = \frac{V_{DD} - V_{D1}}{R} = \frac{5 - 0.738}{1K} = 4.262mA \qquad V_{D2} = V_{D1} + 60mV \times Log\left(\frac{I_{D2}}{I_{D1}}\right) \approx 0.738 + 60mV \times Log\frac{4.262m}{4.3m} \cong 0.738V$$ (iteration 2)

No further iterations are necessary. The circuit used gives $I_D \cong$4.262mA and $V_D \cong$0.738V

Piecewise linear model



We have a signal superimposed to the bias voltage V_D:

$$v_D = V_D + v_d \Rightarrow v_d = v_D - V_D \equiv \Delta V_D \Rightarrow v_D = V_D + \Delta V_D$$

$$i_D \cong I_S e^{\frac{v_D}{V_T}} = I_S e^{\frac{V_D + \Delta V_D}{V_T}} = I_S e^{\frac{V_D}{V_T}} e^{\frac{\Delta V_D}{V_T}} = I_D e^{\frac{\Delta V_D}{V_T}}$$

If the max excursion of the signal is small (that is $\Delta V_D/V_T \ll 1$):

$$i_D = I_D e^{\frac{\Delta V_D}{V_T}} \approx I_D\left(1 + \frac{\Delta V_D}{V_T}\right)$$

$$\boxed{\frac{\Delta V_D}{V_T} \ll 1}$$

*

$$\boxed{\begin{array}{l} e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots \\ \text{for } x \ll 1: \; e^x \cong 1 + x \end{array}}$$

$$i_D \approx I_D + \underbrace{\frac{I_D}{V_T}}_{= g_d}\Delta V_D = I_D + \underbrace{g_d \Delta V_D}_{= \Delta I_D} = I_D + \underbrace{\Delta I_D}_{= i_d}$$

we find out that the response of the diode is about linear and it make sense to approximate the diode characteristic with the tangent line at Q:

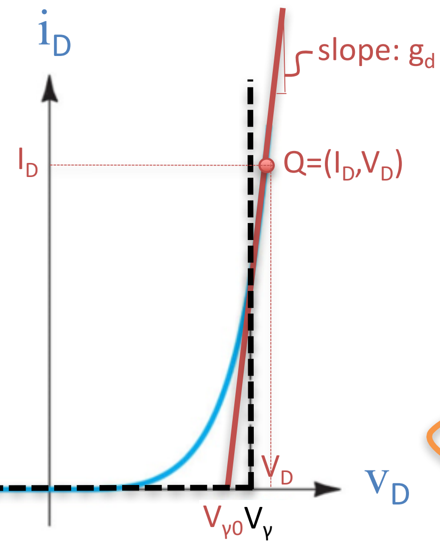
$$\frac{1}{r_d} = g_d = \frac{\Delta I_D}{\Delta V_D} = \frac{i_d}{v_d} = \left.\frac{di_D}{dv_D}\right|_{@V_D}$$

Piecewise linear model

Equation of line given two points:

$$y - y_0 = m(x - x_0)$$

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}$$



slope=ΔI/ΔV

$$i_D = I_S\left(e^{\frac{V_D}{V_T}} - 1\right) \approx I_D + g_d(v_D - V_D) = I_D + \frac{I_D}{V_D - V_{\gamma 0}}(v_D - V_D)$$

$$g_d = \frac{I_D}{V_T} \Rightarrow V_T = V_D - V_{\gamma 0} \Rightarrow V_{\gamma 0} = V_D - V_T$$

alternatively:

$$y = mx + n \Rightarrow i_D \approx \frac{v_D}{r_d} - \frac{V_{\gamma 0}}{r_d}$$

$$y(\bar{x}) \equiv 0 = m\bar{x} + n \Rightarrow n = -m\bar{x}$$

* How small is a small signal ?

$$e^x \cong 1 + x$$

The error of the approx. should be ≤ 10%

$$Error = e^x - 1 - x \le 0.1$$

Solving numerically we se that for x ≤ 0.4 the error is ≤ 0.09

$$\frac{\Delta V_D}{V_T} \le 0.4 \Rightarrow \Delta V_D \le 26mV \times 0.4 \approx 10.4mV$$

Diode Circuits

- … Finally let's start building some circuit
- Applications:
  - Rectifiers
  - Limiting Circuits (a.k.a. Clippers)
  - Level Shifters (a.k.a. Clampers)
  - Detectors
  - Voltage doublers
  - Regulators
  - Switches