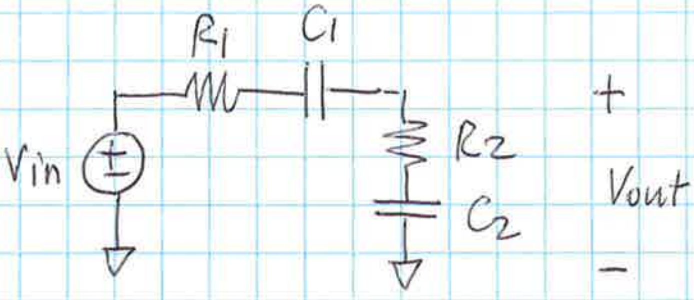
Example: **Solving circuits by physical Intuition**



$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2} = \frac{R_2 + 1/sC_2}{R_1 + 1/sC_1 + R_2 + 1/sC_2} =$$

$$= \frac{(sR_2C_2 + 1)/sC_2}{\dfrac{sR_1C_1C_2 + C_2 + sR_2C_1C_2 + C_1}{sC_1C_2}} =$$

$$= \frac{sR_2C_2 + 1}{\dfrac{sR_1C_1C_2 + C_2 + sR_2C_1C_2 + C_1}{C_1}} =$$

$$= \frac{sR_2C_2 + 1}{sR_1C_2 + \frac{C_2}{C_1} + sR_2C_2 + 1} =$$

$$= \frac{sR_2C_2 + 1}{sC_2(R_1 + R_2) + 1 + \frac{C_2}{C_1}} \cdot \frac{(1 + C_2/C_1)}{(1 + C_2/C_1)} =$$

$$= \frac{(sR_2C_2 + 1)/(1 + C_2/C_1)}{\dfrac{sC_2(R_1 + R_2)}{1 + C_2/C_1} + 1} =$$

$$= \frac{(sR_2C_2+1) \Big/ \frac{C_1+C_2}{C_1}}{\frac{sC_2(R_1+R_2)}{\frac{C_2+C_1}{C_1}} + 1} =$$

$$= \frac{(sR_2C_2+1) \Big/ \frac{C_1+C_2}{C_1}}{s \underbrace{\frac{C_1C_2}{C_1+C_2}}_{\substack{C_1 \text{ series } C_2 \\ = C_{12}}} (R_1+R_2)+1} =$$

$$= \frac{\frac{sR_2C_2+1}{C_1+C_2} \cdot C_1}{sC_{12}(R_1+R_2)+1} =$$

$$= \frac{sR_2C_{12} + \frac{C_1}{C_1+C_2}}{sC_{12}(R_1+R_2)+1} = \frac{sR_2C_{12} + \frac{C_{12}}{C_2}}{sC_{12}(R_1+R_2)+1}$$

$$\text{DC gain} = \frac{C_{12}}{C_2} = \frac{C_1}{C_1+C_2} \qquad (\text{let } s \to 0)$$

$$\text{pole} = -\frac{1}{(R_1+R_2)\cdot C_{12}} \qquad (\text{root denominator})$$

$$\text{zero} = -\frac{1}{R_2C_2} \qquad (\text{root numerator})$$

THIS IS TOO MUCH WORK THERE'S TO BE A BETTER WAY !!

let's look at the circuit:

at $s \to 0$, the impedance of C_1 and C_2 are $\gg$ than R_1 and R_2

the circuit is a capacitive divider

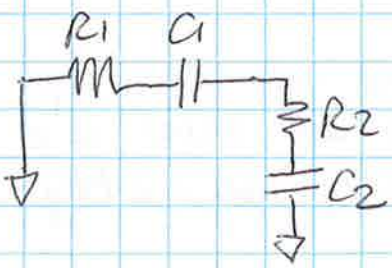
$$\frac{V_{out}}{V_{in}}(s=0) = \frac{C_1}{C_1+C_2}$$

at $s \to \infty$, C_1 and C_2 are 2 shorts the circuit is a resistive divider

$$\frac{V_{out}}{V_{in}}(s \to \infty) = \frac{R_2}{R_1+R_2}$$

↓ match the algebra!

The circuit has one pole (null indepentent sources)



↓ poles do not depends on the sources, are a "natural" characteristic of the circuit

C_1 and C_2 appears in series (they are not indep. of each other

→ their state (i.e. voltage across them) is not indep.

How much is the resistance seen by C_{12}?

→ (R_1+R_2)

$$\omega_{pole} = \frac{1}{\tau} = \frac{1}{(R_1+R_2)C_{12}}$$

The circuit has one zero:

for $s \to \infty$ Vout is not zero (the order of num. and denom must be the same $\rightarrow$ 1 pole implies one zero)

$$V_{out} = 0 = R_2 \cdot I_2 + V_{C2}$$

$\omega_z \equiv$ frequency for which $V_{out} = 0$

$$C_2 R_2 s V_{C2} + V_{C2} = 0 \rightarrow V_{C2}(1 + s R_2 C_2) = 0$$

$$s_z = -\frac{1}{R_2 C_2} \qquad \left(\omega_z = \frac{1}{R_2 C_2} \right)$$

We could have avoided even the little algebra we just did:
once we know ω_p we can infer ω_z using geometry!

$$\frac{C1/(C1+C2)}{R2/(R1+R2)} = wz/wp$$