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Schaum's Outline of Theory and Problems of Feedback and Control Systems 2/e
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15.4 BODE PLOTS OF SIMPLE CONTINUOUS-TIME FREQUENCY RESPONSE FUNCTIONS AND THEIR ASYMPTOTIC APPROXIMATIONS

The constant K_B has a magnitude $|K_B|$, a phase angle of 0° if K_B is positive, and $-180°$ if K_B is negative. Therefore the Bode plots for K_B are simply horizontal straight lines as shown in Figs. 15-1 and 15-2.

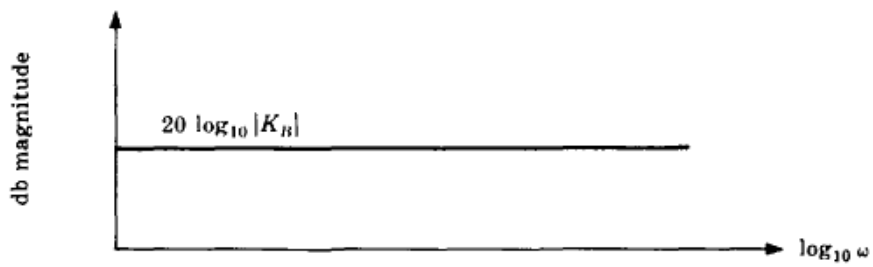

Fig. 15-1

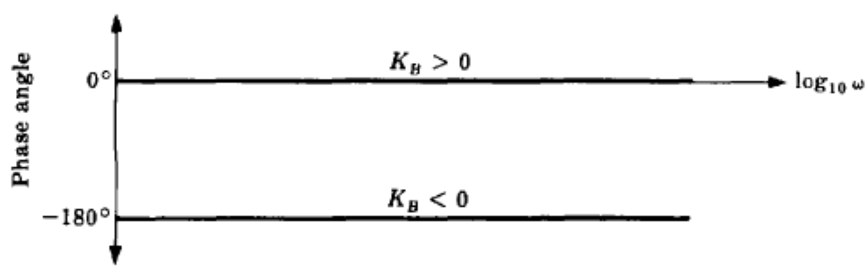

Fig. 15-2

The frequency response function (or sinusoidal transfer function) for a *pole of order l at the origin* is

$$\frac{1}{(j\omega)^l} \tag{15.4}$$

The bode plots for this function are straight lines, as shown in Figs. 15-3 and 15-4.

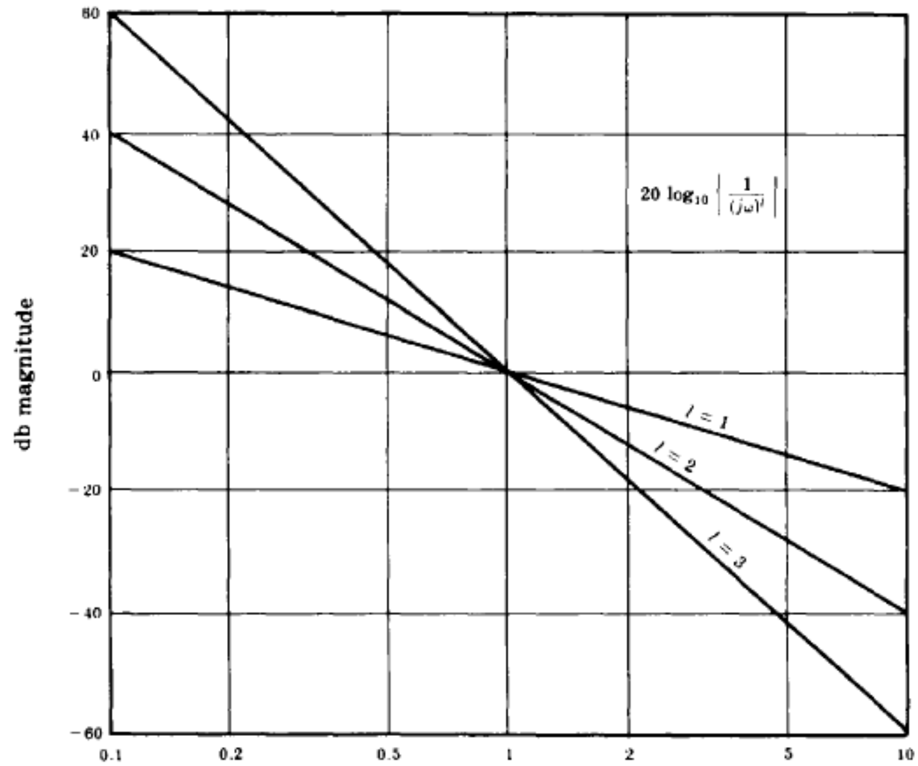

Fig. 15-3

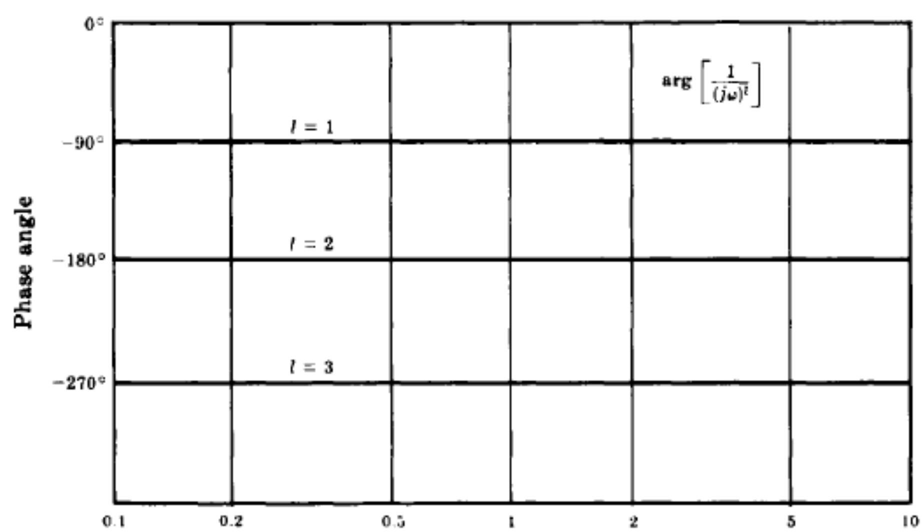

Fig. 15-4

For a *zero of order l at the origin*,

$$(j\omega)^l. \tag{15.5}$$

the Bode plots are the reflections about the 0-db and 0° lines of Figs. 15-3 and 15-4, as shown in Figs. 15-5 and 15-6.

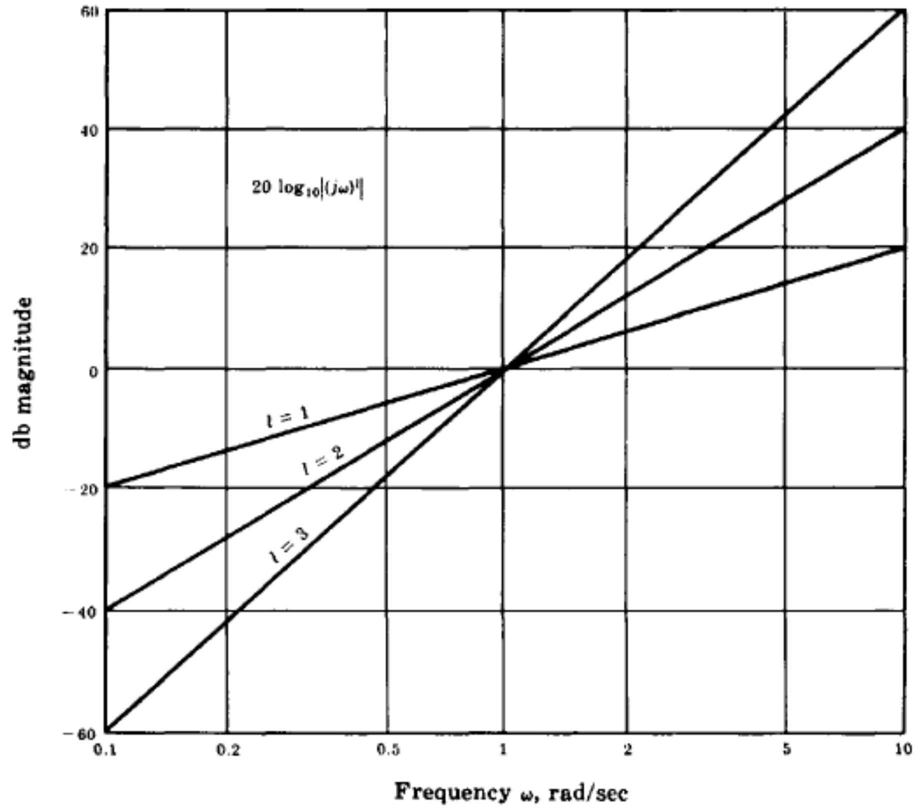

Fig. 15-5

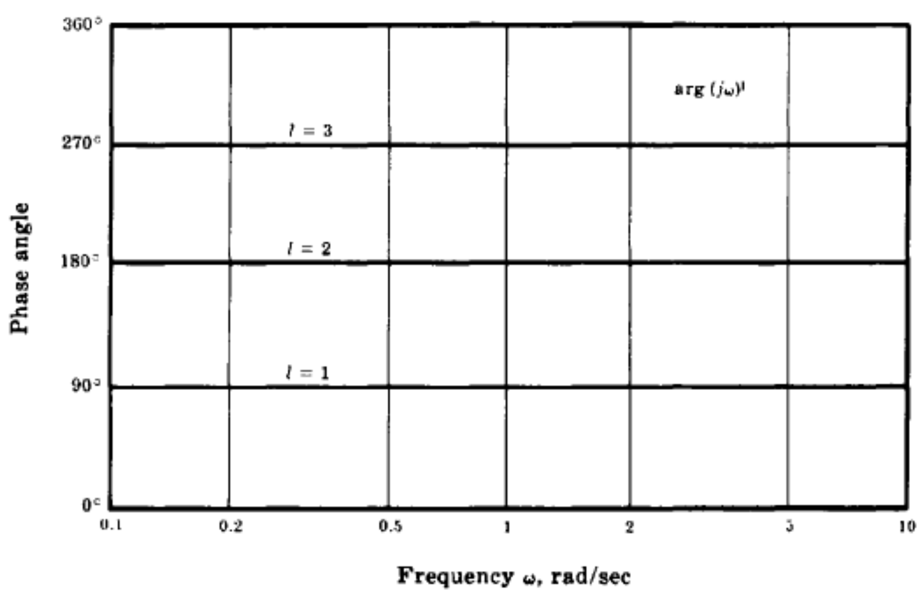

Fig. 15-6

Consider the *single-pole* transfer function $p/(s+p)$, $p>0$. The Bode plots for its frequency response function

$$\frac{1}{1+j\omega/p} \tag{15.6}$$

are given in Figs. 15-7 and 15-8. Note that the logarithmic frequency scale is normalized in terms of p.

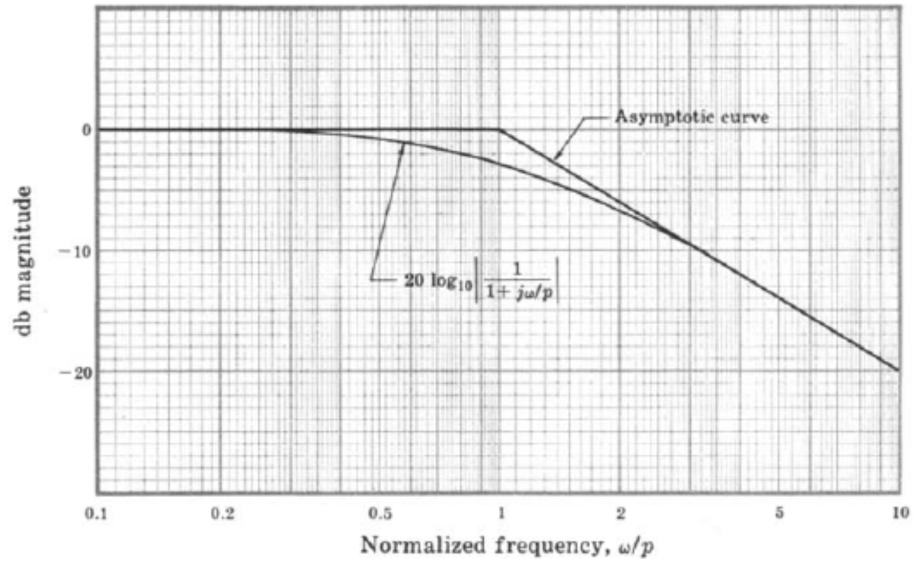

Fig. 15-7

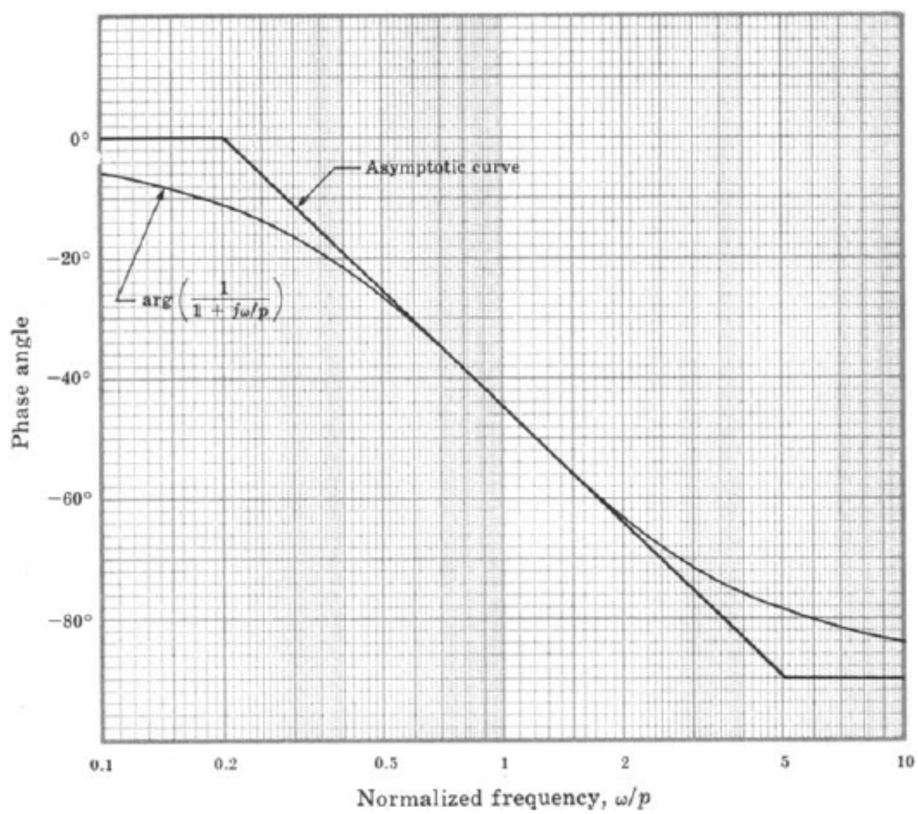

Fig. 15-8

To determine the *asymptotic approximations* for these Bode plots, we see that for $\omega/p \ll 1$, or $\omega \ll p$,

$$20 \log_{10}\left|\frac{1}{1+j\omega/p}\right| \cong 20 \log_{10} 1 = 0 \text{ db}$$

and for $\omega/p \gg 1$, or $\omega \gg p$,

$$20 \log_{10}\left|\frac{1}{1+j\omega/p}\right| \cong 20 \log_{10}\left|\frac{1}{j\omega/p}\right| = -20 \log_{10}\left(\frac{\omega}{p}\right)$$

Therefore the Bode magnitude plot asymptotically approaches a horizontal straight line at 0 db as ω/p approaches zero and $-20\log_{10}(\omega/p)$ as ω/p approaches infinity (Fig. 15-7). Note that this high-frequency asymptote is a straight line with a slope of -20 db/decade, or -6 db/octave when plotted on a logarithmic frequency scale as shown in Fig. 15-7. The two asymptotes intersect at the **corner frequency** $\omega = p$ rad/sec. To determine the phase angle asymptote, we see that for $\omega/p \ll 1$, or $\omega \ll p$,

$$\arg\left(\frac{1}{1+j\omega/p}\right) = -\tan^{-1}\left(\frac{\omega}{p}\right)\bigg|_{\omega \ll p} \cong 0^\circ$$

and for $\omega/p \gg 1$, or $\omega \gg p$,

$$\arg\left(\frac{1}{1+j\omega/p}\right) = -\tan^{-1}\left(\frac{\omega}{p}\right)\bigg|_{\omega \gg p} \cong -90^\circ$$

Thus the Bode phase angle plot asymptotically approaches 0° as ω/p approaches zero, and $-90°$ as ω/p approaches infinity, as shown in Fig. 15-8. A negative-slope straight-line asymptote can be used to join the 0° asymptote and the $-90°$ asymptote by drawing a line from the 0° asymptote at $\omega = p/5$ to the $-90°$ asymptote at $\omega = 5p$. Note that it is tangent to the exact curves at $\omega = p$.

The *errors* introduced by these asymptotic approximations are shown in Table 15-1 for the single-pole transfer function at various frequencies.

Table 15-1. Asymptotic Errors for $\dfrac{1}{1+j\omega/p}$

ω	$p/5$	$p/2$	p	$2p$	$5p$
Magnitude error (db)	-0.17	-0.96	-3	-0.96	-0.17
Phase angle error	$-11.3°$	$-0.8°$	$0°$	$+0.8°$	$+11.3°$

The Bode plots and their asymptotic approximations for the *single-zero* frequency response function

$$1 + \frac{j\omega}{z_1} \tag{15.7}$$

are shown in Figs. 15-9 and 15-10.

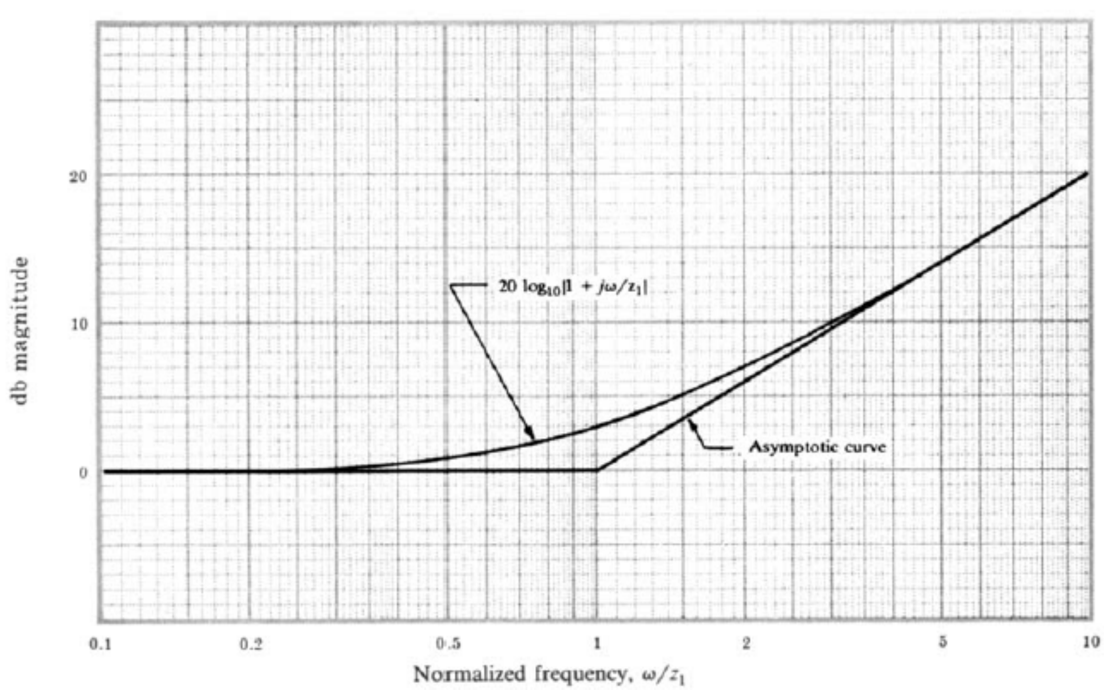

Fig. 15-9

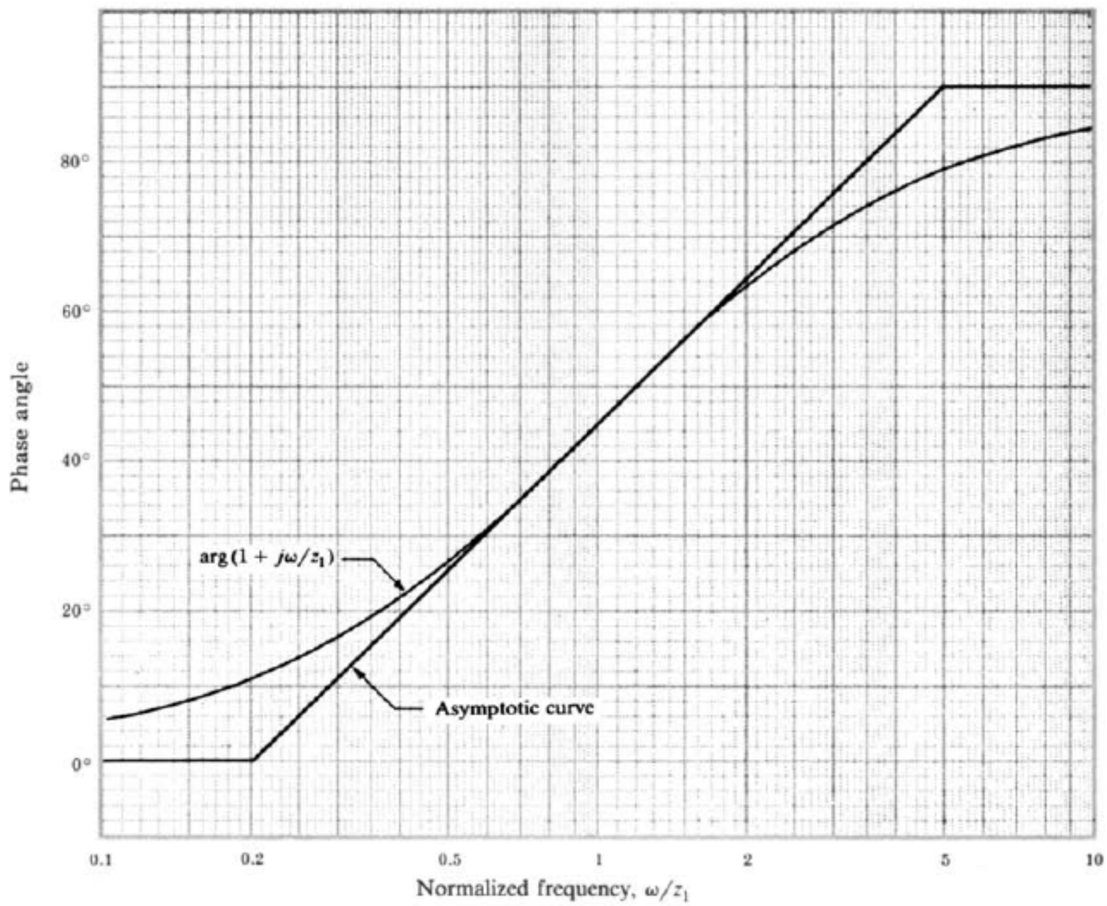

Fig. 15-10

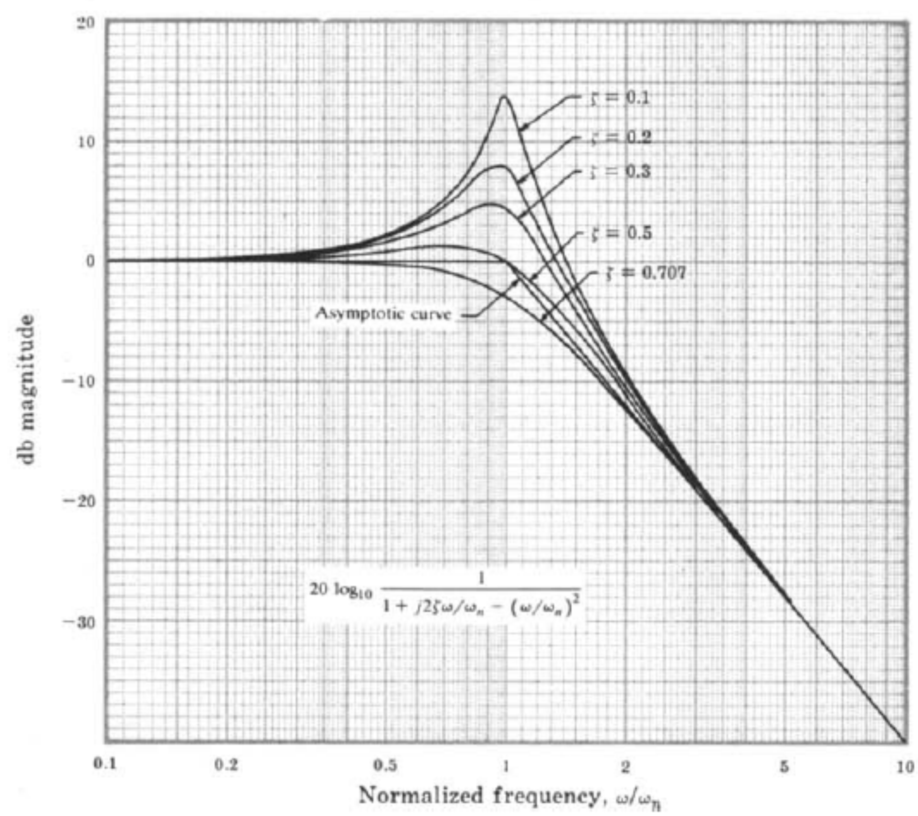

Fig. 15-11

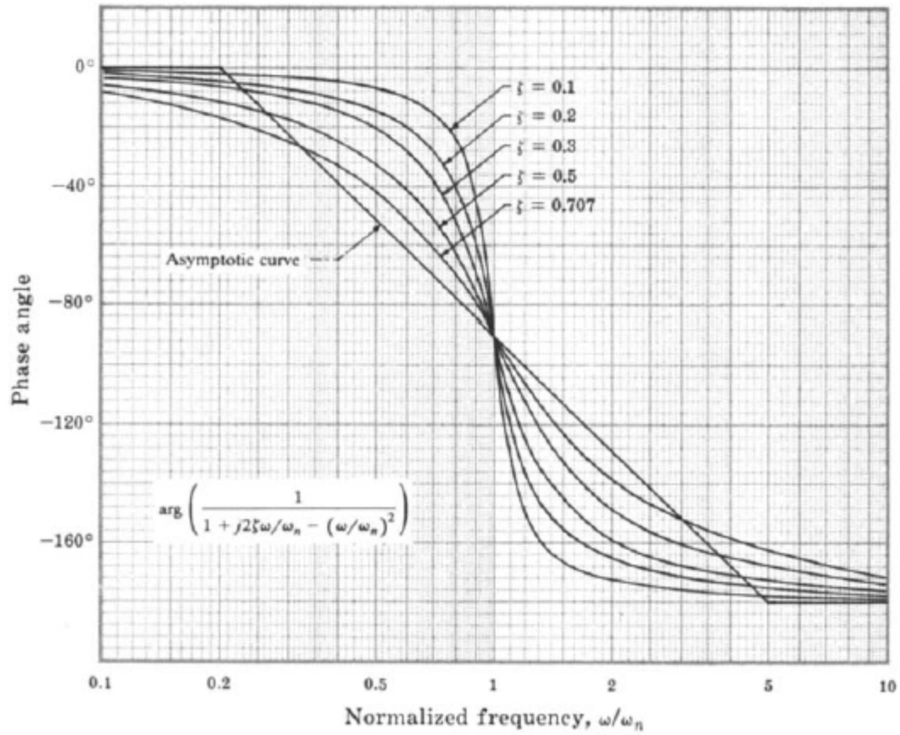

Fig. 15-12

The Bode plots and their asymptotic approximations for the second-order frequency response function with *complex poles*,

$$\frac{1}{1 + j2\zeta\omega/\omega_n - (\omega/\omega_n)^2} \qquad 0 \le \zeta \le 1 \tag{15.8}$$

are shown in Figs. 15-11 and 15-12. Note that the damping ratio ζ is a parameter on these graphs.

The magnitude asymptote shown in Fig. 15-11 has a corner frequency at $\omega = \omega_n$ and a high-frequency slope twice that of the asymptote for the single-pole case of Fig. 15-7. The phase angle asymptote is similar to that of Fig. 15-8 except that the high-frequency portion is at $-180°$ instead of $-90°$ and the point of tangency, or inflection, is at $-90°$.

The Bode plots for a pair of *complex zeros* are the reflections about the 0 db and 0° lines of those for the complex poles.

15.5 CONSTRUCTION OF BODE PLOTS FOR CONTINUOUS-TIME SYSTEMS

Bode plots of continuous-time frequency response functions can be constructed by summing the magnitude and phase angle contributions of each pole and zero (or pairs of complex poles and zeros). The asymptotic approximations of these plots are often sufficient. If more accurate plots are desired, many software packages are available for rapidly accomplishing this task.

For the general open-loop frequency response function

$$GH(j\omega) = \frac{K_B(1 + j\omega/z_1)(1 + j\omega/z_2)\cdots(1 + j\omega/z_m)}{(j\omega)^l(1 + j\omega/p_1)(1 + j\omega/p_2)\cdots(1 + j\omega/p_n)} \tag{15.9}$$

where l is a positive integer or zero, the magnitude and phase angle are given by

$$20\log_{10}|GH(j\omega)| = 20\log_{10}|K_B| + 20\log_{10}\left|1 + \frac{j\omega}{z_1}\right| + \cdots + 20\log_{10}\left|1 + \frac{j\omega}{z_m}\right|$$
$$+ 20\log_{10}\frac{1}{|(j\omega)^l|} + 20\log_{10}\frac{1}{|1 + j\omega/p_1|} + \cdots + 20\log_{10}\frac{1}{|1 + j\omega/p_n|} \tag{15.10}$$

and

$$\arg GH(j\omega) = \arg K_B + \arg\left(1 + \frac{j\omega}{z_1}\right) + \cdots + \arg\left(1 + \frac{j\omega}{z_m}\right)$$
$$+ \arg\left(\frac{1}{(j\omega)^l}\right) + \arg\left(\frac{1}{1 + j\omega/p_1}\right) + \cdots + \arg\left(\frac{1}{1 + j\omega/p_n}\right) \qquad (15.11)$$

The Bode plots for each of the terms in Equations (*15.10*) and (*15.11*) were given in Figs. 15-1 to 15-12. If $GH(j\omega)$ has complex poles or zeros, terms having a form similar to Equation (*15.8*) are simply added to Equations (*15.10*) and (*15.11*). The construction procedure is best illustrated by an example.

EXAMPLE 15.3. The asymptotic Bode plots for the frequency response function

$$GH(j\omega) = \frac{10(1 + j\omega)}{(j\omega)^2\left[1 + j\omega/4 - (\omega/4)^2\right]}$$

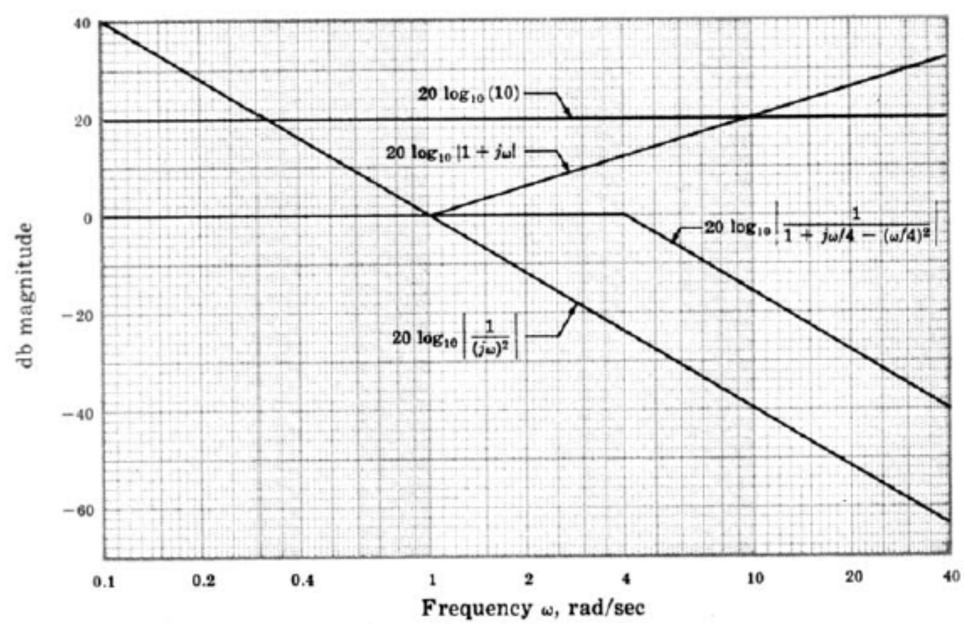

Fig. 15-13

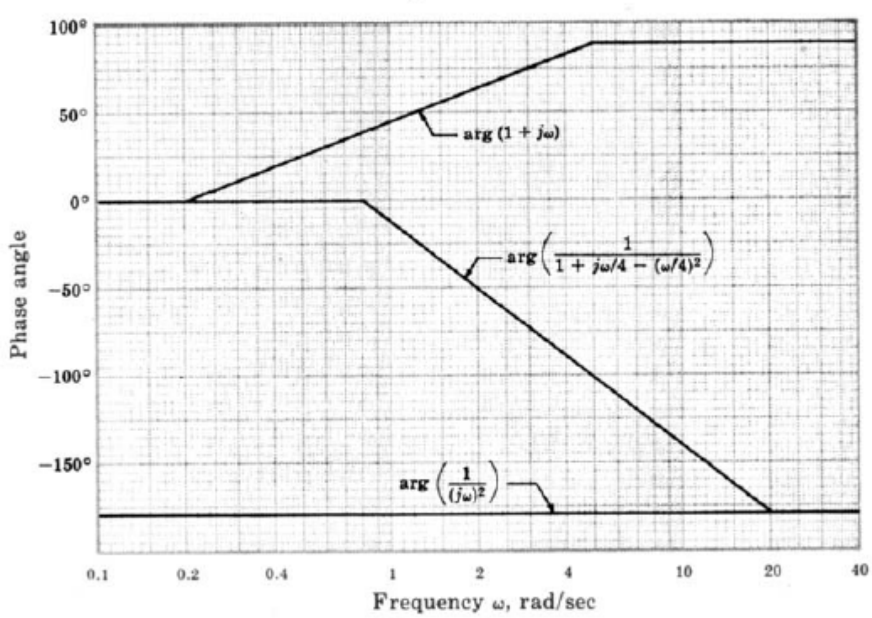

Fig. 15-14

are constructed using Equations (*15.10*) and (*15.11*):

$$20\log_{10}|GH(j\omega)| = 20\log_{10}10 + 20\log_{10}|1+j\omega| + 20\log_{10}\left|\frac{1}{(j\omega)^2}\right| + 20\log_{10}\left|\frac{1}{1+j\omega/4-(\omega/4)^2}\right|$$

$$\arg GH(j\omega) = \arg(1+j\omega) + \arg\left(1/(j\omega)^2\right) + \arg\left(\frac{1}{1+j\omega/4-(\omega/4)^2}\right)$$

The graphs for each of the terms in these equations are obtained from Figs. 15-1 to 15-12 and are shown in Figs. 15-13 and 15-14. The asymptotic Bode plots for $GH(j\omega)$ are obtained by adding these curves, as shown in Figs. 15-15 and 15-16, where computer-generated Bode plots for the frequency response function are also given for comparison with the asymptotic approximations.

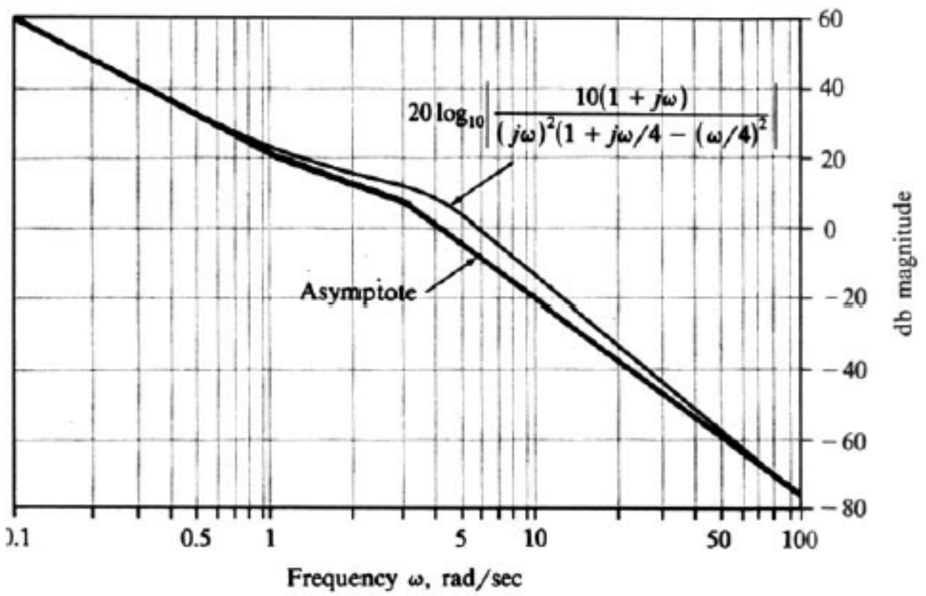

Fig. 15-15

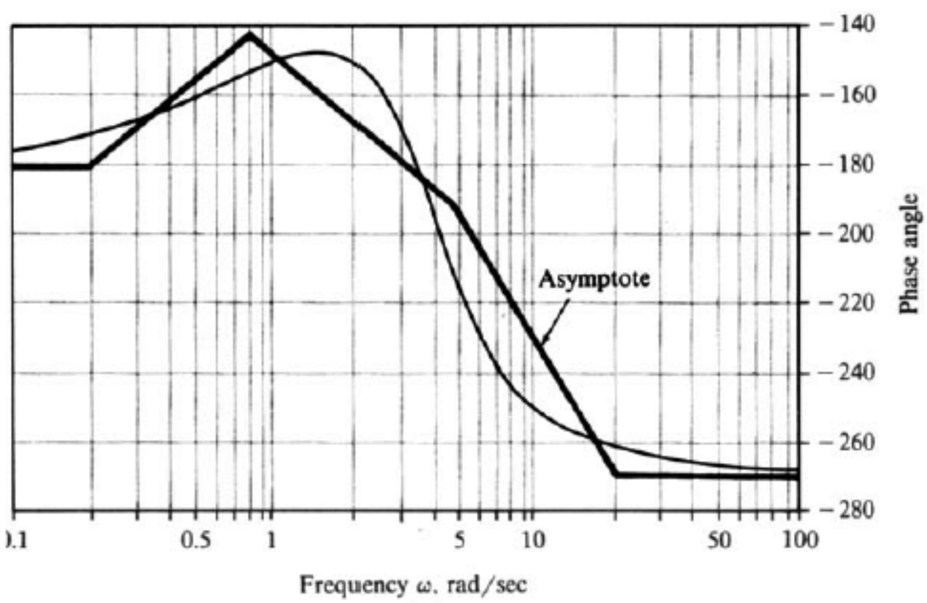

Fig. 15-16

CONSTRUCTION OF BODE PLOTS FOR CONTINUOUS-TIME SYSTEMS

15.6. Construct the asymptotic Bode plots for the frequency response function

$$GH(j\omega) = \frac{1 + j\omega/2 - (\omega/2)^2}{j\omega(1 + j\omega/0.5)(1 + j\omega/4)}$$

The asymptotic Bode plots are determined by summing the graphs of the asymptotic representations for each of the terms of $GH(j\omega)$, as in equations *(15.10)* and *(15.11)*. The asymptotes for each of these terms are shown in Figs. 15-23 and 15-24 and the asymptotic Bode plots for $GH(j\omega)$ in Figs. 15-25 and 15-26. The exact Bode plots generated by computer are shown for comparison.

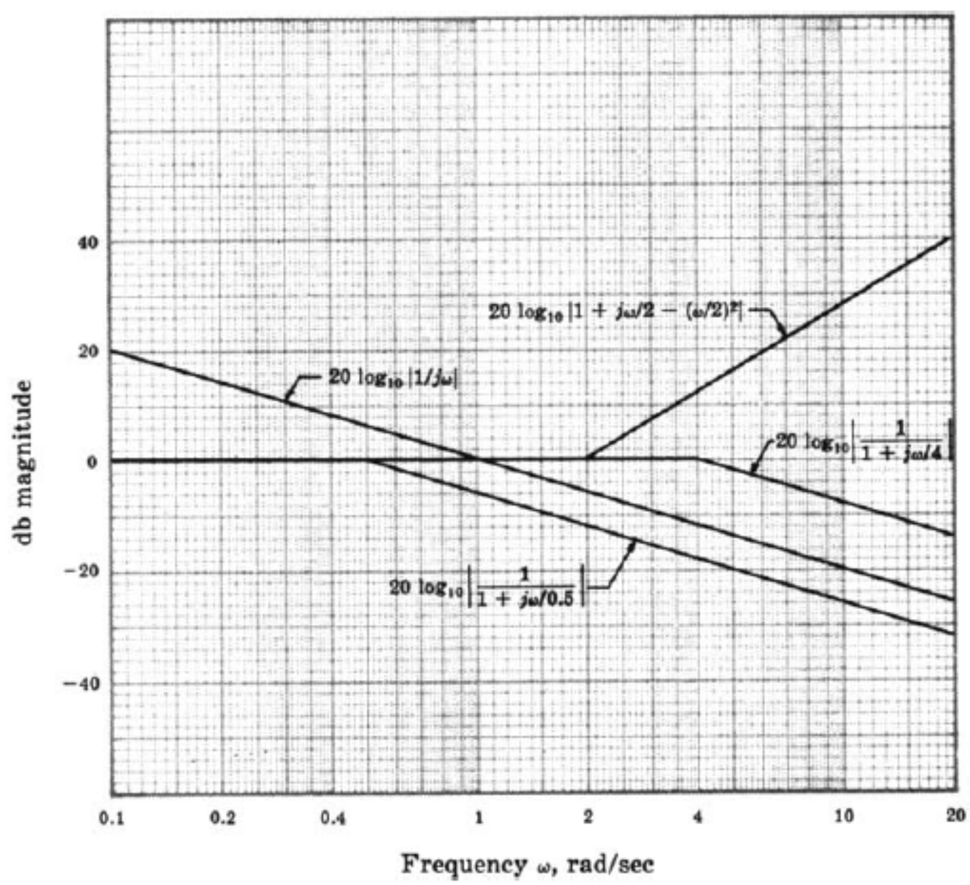

Fig. 15-23

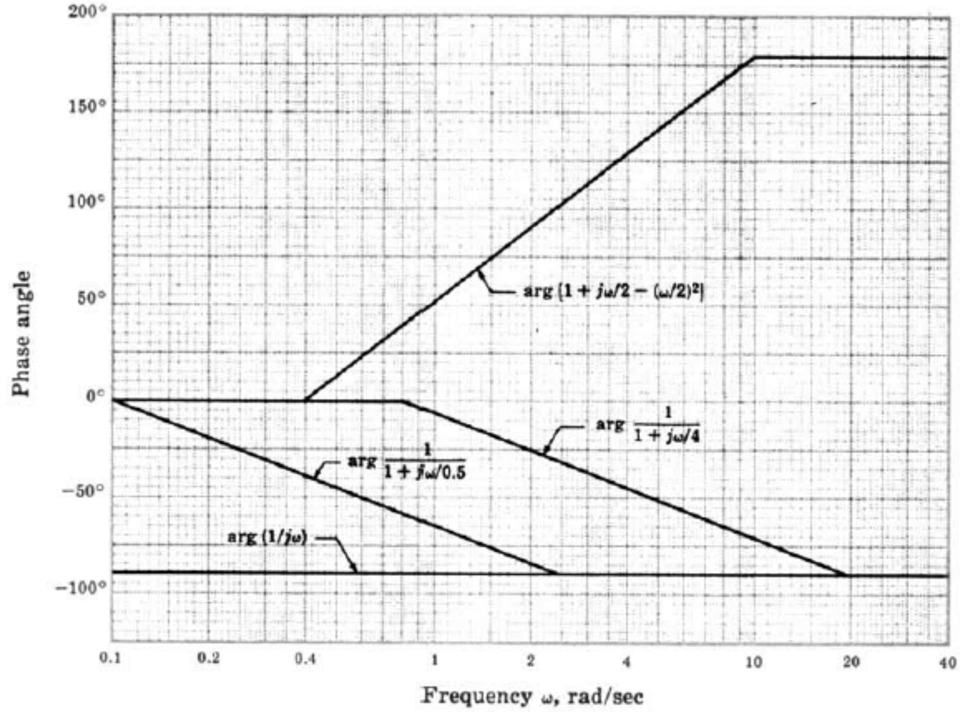

Fig. 15-24

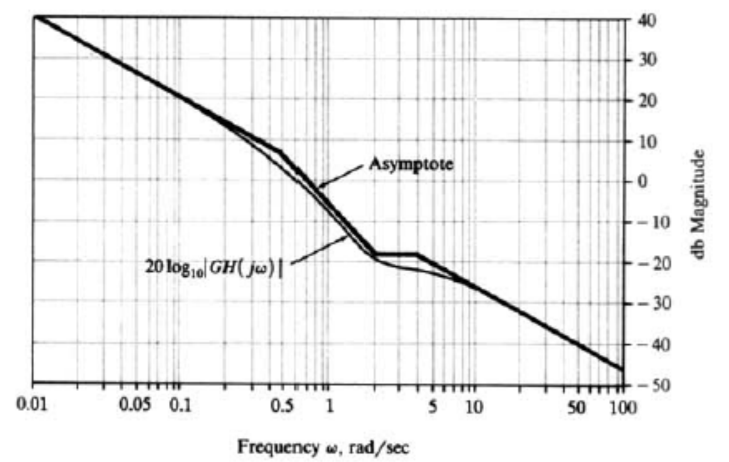

Fig. 15-25

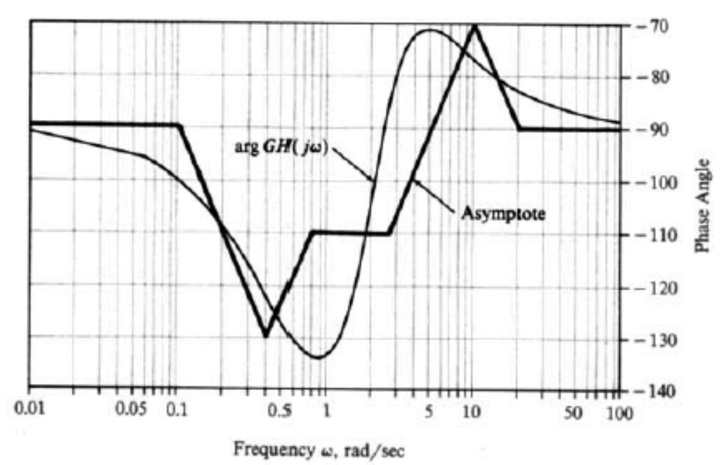

Fig. 15-26

15.7. Construct Bode plots for the frequency response function

Mathcad

$$GH(j\omega) = \frac{2}{j\omega(1 + j\omega/2)(1 + j\omega/5)}$$

The asymptotic Bode plots are constructed by summing the asymptotic plots for each term of $GH(j\omega)$, as in Equation *(15.10)* and *(15.11)*, and are shown in Figs. 15-27 and 15-28. More accurate curves determined numerically by computer are also plotted for comparison.

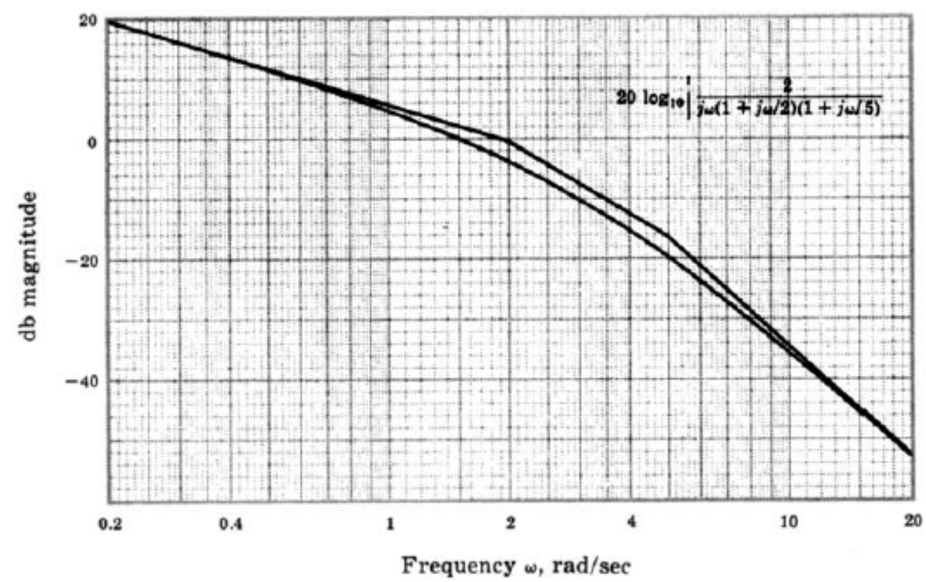

Fig. 15-27

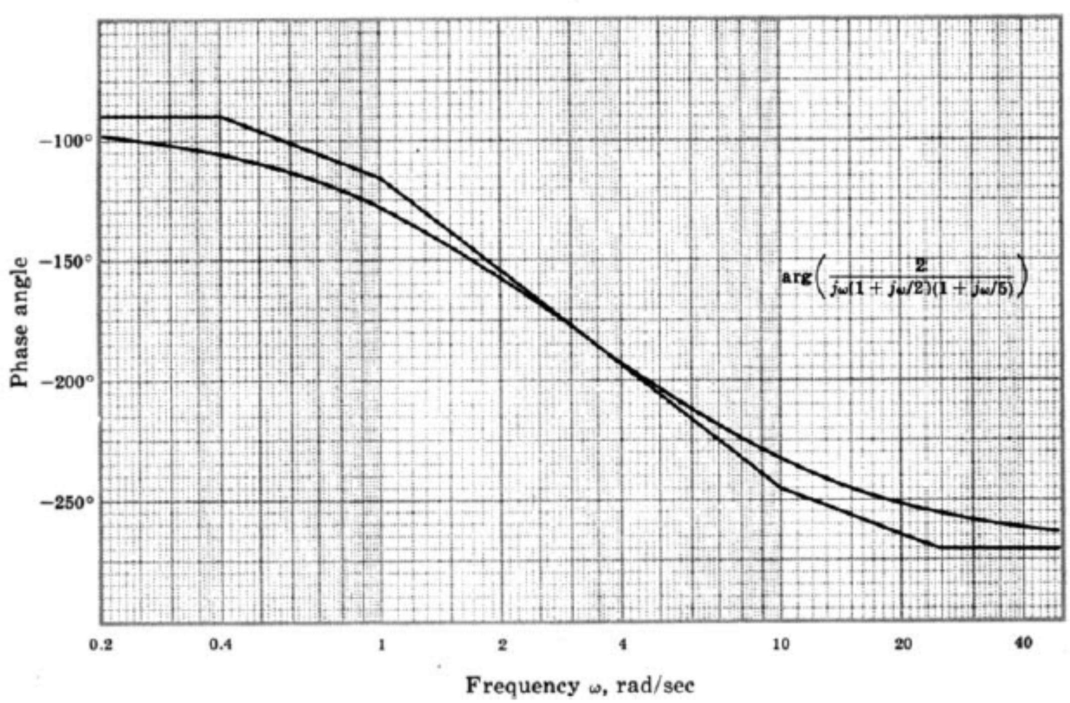

Fig. 15-28

15.8. Construct the Bode plots for the open-loop transfer function $GH = 2(s+2)/(s^2-1)$.

Mathcad

With $s = j\omega$, the Bode form for this transfer function is

$$GH(j\omega) = \frac{-4(1+j\omega/2)}{(1+j\omega)(1-j\omega)}$$

This function has a right-half plane pole [due to the term $1/(1-j\omega)$] which is not one of the standard functions introduced in Section 15.4. However, this function has the same magnitude as $1/(1+j\omega)$ and the same phase angle as $1+j\omega$. Thus for a function of the form $1/(1-j\omega/p)$, the magnitude can be determined from Fig. 15-7 and the phase angle from Fig. 15-10. For this problem the phase angle contributions from the terms $1/(1+j\omega)$ and $1/(1-j\omega)$ cancel each other. The asymptotes for the Bode magnitude plot are shown in Fig. 15-29 along with a more accurate Bode magnitude plot. The Bode phase angle is determined solely from $\arg K_B = \arg(-4) - 180°$ and the zero at $\omega = 2$, as shown in Fig. 15-30.

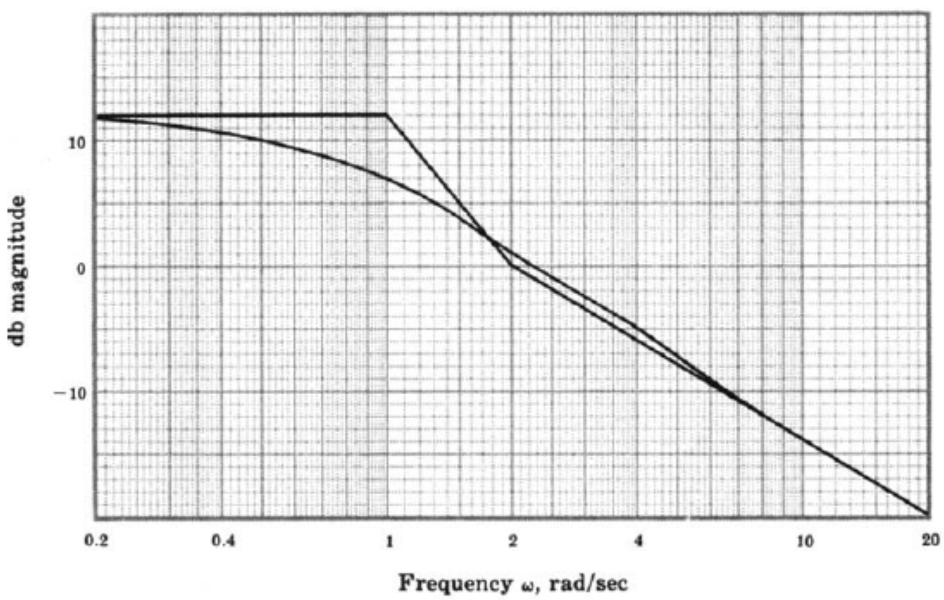

Fig. 15-29

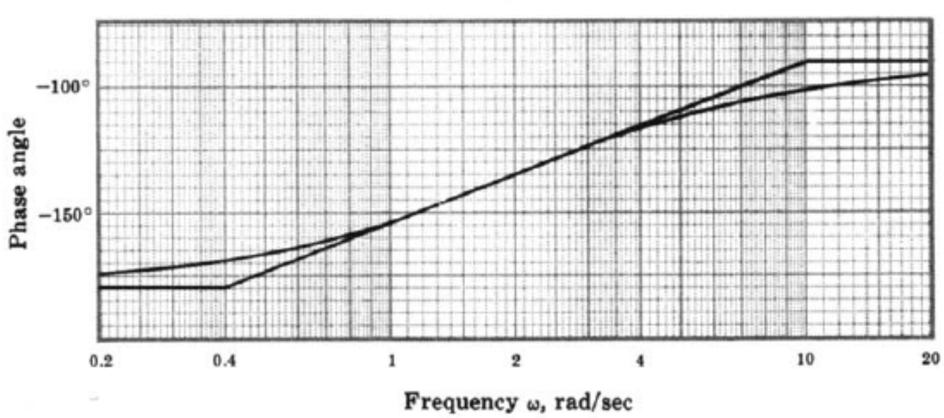

Fig. 15-30