Ali Sheikholeslami

A Capacitor Analogy, Part 1

Welcome to the tenth article in the "Circuit Intuitions" column series. As the title suggests, each article provides insights and intuitions into circuit design and analysis. These articles are aimed at undergraduate students but may serve the interests of other readers as well. If you read this article, I would appreciate your comments and feedback, as well as your requests and suggestions for future articles in this series. Please e-mail your comments to me at: ali@ece.utoronto.ca.

In electronics, we often use analogies to better understand circuit concepts and circuit behaviors. These analogies help bring intangible concepts closer to our understanding until we fully feel at home with them, then we leave the analogies behind. Yet whenever we are encountered with a circuit component in a new configuration, we may resort back to analogies to see if we could understand a similar configuration in an analogous world. If we could, then it is often easy to find a similar solution to the original problem. In this article, we provide an analogy for a capacitor and see how far we could go with this analogy in understanding and solving capacitor circuits.

A capacitor stores charge similar to a glass that stores water in the following sense. The charge (Q) stored on a capacitor is proportional to the voltage across the capacitor (V). Similarly, the amount of water held in a glass of water is proportional to the water height if we assume a fully cylindrical glass. The factor of proportionality in the capacitor is the capacitance (C), and in the glass of water is the cross-section area of the glass.

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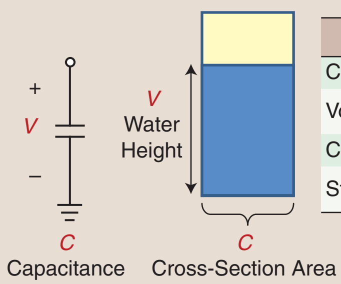
Figure 1 shows a capacitor next to a glass of water. For this analogy to work, we need to make some assumptions. Let us assume both the water mass density (ρ) and the gravity (g) are unity. Then, the weight of the water will be equal in number to the volume of water. Let us denote by Q the total water weight, by C the cross-section area of the glass, and by V the water height. Then we have a one-to-one correspondence between the capacitor and the glass. In both cases, $Q = CV$. This equation tells us that, for a fixed amount of charge, a larger capacitor produces a smaller voltage. This is similar to a glass with a larger cross section, resulting in a lower water height when the amount of water is fixed.

Let us calculate the stored energy in a capacitor charged to V_{DD}. We use the circuit in Figure 2(a), which employs a PMOS transistor to charge up the capacitor from $0V$ to V_{DD}. If the gate voltage transitions from V_{DD} to zero at time zero, the transistor will turn ON, allowing current to flow from the power supply (V_{DD}) to the capacitor, gradually charging the capacitor. Initially, the transistor is in saturation, creating a large current, but as times go on and the voltage across the capacitor increases, the transistor will move to the triode region, providing less current, until the current stops altogether when the capacitor is fully charged to V_{DD}. The total energy stored in the capacitor can be calculated by integrating the instantaneous power delivered to the capacitor. In other words, the stored energy (E_S) can be found as



Capacitor	Equation	Glass
Capacitance	C	Cross-Section Area
Voltage	V	Water Height
Charge	Q (= CV)	Water Weight*
Stored Energy	$E = 1/2\ CV^2$	Potential Energy

FIGURE 1: Charging a capacitor is analogous to filling up a glass with water. With water weight (*), we have assumed unity water density (ρ) and gravity (g).

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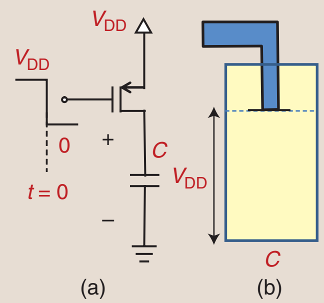

Capacitor	Equation	Glass
Stored Energy (Electrical)	$E_S = 1/2\ CV_{DD}^2$	Stored Energy (Potential)
Wasted Energy	$E_W = 1/2\ CV_{DD}^2$	Wasted Energy

FIGURE 2: In the process of charging a capacitor to V_{DD}, we both store and waste $(1/2)CV_{DD}^2$ of energy. This is similar to the process of filling up a glass with water.

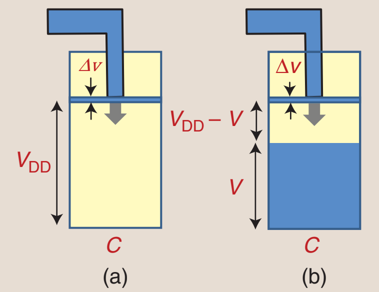

FIGURE 3: The water dropped initially into the glass wastes all of its potential energy. As the water height increases, the newly added water keeps more of its potential energy in the glass and wastes less.

$$E_S = \int_0^T i v dt = \int_0^{V_{DD}} C v dv = \frac{1}{2} C V_{DD}^2.$$

Similarly, we can calculate the energy wasted (E_W) in the transistor in this process:

$$\begin{aligned} E_W &= \int_0^T i(V_{DD} - v) dt \\ &= \int_0^{V_{DD}} C(V_{DD} - v) dv \\ &= \frac{1}{2} C V_{DD}^2. \end{aligned}$$

What is interesting about these results is that E_S and E_W are equal, and their values have nothing to do with the transistor parameters. In fact, even if we replace the transistor with a simple resistor (linear or nonlinear, small or large), we would get the same results for the energy stored and for the energy wasted. Why is this the case? And why do we have to waste an amount of energy equal to the amount stored? Is this a fundamental law of nature? Can we avoid this waste of energy? If so, then how? I encourage you to pause and think about these questions before continuing.

To answer these questions, let us resort to our analogy and see if we can draw any insight from it. Figure 2(b) shows a glass of water being filled with a water hose from the top (at the height of V_{DD}). Initially, the glass is empty, and eventually it will be full. Once full, the potential energy stored in the glass can be calculated as water weight (CV_{DD}) times its average height (or equivalently the height of its center of mass, which is $V_{DD}/2$), resulting in the same equation as E_S shown above. Alternatively, as shown in Figure 3, we could have arrived at the same solution by simply integrating the incremental potential energy of many slabs of water, each of weight $C\Delta V$ at a height V (from zero to V_{DD}). This approach would be very similar to the one we used for the capacitor. How about the energy lost during this process? Note that as the water is dropped from the water hose (sitting at height V_{DD}) to find its new height (say V), it loses a potential energy equal to $C\Delta V(V_{DD}\text{-}V)$. Again, if we integrate this loss of energy with V changing from zero to V_{DD}, we will get $(1/2)CV_{DD}^2$.

It is interesting to explore this analogy further to see if what we said earlier about charging the capacitor is also true for the glass of water. For one, the final potential energy stored in the glass of water is independent of how we fill the glass. In other words, we could have used a hose with larger diameter or a smaller diameter; we could have squeezed the hose and then release it; we could have had a constant current of water or a current that varies over time or with the height of the water in the glass! In all these cases, the stored energy will be the same simply because we have a glass full of water at the end, and it will have the same stored potential energy.

How about the energy wasted? Where does the lost energy go to? You can imagine as we drop a slab of water ($C\Delta V$)from a height of V_{DD}, its potential energy will turn into kinetic energy and cause a splash in the glass. The kinetic energy that manifests itself as splashes will eventually turn to heat, warming the water and the glass. However, we note that there will be a larger splash when the glass is completely empty and less splash when the glass is almost full. In other words, the water poured initially contributes more to the splash (loss) and less to the stored energy (as it is placed at the bottom of the glass), whereas the water poured toward the end contributes more to the energy stored (as it is placed close to the top of the glass) and less to the energy wasted.

Can we use this analogy to come up with a method of charging the capacitor that results in less waste? We said the waste in potential energy manifests itself in kinetic energy or water splash. How could we fill a glass with a water hose and not cause any splash? A bartender would tell you that you need to initially bring the hose close to the bottom of the glass and then gently pull it upward as you fill the glass! You can imagine that this will result in less splash in the water and, hence, in less waste of energy!

To charge the capacitor through a series resistor that is connected to the power supply yet without wasting energy, we should gradually increase our power supply voltage from zero to V_{DD}, instead of raising it from zero to V_{DD} in one step. As a simple exercise, imagine we increase our power supply voltage from zero to V_{DD} in two steps: In the first step, we raise it from

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She has been an SSCS member since it became a Society in the early 1990s. She is a member of the Women in Engineering Committee as the Society liaison for SSCS. This year, Wanda and her team are working on ways to get more female members involved in volunteer opportunities with the Society and planning more activities for women at upcoming conferences. "We want to get a baseline of female participation in Chapter activities, conferences, and publications for our Society members," she says.

Wanda's current role is president of Design Connect Create, a nonprofit that offers physics camps to young women prior to their first physics class in high school. Design Connect Create strives to give women the confidence and preparation to be successful in physics, which is a gateway course to engineering. The program started in Dallas's inner-city high schools, and this year it expanded to Fort Worth, Austin, and Houston.

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Wanda says, "The one thing I love about the program is when the students struggle to learn a new concept. I like seeing the excitement in their faces when they reach that 'ah ha' moment, when the light bulb comes on."

She says one of the most satisfying parts of the program is seeing the girls excel. "The most rewarding part of my job is seeing the camp participants do better in pre-AP physics and AP physics after they have participated in the camp or when the girls that go on to major in engineering come back to the camp during college to be a teacher's assistant."

Prior to her role at Design Connect Create, she was the executive director at High-Tech High Heels, a nonprofit with the mission of closing the gender gap in STEM through programs that inspire young women in high school to pursue STEM degrees in college.

Outside of work, Wanda likes to run, do yoga, and spend time at the beach. Every summer, her family goes to the beach in Texas with five other families of close college friends. They spend a week building sand sculptures each day, to the amazement of many on the beach.

"It's surprising what happens when you get lots of engineers working on sand castles," she says.

—Abira Sengupta

If you have suggestions regarding SSCS's Women in Engineering initiative, please e-mail Wanda at wandakgass@gmail.com.

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zero to $V_{DD}/2$, and we keep it at this level until the capacitor is charged to $V_{DD}/2$. Then we raise it further to V_{DD} in the second step. In this way, the total energy stored at the end of the second step would be the same as before, but the total energy wasted becomes $(1/4)CV_{DD}^2$, which is half of the total energy stored. If we ramp the power supply in N steps instead, the total energy stored will be the same as before but the total energy wasted becomes $1/N$ of $(1/2)CV_{DD}^2$. If we assume it takes $5RC$ to charge the capacitor when the power supply is set to V_{DD} in one step, then it will take $5NRC$ when we ramp the power supply in N steps. As we increase N, the wasted energy will approach zero but the time it takes to charge the capacitor will approach infinity.

In closing, it should be mentioned that, like any analogy, this one is not without limitations. For example, this analogy is only valid if we assume one side of the capacitor is grounded or tied to a fixed voltage. This is because we assume the bottom of a glass sits on the ground or at a fixed height. Also, unlike an ideal capacitor, which is a loss-less lumped element, the glass is a distributed component with both lossless and lossy components. The glass walls, for example, absorb some of the water's kinetic energy, acting similar to a series resistance in the RC circuit. In other words, the glass would be a better analogy for a series RC circuit than for an ideal capacitor.

To summarize, analogies are powerful tools in the early stages of understanding of less tangible concepts. We described one analogy for capacitors in this article in an attempt to make them as tangible, and enjoyable, as a glass of water. We will further explore this analogy in the next article in this series.