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Miller's Approximation

Welcome to the seventh article in this column series. As the title suggests, each article provides insights and intuitions into circuit design and analysis. These articles are aimed at undergraduate students but may serve the interests of other readers as well. If you read this article, I would appreciate your comments and feedback, as well as your requests and suggestions for future articles in this series. Please e-mail your comments to me at: ali@ece.utoronto.ca.

In the previous article, we presented an intuitive view of Miller's theorem, especially as it applies to resistors. In this article, we use Miller's theorem to estimate the bandwidth of an amplifier with a capacitor between its input and output nodes.

Figure 1 shows an ideal voltage amplifier (i.e., one with infinite input impedance and zero output impedance) with a constant voltage gain of "$-A_0$" and a capacitor C_{12} between its input and output nodes. Miller's theorem says that we can replace C_{12} with two capacitors, C_{1M} and C_{2M}, connected from the input and output node, respectively, to ground, where,

$$C_{1M} = C_{12}(1 + A_0) \tag{1}$$
$$C_{2M} = C_{12}(1 + 1/A_0). \tag{2}$$

If we assume $A_0 \gg 1$, then Miller's theorem tells us that the capacitor between the two nodes appears as much larger at the input node (by a factor of $\sim A_0$) but as the same capacitor (by a factor of ~ 1) at the output node. This makes intuitive sense because a small voltage increment at the input results in a much larger decrement at the output, which in turn attracts a large amount of charge on the capacitor plates, as if the capacitor were much larger! From the perspective of the output node, however, a change in the output voltage corresponds to a much smaller change at the input. We can then simply assume the input node is grounded. This is equivalent to saying the capacitor seen from the output is the same as C_{12}.

When the amplifier is ideal but its gain (A) is frequency dependent (i.e., not constant) or when the amplifier is nonideal (e.g., has a finite output impedance), there may be confusion as how to apply Miller's theorem or how useful it may be. We focus on this in the remainder of this article.

First, let us examine a slightly generalized case where the amplifier is ideal but it has a frequency-dependent gain $A_v(j\omega)$, instead of constant $-A_0$, with a single pole frequency, f_p. In other words, assume

$$A_v(j\omega) = \frac{-A_0}{1 + \frac{jf}{f_p}}.$$

Note that since this is an ideal amplifier, adding C_{12} across it will not change the voltage transfer function. We can therefore apply Miller's theorem (1), to arrive at the following equation:

$$C_{1M} = C_{12}(1 + A_0)\frac{1 + \frac{jf}{f_p(1 + A_0)}}{1 + \frac{jf}{f_p}}.$$

This equation simply states that C_{1M} is now frequency dependent. At low frequencies (below f_p), C_{12} is simply multiplied by the dc gain of the amplifier. At midfrequencies (between f_p and $\sim A_0 f_p$), when the voltage gain drops, C_{1M} drops also. Beyond the amplifier's unity-gain frequency (i.e., $A_0 f_p$), C_{1M} approaches C_{12}. This makes intuitive sense because, at very high frequencies, the gain of the amplifier will approach zero and the output node becomes grounded, producing C_{12} at the input node.

We do not need to derive an equation for the output node capacitance (C_{2M}) in this case because C_{2M} will have no impact on V_2, given that the amplifier is assumed to have zero output impedance.

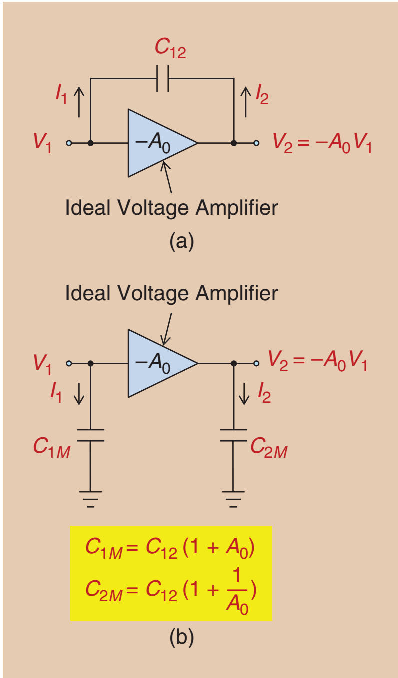

FIGURE 1: Miller's theorem: (a) capacitance C_{12} is connected between the input and output nodes of an ideal voltage amplifier with a constant gain of $-A_0$, and (b) Miller's equivalent circuit.

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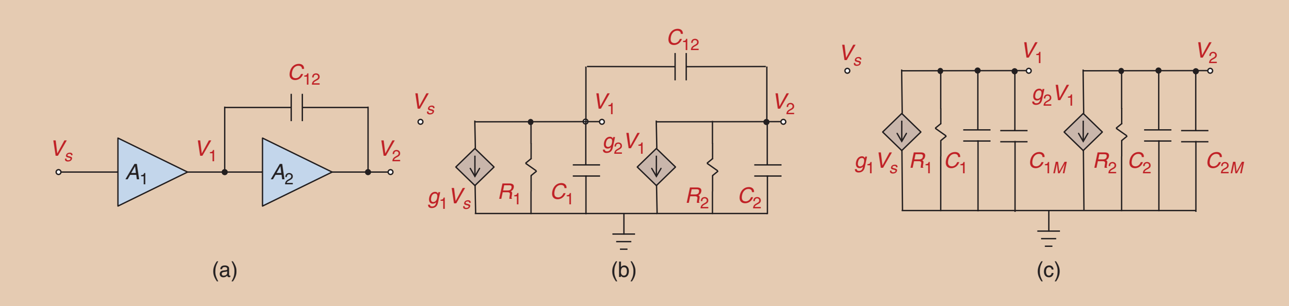

FIGURE 2: Miller's approximation: (a) a two-stage amplifier with a Miller capacitance across the second stage, (b) a small-signal equivalent circuit for (a), and (c) a small-signal equivalent circuit where C_{12} is replaced by C_{1M} and C_{2M} according to Miller's theorem.

To summarize, a C_{12} across a frequency-dependent ideal amplifier may only impact the capacitive load seen at the input node of the amplifier but will have no impact on the output node of the amplifier.

Now, let us consider a more general case where the amplifier is nonideal (i.e., it has a finite input impedance and a nonzero output impedance). We further assume this amplifier is driven by another nonideal amplifier as shown in Figure 2(a).

Let us denote by $A_{v1}(j\omega)$ and $A_{v2}(j\omega)$ the voltage transfer functions of the two stages *without* the presence of C_{12}. A linear model for this two-stage amplifier along with the added C_{12} is shown in Figure 2(b). In this figure, R_1 and C_1 represent the total resistance and capacitance of node 1, respectively, and g_1 represents the transconductance of the first stage. R_2, C_2, and g_2 are the corresponding parameters of the second stage.

An important observation in this case is that the addition of C_{12} will change $A_{v2}(j\omega)$ to $A'_{v2}(j\omega)$, which is unknown and needs to be determined. However, to replace C_{12} with its Miller's equivalents, as shown in Figure 2(c), we need $A'_{v2}(j\omega)$ in the first place. What can we do?

In Miller's approximation, we simply use the dc gain of the second stage (A_{v20}) in (1) and (2) to find an estimate of C_{1M} and C_{2M}. We then proceed to write an expression for $A'_{v1}(j\omega)$ and $A'_{v2}(j\omega)$.

How valid is this approximation? $A'_{v}(j\omega)\,(= A'_{v1}(j\omega)A'_{v2}(j\omega))$ has two poles corresponding to the two capacitive nodes in the circuit and a zero due to C_{12}. We do not know the exact locations of the poles, but we do know that in the vicinity of the first pole (i.e., the dominant pole) $A'_{v2}(j\omega)$ can be approximated by the dc gain of $A_{v2}(j\omega)$, A_{v20}. The actual gain at the dominant pole frequency is 3 dB lower, but we accept this approximation. Therefore, in calculating the location of the first pole, Miller's approximation does provide a reasonable estimate, as follows:

$$f_{p1} = \frac{1}{2\pi R_1(C_1 + C_{12}(1 + A_{v20}))}.$$

At the frequency of the second pole, however, the gain has already dropped substantially from the dc gain, and hence Miller's approximation does not yield a good estimate for C_{2M}, and consequently for f_{p2}. In fact, if $f_{p2} \gg f_{p1}$, our estimate of f_{p2} may be off by an order of magnitude.

Miller's approximation also misses the zero that exists in the circuit of Figure 2(b). A quick inspection of this circuit reveals that a zero lies at a frequency where the current through C_{12} becomes equal to g_2V_1. When this occurs, the current through the parallel combination of C_2 and R_2 becomes zero, creating a zero in the transfer function. In other words, we can write

$$f_z = \frac{g_2}{2\pi C_{12}}.$$

The existence of the zero makes intuitive sense because, as we increase the frequency, there will be a point where the impedances of C_{12} and C_2 will be much smaller than R_2. At these frequencies, the transfer function of the second stage becomes that of a capacitive divider, which has a constant gain and does not drop with increasing frequency, a clear sign of the existence of a zero.

But none of these points should imply that Miller's *theorem* is inaccurate; they only tell us the limitation of Miller's *approximation*. In fact, one can show that if we use the proper frequency-dependent gain in calculating C_{1M} and C_{2M} (as we did in the case of an ideal amplifier), we can accurately find the original poles. But this will defeat the purpose of using Miller's theorem for a simple, quick estimation of the circuit bandwidth.

If Miller's approximation cannot be used to derive an expression for the second pole of the circuit in Figure 2(b), what can we do instead? The answer will actually depend on the relative ratios of the capacitors. Here, we provide a quick and intuitive method for the case when $C_{12} \gg C_1$ and C_2. In this case, C_{12}, which is referred to as the *Miller compensation capacitor*, improves the circuit stability when placed in a feedback loop [1], [2].

Given $C_{12} \gg C_1$ and C_2, as we increase frequency, C_{12} will short the two nodes together, creating a total capacitance of $C_1 + C_2$ in parallel with a total resistance $1/g_2$, where we have assumed $1/g_2$ is much smaller than R_1 and R_2. Therefore, we can write an expression for the second pole as follows:

$$f_{p2} = \frac{g_2}{2\pi(C_1 + C_2)}.$$

(Continued on p. 13)

small-signal bandwidth must be far greater than the input frequency.

2) To which node(s) should the n-wells of M_3 and M_8 in Figure 10 be connected?

   They should be connected to node P to ensure the source and drain junctions of these transistors are not forward biased.

3) How high can V_X in Figure 10 go to avoid stressing M_{14}?

   When M_{14} is off, its source voltage reaches approximately $V_{DD} - V_{TH}$. For the source-drain potential difference to remain less than V_{DD}, V_X must not exceed $2V_{DD} - V_{TH}$.

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EDITOR'S NOTE

(Continued from p. 4)

- Willy Sansen, in "Minimum Power in Analog Amplifying Blocks: Presenting a Design Procedure," answers questions he received from his 2015 ISSCC plenary talk.
- Behzad Razavi continues his column series "A Circuit for All Seasons" by providing an article that discusses the bridged T-coil. This article fits well into this issue's feature of wireline communications due to the use of the T-coil for extending the bandwidth of a circuit.
- Ali Sheikholeslami provides another piece in his well-received series, "Circuit Intuitions." In this issue, he continues discussing Miller's theorem, its uses and shortcomings when analyzing circuits. As usual (and the e-mail we receive would support this), the article provides useful insight into circuit analysis and design.
- Finally, Marcel Pelgrom discusses "The Next Hype" in his column, which is always an entertaining article that provokes thought. It's one of my favorite reads in each magazine issue. I hope you agree!

We hope you enjoy reading *IEEE Solid-State Circuits Magazine.* Please send comments to me at rjacobbaker@gmail.com.

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CIRCUIT INTUITIONS

(Continued from p. 8)

This equation, along with equations for f_{p1} and f_z, can now be used to form the equation for the overall voltage transfer function of the two-stage amplifier.

It is worth noting that as we increase C_{12}, f_{p1} and f_{p2} (as found by their respective equations) will move farther apart, a phenomenon referred to as pole splitting [1], [2].

In summary, Miller's approximation uses the dc gain of the amplifier to provide a relatively accurate estimation of its dominant pole (i.e., the circuit bandwidth). This approximation, however, becomes inaccurate when determining the second pole of the amplifier; other intuitive methods exist for this purpose.

For further discussions and intuition into Miller's theorem, we refer the readers to [3].

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